

Inventory Model of Deteriorating Items for Linear Holding Cost with Time Dependent Demand

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Abstract: The objective of this model is to investigate the inventory system for perishable items with time proportional deterioration rate. The Economic order quantity is determined for minimizing the average total cost per unit time. The model is developed for time dependent demand rate with finite time horizon and linear holding cost with shortage. The result is illustrated with numerical example.

Keyword: Time dependent Demand, Optimal control, Inventory system, Shortage.

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1. INTRODUCTION

Deterioration is defined as decay or damage so that the item cannot be used for its original purpose. Such items, widely exist in our daily life, are fresh products, fruits, vegetables, seafood, etc. These items decrease in quantity or utility during their delivery and storage stage periods. To improve the quality of the product and to reduce deteriorating loss in the supply chain, the emphasis is on the whole process life cycle management which includes production, storage, transportation, retailing etc. Accompany by the increasing of the variety and quantity of the deteriorating fashion goods, consumer's appetite for high quality perishable items are continually upgrading rapidly, so that the topic of deteriorating inventory system management became popular in the field of research and business.

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Several researchers had studied stock deterioration over the years. Ghare and Schrader [3] were among the first authors to consider the role of deterioration in inventory modeling. Other authors, such as Covert and Philip [1], Kang and Kim [4] and Raafat et.al. [5], assumed either instantaneous re-supply or a finite production rate with different assumptions on the occurrence of deterioration. Wee [7] studied a replenishment policy with price-dependent demand and a varying rate of deterioration.

In the early 1970s, Silver and Meal [6] proposed an approximate solution procedure for the general case of a deterministic, time-varying demand pattern. Donaldson [2] then considered an inventory model with a linear trend in demand. After Donaldson, numerous research works had been carried out incorporating time varying demand into inventory models under a variety of circumstances.

There are certain inventories, like fertilizer, which deteriorate and whose demand is time dependent, come to zero inventory at some time before the cycle is completed. In this paper, an attempt has been made to develop an inventory model for perishable items with time proportional deterioration rate and the time dependent demand pattern over a finite planning horizon. Nature of the model is discussed for shortage state. Optimal solution for the proposed model is derived and the applications are investigated with the help of numerical example.

2. ASSUMPTIONS AND NOTATIONS

Following assumptions are made for the proposed model:

- i. Single inventory is used.
- ii. Lead time is zero.
- iii. Shortages are allowed and are completely backlogged.
- iv. Time-dependent Demand rate is considered.
- v. Deterioration is considered to be Time-proportional.
- vi. Replenishment rate is infinite but size is finite.
- vii. Time-horizon is finite.
- viii. There is no repair of deteriorated items occurring during the cycle.

Following notations have been used for the model:

$I(t)$ = On hand inventory level at any time t ,

$R(t) = a e^{bt}$ is the time-dependent demand rate at any time t , $a \geq 0, 0 < b < 1$.

$\theta(t)$ = Instantaneous rate of deterioration of the on-hand inventory given by $\alpha \cdot t$ where $0 < \alpha < 1$.

$I(0) = Q$ = initial inventory = Inventory at time $t = 0$.

S = Inventory at time $t = T$.

T = Duration of a cycle.

A = The Ordering cost per order.

c_p = The purchase cost per unit item.

c_d = The deterioration cost per unit item.

$H(t)$ = The holding cost per unit time, $H(t) = l + m t$.

c_b = The shortage cost per unit item.

γc_p = The salvage value associated to the deteriorated units during the cycle
time

where $0 \leq \gamma < 1$.

U = The total average cost of the system.

3. FORMULATION

In this model, it is assumed that the amount of stock starts to deplete in the period $[0, t_1]$ due to the combined effect of demand and deterioration. By this process, the stock reaches zero at time t_1 . In the interval $[t_1, T]$ there is no deterioration. Since in this interval demand is considered, this leads to predict to order the inventory in the next cycle. Thus, the inventory level at any instant of time during $[0, t_1]$ is described as follows.

