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On Weighted Generalized Residual Information Measure

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Abstract: In this paper, we have proposed the concept of weighted generalized residual entropy of order α and type β, and have shown that the proposed measure characterizes the distribution function uniquely.

Keywords: Shannon Entropy; Residual Entropy; Lifetime Distribution; Weighted Distribution.

1. INTRODUCTION

Shannon entropy and its applications is an essential and well known concept in the area of information theory. For an absolutely continuous random variable X with probability density function $f(x)$ it is defined as,

$$H(X) = -\int_0^\infty f(x)\log f(x)dx, \tag{1}$$

where '*log*' denotes the natural logarithm. The measure (1) is called differential entropy. One of the main drawbacks of this measure is that it may not always be non negative and if it is negative then $H(X)$ is no longer an uncertainty measure. Kinchin [11] removed this limitation by introducing a convex function ϕ, and defined the generalized entropy measure as

$$H^{\phi}(X) = -\int_0^\infty f(x)\phi(f(x))dx, \tag{2}$$

where $\phi(1) = 0$. For two particular choices of ϕ,(2) can be expressed respectively as the Harvard and Charvat [9] and Renyi [17] entropy measures, given respectively by

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$$H_1^{\alpha}(X)=\frac{1}{\alpha-1}(1-\int_0^{\infty} f^{\alpha}(x)dx),\alpha>0\, and\, \alpha\neq 1 \quad (3)$$

and

$$H_2^{\alpha}(X)=\frac{1}{1-\alpha}\log(\int_0^{\infty} f^{\alpha}(x)dx),\alpha>0\, and\, \alpha\neq 1. \quad (4)$$

If $\alpha\to 1,$ then equations (3) and (4) reduce to (1). Another generalized the entropy of order α and type β, refer to Verma [21], is defined by

$$H^{(\alpha,\beta)}(X)=\frac{1}{\beta-\alpha}\log(\int_0^{\infty} f^{\alpha+\beta-1}(x)dx),(\beta-1)<\alpha<\beta\, and\, \beta\geq 1. \quad (5)$$

When $\beta=1$ and $\alpha\to 1,$ then (5) reduces to (1). It may be noted that although (1) is negative for some distributions, but by choosing appropriate value of α for (3) and (4), and of α and β for (5), these generalized entropies can be made non negative.

In survival analysis and life testing, one has information about the current age of the component under consideration. In such cases, the age must be taken into account while measuring uncertainty. Shannon entropy is unsuitable in such situations and must be modified to take the age into account. As a solution, Ebrahimi [5] introduced the concept of residual entropy to measure the uncertainty of such systems. For a random lifetime *X*, at time *t* the residual entropy is defined as the differential entropy of $[X-t\,|\,X>t]$, and is given by

$$H(X;t)=-\int_t^{\infty}\frac{f(x)}{\bar{F}(t)}\log\left(\frac{f(x)}{\bar{F}(t)}\right)dx, \quad (6)$$

where $\bar{F}(t)=1-F(t)$ is the survival function of *X*. In the same spirit, for a system surviving up to age *t*, Nanda and Paul [13] introduced generalized form of *H*(*X;t*) and redefined (3) and (4), respectively as

$$H_1^{\alpha}(X;t)=\frac{1}{\alpha-1}\left(1-\int_t^{\infty}\left(\frac{f(x)}{\bar{F}(t)}\right)^{\alpha}\right)dx \quad (7)$$

and

$$H_2^{\alpha}(X;t)=\frac{1}{1-\alpha}\int_t^{\infty}\log\left(\frac{f(x)}{\bar{F}(t)}\right)^{\alpha}dx. \quad (8)$$

When $\alpha\to 1$, (7) and (8) reduce to (6). Similarly, Baig and Dar [1] redefined (5) and introduced the concept of generalized residual entropy of order α and type β given by

$$H^{(\alpha,\beta)}(X;t)=\frac{1}{\beta-\alpha}\log\left(\int_t^{\infty}\frac{f^{\alpha+\beta-1}(x)}{\bar{F}^{\alpha+\beta-1}(t)}dx\right),(\beta-1)<\alpha<\beta \, and \, \beta\geq 1. \quad (9)$$

If $\beta=1$ and $\alpha\rightarrow 1$, (9) reduces to (6).