Let $I(t)$ be the on-hand inventory at time t . Then at time $t + \Delta t$, the on-hand inventory in the interval $[0, t_1]$ will be

$$I(t + \Delta t) = I(t) - \theta I(t) \Delta t - R(t) \Delta t,$$

where θ and $R(t)$ are as defined in the notation. Dividing by Δt and then taking the limit as $\Delta t \rightarrow 0$, we get

$$\frac{dI(t)}{dt} + \alpha t I(t) = -a e^{bt}; \quad 0 \leq t \leq t_1. \quad (3.1)$$

In the present case, the boundary conditions are $I(0) = Q, I(t_1) = 0$. For the next interval $[t_1, T]$, we have

$$I(t + \Delta t) = I(t) - R(t) \Delta t.$$

Dividing by Δt and then taking as $\Delta t \rightarrow 0$ we get

$$\frac{dI(t)}{dt} = -a e^{bt}; \quad t_1 \leq t \leq T. \quad (3.2)$$

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The boundary conditions are $I(t_1) = 0, I(T) = S$.

On solving equation (3.1) with boundary condition $I(t_1) = 0$ we have

$$I(t) = \left(1 - \frac{\alpha t^2}{2}\right) \left[t_1 + \frac{\alpha}{6} t_1^3 + \frac{b}{2} t_1^2 - \left(t + \frac{\alpha}{6} t^3 + \frac{b}{2} t^2 \right) \right]; 0 \leq t \leq t_1 . \quad (3.3)$$

On solving equation (3.2) with boundary condition $I(t_1) = 0$. we have

$$I(t) = \frac{a}{b} \{ e^{bt_1} - e^{bt} \} ; t_1 \leq t \leq T . \quad (3.4)$$

Using $I(T) = S$ in equation (3.4), we have

$$S = \frac{a}{b} \{ e^{bt_1} - e^{bT} \} .$$

Using $I(0) = Q$ in equation (3.3) , we have

$$Q = \left[t_1 + \frac{\alpha}{6} t_1^3 + \frac{b}{2} t_1^2 \right] . \quad (3.6)$$

The number of deteriorated units during one cycle time is given by.

$$D(T) = Q - \int_0^T R(t) dt = t_1 + \frac{\alpha}{6} t_1^3 + \frac{b}{2} t_1^2 - \frac{a}{b} (e^{bT} - 1) . \quad (3.7)$$

From of equation (3.4) the amount of shortage during the time interval $[t_1, T]$ is

$$\int_{t_1}^T I dt = \frac{a}{b} \left\{ e^{bt_1} (T - t_1) - \frac{1}{b} (e^{bT} - e^{bt_1}) \right\} . \quad (3.8)$$

Using the above equations into consideration the different costs will be as follows:

1. Purchasing cost per cycle

$$C_p I(0) = C_p \left[t_1 + \frac{\alpha}{6} t_1^3 + \frac{b}{2} t_1^2 \right]. \quad (3.9)$$

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2. Holding cost per cycle

$$\int_0^{t_1} H(t) I(t) dt = \frac{l}{2} t_1^2 + \left(\frac{lm}{3} - \frac{m}{6} \right) t_1^3 + \left(\frac{l\alpha}{12} + \frac{m^2}{8} \right) t_1^4 + \left(\frac{m\alpha}{24} - \frac{lm\alpha}{30} \right) t_1^5 - \frac{m^2\alpha}{48} t_1^6. \quad (3.10)$$

3. Deterioration cost per cycle

$$C_d D(T) = C_d \left\{ t_1 + \frac{\alpha}{6} t_1^3 + \frac{b}{2} t_1^2 - \frac{a}{b} (e^{bT} - 1) \right\}. \quad (3.11)$$

4. Shortage cost per cycle

$$-C_b \int_{t_1}^T I dt = -\frac{a C_b}{b} \left\{ e^{b t_1} (T - t_1) - \frac{1}{b} (e^{bT} - e^{b t_1}) \right\}. \quad (3.12)$$

5. Ordering cost

$$OC = A. \quad (3.13)$$

6. Salvage value of deteriorated items

$$SV = \gamma C_p \left\{ t_1 + \frac{\alpha}{6} t_1^3 + \frac{b}{2} t_1^2 - \frac{a}{b} (e^{bT} - 1) \right\}. \quad (3.14)$$