In recent years, a lot of work has been focused on $H(X;t)$. Di Crescenzo and Longobardi [3] introduced the notions of "weighted residual entropy", that are suitable to describe dynamic information of random lifetimes. Baig and Dar [1] characterized some life time models and defined new classes of life time distributions using generalized information measure for residual lifetime distributions. Taneja et. al. [20]considered a dynamic measure of inaccuracy between two residual lifetime distributions. Kumar et. al. [12] introduced a length biased weighted residual inaccuracy measure between two residual lifetime distributions over the interval (t, ∞) and derived a lower bound to the weighted residual inaccuracy measure. Sunoj and Linu [19] extended cumulative residual Renyi's entropy into the bivariate set-up and proved certain characterizing relationships to identify different bivariate lifetime models. Psarrakos and Navarro [15] considered dynamic generalized cumulative residual entropy using the residual lifetime. Das [4] extended the concept of weighted generalized entropy, based upon the concept of generalized entropies given by Havard and Charvat [9] and Renyi [17] and discussed the properties of weighted generalized residual entropy.

In this paper, we propose the concept of weighted generalized residual entropy of order α and type β and show that the proposed measure characterizes the distribution function uniquely. Section 2 is devoted to weighted generalized entropy along with an example. In Section 3 we introduce weighted generalized form of residual entropy and prove a characterization theorem for this. In Section 4, some concluding remarks with future aspects have been included.

In this paper, the terms "increasing" and "decreasing" is not used in strict sense and following notions are used:

X : An absolutely continuous non-negative random variable representing lifetime of the system;

$F(x)$: $P[X \leq x]$, probability distribution function of X;

$\bar{F}(t)$: $P[X > x] = 1 - F(t)$, survival function of X ;

$f(x)$: Derivative of F (x), probability density function of X;

$r_x(t)$: $\frac{f(t)}{\bar{F}(t)}$ the hazard function, or failure rate, of X;

$[A|B]$: Any random variable whose distribution is identical to the conditional distribution of A given B;

2. WEIGHTED GENERALIZED ENTROPY

Fisher [6] and Rao [16] introduced the concept of weighted distribution and provided an approach to deal with model specification and data interpretation problems. Fisher [6] studied the influence of methods of ascertainment on the distribution form of recorded observation, and Rao [16] modelled statistical data by weighted distributions where standard distributions were not appropriate due to various reasons like unobserved or damaged values etc. For more results on weighted distributions refer to Oluyede and Terbeche [14], Ghitany and Al-Mutairi [7], Kareema and Ahmad [10].

The probability function of weighted random variable X_w associated to the random variable X with weight function $w(x)$, probability density function $f(x)$ and survival function $\bar{F}(x)$, is defined by

$$f_w(x) = \frac{w(x)}{E(w(x))} f(x), 0 \leq x < \infty, \tag{10}$$

where $w(x)$ is positive for all value of $x \geq 0$ and $0 < E(w(X)) < \infty$. On particular choices of weight function $w(x)$ we have different weighted models. For example, when $w(x)=x$, resulting distribution is called length-biased distribution and the associated probability density function of length biased random variable X_w is defined as

$$f_w(x) = \frac{x}{E(X)} f(x), \tag{11}$$

and the corresponding length biased survival function is defined as

$$\bar{F}_w(x) = \frac{E(X \mid X > x)}{E(X)} \bar{F}(x). \tag{12}$$

In agreement with Belis and Guiasu [2] and Guiasu [8], weighted Shannon entropy can be obtained from (1), by applying weight to the probability density function and is defined by

$$H_w(x) = \int_0^\infty f_w(x) \log f_w(x) dx.$$

On substituting the values of weighted functions, we get

$$H_w(x) = -\frac{E(X \log X)}{E(X)} + \frac{E(X) \log E(X)}{E(X)} - \frac{1}{E(X)} \int_0^\infty x f(x) \log f(x) dx. \tag{13}$$

Similarly from (5), weighted generalized entropy of order α and type β is given by

$$H_w^{(\alpha,\beta)}(X) = \frac{1}{\beta-\alpha}\log\left(\int_0^\infty f_w^{\alpha+\beta-1}(x)dx\right). \tag{14}$$

Substituting for $f_w(x)$ from (11) in (14), we get

$$H_w^{(\alpha,\beta)}(X) = \frac{1}{\beta-\alpha}\log\left(\int_0^\infty \frac{x^{\alpha+\beta-1}f^{\alpha+\beta-1}(x)}{E(X)^{\alpha+\beta-1}}dx\right). \tag{15}$$

When $\beta = 1$ and $\alpha \to 1$, then (15) reduce weighted Shannon entropy (13).