The average total cost per unit time of the model will be

$$U(t_1) = \frac{1}{T} \left[C_p \left\{ t_1 + \frac{\alpha}{6} t_1^3 + \frac{b}{2} t_1^2 \right\} + (C_d - \gamma C_p) \left\{ t_1 + \frac{\alpha}{6} t_1^3 + \frac{b}{2} t_1^2 - \frac{a}{b} (e^{bT} - 1) \right\} + \frac{l}{2} t_1^2 + \left(\frac{lm}{3} - \frac{m}{6} \right) t_1^3 + \left(\frac{l\alpha}{12} + \frac{m^2}{8} \right) t_1^4 + \left(\frac{m\alpha}{24} - \frac{lm\alpha}{30} \right) t_1^5 - \frac{m^2\alpha}{48} t_1^6 + A - \frac{a C_b}{b} \left\{ e^{b t_1} (T - t_1) - \frac{1}{b} (e^{bT} - e^{b t_1}) \right\} \right]. \quad (3.15)$$

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As the necessary conditions for minimization of $U(t_1)$ is

$$\frac{\partial U(t_1)}{\partial t_1} = 0. \quad (3.16)$$

and the sufficient condition for minimization of $U(t_1)$ is

$$\frac{\partial^2 U(t_1)}{\partial t_1^2} > 0. \quad (3.17)$$

from (3.16), we have

$$\begin{aligned} & (C_p + C_d - \gamma C_p) \left\{ 1 + \frac{\alpha}{2} t_1^2 + b t_1 \right\} + l t_1 + \left(l m - \frac{m}{2} \right) t_1^2 + \\ & \left(\frac{l \alpha}{3} + \frac{m^2}{2} \right) t_1^3 + \left(\frac{5 m \alpha}{24} - \frac{l m \alpha}{6} \right) t_1^4 - \frac{m^2 \alpha}{8} t_1^5 - a C_b e^{b t_1} (T - t_1) = 0. \end{aligned} \quad (3.18)$$

But as it is difficult to solve the problem by deriving a closed equation of the solution of equation (3.15), Mat lab Software has been used to determine optimal t_1^* and it is verified for the same value of t_1^* , the cost $U(t_1^*)$ is minimized.

4. COMPUTATIONAL ALGORITHM

Step-1: Start.

Step-2: Initialize the value of the variables $A, l, m, T, c_p, c_b, c_d, \alpha, a, b, \gamma$.

Step-3: Evaluate $U(t_1)$.

Step-4: Evaluate $\frac{\partial U(t_1)}{\partial t_1}$.

Step-5: Solve the equation $\frac{\partial U(t_1)}{\partial t_1} = 0$.

Step-6: Choose the solution from Step-5.

Step-7: Evaluate $\frac{\partial^2 U(t_1)}{\partial t_1^2}$.

Step-8: If the value of Step-7 is greater than zero then this solution is optimal(minimum) and go to Step-10.

Step-9: Otherwise go to Step-6.

Step-10: End.

5. EXAMPLE

The values of the parameters are considered as follows:

$$c_p = \$20 / \text{unit}, c_b = \$12 / \text{unit} / \text{year}, c_d = \$8 / \text{unit}, \\ \alpha = 0.1, a = 100, b = 0.2, \gamma = 0.1,$$

$$A = 50, l = 2, m = 1.5, T = 1 \text{ Year}.$$

According to equation (3.18), we obtain the optimal $t_1^* = 0.4873$. In addition, the optimal $I^*(0) = 0.513$ units. Moreover, from equation (3.15), we have the minimum average total cost per unit time as $U^* = 291.663$

6. CONCLUSION

Here an EOQ model is derived for perishable items with time dependent demand pattern. Time proportional deterioration rate is used. In this model, shortages are allowed. The model is studied for minimization of total average cost. The importance of the model is that it predicts the amounts of the inventory required, ordering for the next cycle. This model can be used for procuring fertilizers. Sometimes spare parts of the automobiles also come under such model where the deterioration is very low. The model has been verified by numerical example.

The model can further be studied for different demand situations and for multiple items under identical conditions. This can also be extended for different deterioration conditions and also for discounted cash flow approach.

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