Example 2.1 Let *X* and *Y* be the random variable with probability density functions,

$$f(x) = \begin{cases} 2x, & 0 \le x < 1 \\ 0, & otherwise \end{cases}$$

and

$$f(y) = \begin{cases} 2(1-y), & 0 \le y < 1 \\ 0, & otherwise. \end{cases}$$

For α= 3/2 and β=2, generalized entropy of order α and type β of random variables X and Y is, $H^{(\alpha,\beta)}(X) = H^{(\alpha,\beta)}(Y) = 0.96021$. Further, weighted generalized entropy of order α and type β of random variables *X* and *Y* is $H_w^{(\alpha,\beta)}(X) = 1.90954$ and $H_w^{(\alpha,\beta)}(Y) = 0.604189$. We observed that in this case generalized entropies of order α and type β of random variables *X* and *Y* are same but weighted generalized entropies of order α and type β about the predictability of *X* is more than that of *Y*.

3. WEIGHTED GENERALIZED RESIDUAL ENTROPY

Di Crescenzo and Longobardi [3] introduced the concept of weighted generalized residual entropy and is given by

$$H_w(X;t) = -\frac{1}{E(X\,|\,X>t)}\int_t^\infty x\frac{f(x)}{\bar{F}(t)}\log\left(x\frac{f(x)}{E(X\,|\,X>t)\bar{F}(t)}\right)dx. \tag{16}$$

The weighted residual entropy functions of first and second kind corresponding to (7) and (8), according to S. Das [4], are respectively given as

$$H_{w1}^{\alpha}(X;t) = \left(\frac{1}{\alpha-1}\right)\left(1 - \frac{1}{[E(X\,|\,X>t)]^\alpha}\int_t^\infty x^\alpha \frac{f^\alpha(x)}{\bar{F}^\alpha(t)}dx\right) \tag{17}$$

and

$$H_{w2}^{\alpha}(X;t)=\left(\frac{1}{1-\alpha}\right)\log\left(\int_t^{\infty}x^{\alpha}\frac{f^{\alpha}(x)}{\overline{F}^{\alpha}(t)}dx\right)-\frac{\alpha}{1-\alpha}\log(E(X\mid X>t)). \quad (18)$$

When $\alpha\to 1$, (17) and (18) reduce to (16).

Next, we introduce the concept of weighted generalized residual entropy of order α and type β, given by

$$H_w^{(\alpha,\beta)}(X;t)=\frac{1}{\beta-\alpha}\log\left(\frac{1}{[E(X\mid X>t)]^{\alpha+\beta-1}}\int_t^{\infty}x^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(x)}{\overline{F}^{\alpha+\beta-1}(t)}dx\right). \quad (19)$$

When $\beta=1$ and $\alpha\to 1$, then (19) reduces to (16). Further, we have

$$\begin{aligned}\int_t^{\infty}x^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(x)}{\overline{F}^{\alpha+\beta-1}(t)}dx &= (\alpha+\beta-1)\int_t^{\infty}\left(\int_0^{x}y^{\alpha+\beta-2}dy\right)\frac{f^{\alpha+\beta-1}(x)}{\overline{F}^{\alpha+\beta-1}(t)}dx.\\ &=(\alpha+\beta-1)\left[\int_t^{\infty}\left(\int_0^{t}y^{\alpha+\beta-2}dy-\int_t^{x}y^{\alpha+\beta-2}dy\right)\frac{f^{\alpha+\beta-1}(x)}{\overline{F}^{\alpha+\beta-1}(t)}dx\right].\\ &=(\alpha+\beta-1)\left[\int_t^{\infty}\left(\int_0^{t}y^{\alpha+\beta-2}dy\right)\frac{f^{\alpha+\beta-1}(x)}{\overline{F}^{\alpha+\beta-1}(t)}dx-\int_t^{\infty}\left(\int_t^{x}y^{\alpha+\beta-2}dy\right)\frac{f^{\alpha+\beta-1}(x)}{\overline{F}^{\alpha+\beta-1}(t)}dx\right]. \quad (20)\\ &=(\alpha+\beta-1)\left[\frac{t^{\alpha+\beta-1}}{\alpha+\beta-1}\int_t^{\infty}\left(\frac{f^{\alpha+\beta-1}(x)}{\overline{F}^{\alpha+\beta-1}(t)}\right)dx-\right.\\ &\qquad\left.\frac{1}{\overline{F}^{\alpha+-1}(t)}\int_{y=t}^{\infty}\int_{x=y}^{\infty}y^{\alpha+\beta-2}f^{\alpha+\beta-1}(x)dx\,dy\right].\end{aligned}$$

Equations (9) and (19) can respectively be rewritten as

$$\int_t^{\infty}f^{\alpha+\beta-1}(x)dx=\overline{F}^{\alpha+\beta-1}(t)e^{(\beta-\alpha)H^{(\alpha,\beta)}(X;t)} \quad (21)$$

and

$$H_w^{(\alpha,\beta)}(X;t)=\frac{1}{\beta-\alpha}\log\left(\int_t^{\infty}x^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(x)}{\overline{F}^{\alpha+\beta-1}(t)}dx\right)-\frac{\alpha+\beta-1}{\beta-\alpha}\log(E(X\mid X>t)). \quad (22)$$

Substituting the results from (20) and (21) in (22), weighted generalized residual entropy of order α and type β can be rewritten as

$$\begin{aligned}&H_w^{(\alpha,\beta)}(X;t)=\\ &\frac{1}{\beta-\alpha}\log\left(t^{\alpha+\beta-1}e^{(\beta-\alpha)H^{(\alpha,\beta)}(X;t)}+(\alpha+\beta-1)\int_t^{\infty}y^{\alpha+\beta-2}\frac{\overline{F}^{\alpha+\beta-1}(y)}{\overline{F}^{\alpha+\beta-1}(t)}e^{(\beta-\alpha)H^{(\alpha,\beta)}(X;y)}dy\right) \quad (23)\\ &-\frac{\alpha+\beta-1}{\beta-\alpha}\log(E(X\mid X>t)).\end{aligned}$$

Next, we show that the proposed weighted generalized residual entropy of order α and type β characterizes the distribution function uniquely. In this context we prove the following result.

Theorem: *Let X be an absolutely continuous random variable having probability density function f(x) and survival function* $\bar{F}(t)$. *If* $H^{(\alpha,\beta)}(X;t)$ *is increasing in t, then* $H_w^{(\alpha,\beta)}(X;t)$ *uniquely determines the survival function* $\bar{F}(t)$.

Proof: Rewriting (23) as

$$e^{(\beta-\alpha)}H_w^{(\alpha,\beta)}(X;t)=\frac{I(t)}{[g(t)]^{\alpha+\beta-1}}, \tag{24}$$

where $g(t)=E(X\,|\,X>t)$, and

$$I(t)=t^{\alpha+\beta-1}e^{(\beta-\alpha)H^{(\alpha,\beta)}(X;t)} +(\alpha+\beta-1)\int_t^{\infty}y^{\alpha+\beta-2}\frac{\bar{F}^{\alpha+\beta-1}(y)}{\bar{F}^{\alpha+\beta-1}(t)}e^{(\beta-\alpha)H^{(\alpha,\beta)}(X;y)}dy. \tag{25}$$

Differentiating (25) w.r.t. t, we have

$$I'(t)=t^{\alpha+\beta-1}(\beta-\alpha)\frac{d}{dt}H^{(\alpha,\beta)}(X;t)e^{(\beta-\alpha)H^{(\alpha,\beta)}(x;t)} +(\alpha+\beta-1)^2 r_x(t)\left(\int_t^{\infty}y^{\alpha+\beta-2}\frac{\bar{F}^{\alpha+\beta-1}(y)}{\bar{F}^{\alpha+\beta-1}(t)}e^{(\beta-\alpha)H^{(\alpha,\beta)}(X;y)}dy\right), \tag{26}$$

where $r_x(t)=\frac{f(t)}{\bar{F}(t)}$. Differentiating (24) w.r.t. t, we get

$$\frac{d}{dt}e^{\beta-\alpha}H_x^{(\alpha,\beta)}(X;t)=-\frac{(\alpha+\beta-1)g'(t)}{[g(t)]^{\alpha+\beta}}I(t)+\frac{I'(t)}{[g(t)]^{\alpha+\beta-1}}. \tag{27}$$

Rearranging (19), we have

$$e^{(\beta-\alpha)}g(t)H_w^{(\alpha,\beta)}(X;t)=\frac{1}{[g(t)]^{\alpha+\beta-1}}\int_t^{\infty}x^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(x)}{\bar{F}^{\alpha+\beta-1}(t)}dx. \tag{28}$$

Differentiating (28) w.r.t. t, we have following result,

$$\frac{d}{dt}e^{(\beta-\alpha)}H_w^{(\alpha,\beta)}(X;t)=-\frac{(\alpha+\beta-1)g'(t)}{[g(t)]^{\alpha+\beta}}\int_t^{\infty}y^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(x)}{\bar{F}^{\alpha+\beta-1}(t)}dx +\frac{(\alpha+\beta-1)r_x(t)}{[g(t)]^{\alpha+\beta-1}}\int_t^{\infty}y^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(x)}{\bar{F}^{\alpha+\beta-1}(t)}dy-\frac{t^{\alpha+\beta-1}}{[g(t)]^{\alpha+\beta-1}}[r_x(t)]^{\alpha+\beta-1}. \tag{29}$$

Using the results from equations (26), (27) and (29) and then rearranging the terms, we get

$$\frac{(\alpha+\beta-1)[g'(t)]}{[g(t)]^{\alpha+\beta}}\left[\int_t^\infty y^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(y)}{\overline{F}^{\alpha+\beta-1}(t)}dy-I(t)\right]$$
$$+\frac{(\beta-\alpha)t^{\alpha+\beta-1}}{[g(t)]^{\alpha+\beta-1}}\frac{d}{dt}H^{(\alpha,\beta)}(X;t)e^{(\beta-\alpha)H^{(\alpha,\beta)}(X;t)}$$
$$+\frac{(\alpha+\beta-1)r_x(t)}{[g(t)]^{\alpha+\beta-1}}\left[(\alpha+\beta-1)\int_t^\infty y^{\alpha+\beta-2}\frac{\overline{F}^{\alpha+\beta-1}(y)}{\overline{F}^{\alpha+\beta-1}(t)}\left[e^{(\beta-\alpha)H^{(\alpha,\beta)}(X;y)}\right]dy\right.$$
$$\left.-\int_t^\infty y^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(y)}{\overline{F}^{\alpha+\beta-1}(t)}dy\right]+\frac{t^{\alpha+\beta-1}}{[g(t)]^{\alpha+\beta-1}}[r_x(t)]^{\alpha+\beta-1}=0$$

For fixed $t>0, r_x(t)$ is the solution of $A(x)=0$, where $A(x)$ is,

$$A(x)=\frac{(\alpha+\beta-1)[g'(t)]}{g(t)^{\alpha+\beta}}\left[\int_t^\infty y^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(y)}{\overline{F}^{\alpha+\beta-1}(t)}dy-I(t)\right]$$
$$+\frac{(\beta-\alpha)t^{\alpha+\beta-1}}{[g(t)]^{\alpha+\beta-1}}\frac{d}{dt}H^{(\alpha,\beta)}(X;t)e^{(\beta-\alpha)H^{(\alpha,\beta)}(X;t)}+ \tag{30}$$
$$\frac{(\alpha+\beta-1)x}{[g(t)]^{\alpha+\beta-1}}\left[(\alpha+\beta-1)\int_t^\infty y^{\alpha+\beta-2}\frac{\overline{F}^{\alpha+\beta-1}(y)}{\overline{F}^{\alpha+\beta-1}(t)}[e^{(\beta-\alpha)H^{(\alpha,\beta)}(X;y)}]dy\right.$$
$$\left.-\int_t^\infty y^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(y)}{\overline{F}^{\alpha+\beta-1}(t)}dy\right]+\frac{t^{\alpha+\beta-1}}{[g(t)]^{\alpha+\beta-1}}x^{\alpha+\beta-1}.$$

Differentiating (30) w.r.t. x, we have

$$A'(x)=\frac{(\alpha+\beta-1)t^{\alpha+\beta-1}}{[g(t)]^{\alpha+\beta-1}}x^{\alpha+\beta-2}-\frac{(\alpha+\beta-}{[g(t)]^{\alpha+(}}$$
$$\left[\int_t^\infty y^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(y)}{\overline{F}^{\alpha+\beta-1}(t)}dy-(\alpha+\beta-1)\right.$$
$$\left.\times\int_t^\infty y^{\alpha+\beta-2}\frac{\overline{F}^{\alpha+\beta-1}(y)}{\overline{F}^{\alpha+\beta-1}(t)}[e^{(\beta-\alpha)H^{(\alpha,\beta)}(X;y)}]dy\right].$$

Further, for $A'(x)=0$, the value of x can be obtained as,

$$x=\left[\frac{1}{t^{\alpha+\beta-1}}\left(\int_t^\infty y^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(y)}{\bar{F}^{\alpha+\beta-1}(t)}dy\right.\right.$$

$$\left.\left.-(\alpha+\beta-1)\int_t^\infty y^{\alpha+\beta-2}\frac{\bar{F}^{\alpha+\beta-1}(y)}{\bar{F}^{\alpha+\beta-1}(t)}e^{(\beta-\alpha)H^{(\alpha,\beta)}(X;y)}dy\right)\right]^{\frac{1}{\alpha+\beta-2}}.$$

For x=0, from equation (30) we have,

$$A(0)=\frac{(\alpha+\beta-1)g'(t)}{[g(t)]^{\alpha+\beta}}\left[\int_t^\infty y^{\alpha+\beta-1}\frac{f^{\alpha+\beta-1}(y)}{\bar{F}^{\alpha+\beta-1}(t)}dy-I(t)\right]$$
$$+\frac{(\beta-\alpha)t^{\alpha+\beta-1}}{[g(t)]^{\alpha+\beta-1}}\frac{d}{dt}H^{(\alpha,\beta)}(X;t)e^{(\beta-\alpha)H^{(\alpha,\beta)}}(X;t).$$

To prove the theorem we discuss the following two cases:
Case I: For $\beta>1,\beta-1<\alpha<\beta,$ we have $A(0)>0$. Further, $H^{(\alpha,\beta)}(X;t)$ is increasing in t and $A(\infty)=\infty,$ then

$$A''(x)=(\alpha+\beta-1)(\alpha+\beta-2)\frac{t^{\alpha+\beta-1}}{[g(t)]^{\alpha+\beta-1}}x^{\alpha+\beta-3},$$

is also non-negative. Therefore, $A'(x)$ is increasing in t and $A'(t_0)=0, A'(\infty)=\infty.$ Also, $A(x)\le 0$ for $0<x\le t_0$ and $A'(x)\ge 0$ for $x\ge t_0$. Thus, we can say that $x=r_x(t)$ is the unique solution of $A(x)=0.$

Case II: Similarly, for $\beta<1,\beta-1<\alpha<\beta$, we have $A(0)<0$. Further, if $H^{(\alpha,\beta)}(X;t)$ is increasing in t and $A(\infty)=-\infty,$ then proceeding as in case I, we can interpret that $x=r_x(t)$ is unique solution of $A(x)=0.$
Therefore, from these two cases it can be concluded that if $H^{(\alpha,\beta)}(X;t)$ is increasing in $t>0$ and $A(x)=0$, then $x=r_x(t)$ is the unique solution of $A(x)=0.$ Thus, $H_x^{(\alpha,\beta)}(X;t)$ determines $x=r_x(t)$ uniquely. Hence, the result follows as $x=r_x(t)$ uniquely determine $\bar{F}(t).$

Next, we define *a class of non-parametric class of distribution* on the basis of the weighted residual entropy introduced. Based on the monotonicity

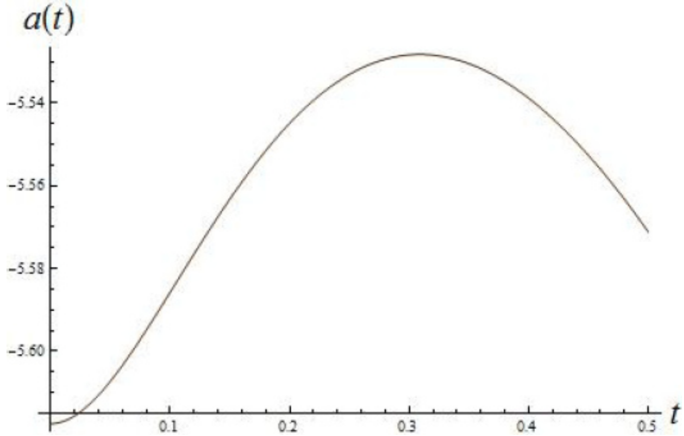

Figure 1:

property of weighted entropies (17) and (18), Di Crescenzo and Longobardi [3] defined two non-parametric classes of distributions. Similarly, here we introduce a non-parametric class of distribution.

Definition 3.1 *A random variable X is said to have decreasing (or increasing) weighted uncertainty residual life of order* α *and type* β *if* $H_x^{(\alpha,\beta)}(X;t)$ *is decreasing (or increasing) in* $t \geq 0$.

There exist distributions which are not monotone in terms of $H_w^{(\alpha,\beta)}(X;t)$, it can be shown by following counter example.

Counter example 3.1 *Let X be a random variable with probability density function* $f(x) = \dfrac{2}{(1+x)^3}, x \geq 0.$

Then the corresponding survival function is

$$\bar{F}(x) = \frac{1}{(1+x)^2}, x \geq 0.$$

If $\alpha = 3/2$ *and* $\beta = 2$, *for* $x \geq 0$, *we see that* $H_w^{(\alpha,\beta)}(X;t) = a(t)$, *say, is not monotone in* $0 \leq t \leq 0.5$ *as shown in Figure 1.*

4. CONCLUSION

In this paper, we have proposed and studied the concept of weighted generalized residual entropy of order α and type β. This proposed residual information measure characterizes the distribution function uniquely. Weighted generalized information measures play an essential role in modelling of statistical data which includes certain amount of damaged or unobserved values. The measure introduced here can be of interest in such type of problems.

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APPENDIX

CALCULATIONS

Input: **(1/(2-3/2))Log[Integrate[(2x)^(3/2+2-1),{x,0,1}]]**

Output: 2 Log[(8 $\sqrt{2}$)/7]

Input: **N[2 Log[(8 $\sqrt{2}$)/7]]**

Output: 0.96021

Input: **(1/(2-3/2))Log[(1/(Integrate[(2x^2),{x,0,1}])^(3/2+2-1)) Integrate[x^(3/2+2-1)(2x)^(3/2+2-1),{x,0,1}]]**

Output: 2 Log[(3 $\sqrt{3}$)/2]

Input: **N[2 Log[(3 $\sqrt{3}$)/2]]**

Output: 1.90954

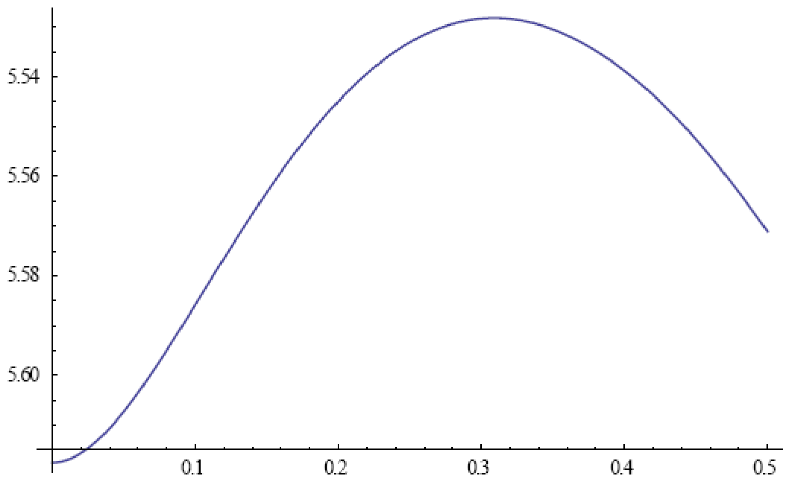
Input: **(1/(2-3/2))Log[Integrate[(2(1-x))^(3/2+2-1),{x,0,1}]]**

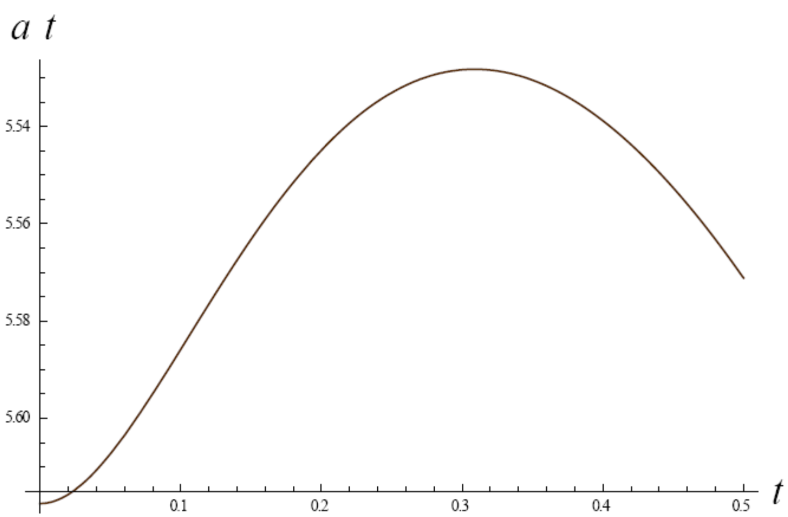
Output: 2 Log[(8 $\sqrt{2}$)/7]

Input: **N[2 Log[(8 $\sqrt{2}$)/7]]**

Output: 0.96021

Input: **(1/(2-3/2))Log[(1/(Integrate[(2x(1-x)), {x, 0,1}])^(3/2+2-**





1))Integrate[x^(3/2+2-1)(2(1-x))^(3/2+2-1),{x,0,1}]]

Output: 2 Log[45/128 $\sqrt{\frac{3}{2}}$]

***Input:* N[2 Log[45/128 $\sqrt{\frac{3}{2}}$]]**

Output: 0.604189

***Input:* Integrate[6x/(1+x)^4,{x,t,Infinity}]**

Output: ConditionalExpression[(1+3 t)/(1+t)^3,Im[t]^{1}0||Re[t]>-1]

Input: **(1/(2-3/2))Log[(1/Integrate[x (2/(1+x)^3), {x, t, Infinity}])^(3/2+2-1)(Integrate[(x (2/(1+x)^3))^(3/2+2-1),{x,t,Infinity}])]**

Output: ConditionalExpression[2 Log[($4^{\sqrt{2}}$ ((1+t)^2/(1+2 t))$^{5/2}$ (32+2 t (80+t (160+t (160-

$$429\sqrt{\frac{t}{(I+t)^{3}}}-2\ \mathrm{t}\,(-40+143\sqrt{\frac{t}{(I+t)^{3}}}$$

$$+4\ \mathrm{t}\,(-2+13\sqrt{\frac{t}{(I+t)^{3}}}+2\ \mathrm{t}\sqrt{\frac{t}{(I+t)^{3}}}))\,/\,3003(1+\mathrm{t})^{5}],$$

$$\mathrm{Re[t]}\neq 0\,\mathit{and}\,\mathrm{IM\ [t]}\neq 0$$

Input: **Plot[%,{t,0,0.5}]**

Output:

Input: **Plot[%2,{t,0,0.5},PlotStyleDirective[RGBColor[0.32,0.15,0.],Opacity[1.],AbsoluteThickness[1.195]],AxesLabel{Style[t,Large],Style[a[t],Large]}]**

Output: