DOI: 10.15415/mjis.2016.42011

Sequences of Fuzzy Soft Multi Points

ANJAN MUKHERJEE[1] AND AJOY KANTI DAS[2]

[1]Department of Mathematics, Tripura University, Suryamaninagar,Agartala-799022, Tripura, INDIA

[2]Department of Mathematics, Iswar Chandra Vidyasagar College, Belonia -799155, Tripura, INDIA

***Email: anjan2002_m@yahoo.co.in**

Received: October 27, 2014| Revised: September 27, 2015| Accepted: February 11, 2016

Published online: March 30, 2016

Abstract In this paper we introduce a new sequence of fuzzy soft multi points in fuzzy soft multi topological space. The concepts of subsequence and convergence sequence of fuzzy soft multi points are proposed. The notion of cluster fuzzy soft multi points of sequences are also introduced. Some basic properties regarding the above concepts are explored.

Keywords: Fuzzy Set, Soft multi set, Fuzzy soft set, Fuzzy soft multi set, Fuzzy soft multi topological space.

1. INTRODUCTION

Theory of fuzzy sets, soft sets and soft multisets are powerful mathematical tools for modeling various types of vagueness and uncertainty. In 1999, Molodtsov[5] initiated soft set theory as a completely generic mathematical tool for modeling uncertainty and vague concepts. Later on Maji et al.[4] presented some new definitions on soft sets and discussed in details the application of soft set in decision making problem. Combining soft sets [5] with fuzzy sets[7], Maji et al. [3] defined fuzzy soft sets. Alkhazaleh et al. [1] as a generalization of Molodtsov's soft set, presented the definition of a soft multi set, thereafter in 2012, Alkhazaleh and Salleh[2] introduced the concept of fuzzy soft multiset theory and studied the application of these sets. Recently, Mukherjee and Das[6] studied the concepts of soft fuzzy soft multi topological spaces in details.

The aim of this paper is to introduce a new sequence of fuzzy soft multi points in fuzzy soft multi topological spaces and their basic properties are studied. The concepts of subsequence and convergence sequence of fuzzy soft

Mathematical Journal of Interdisciplinary Sciences
Vol. 4, No. 2,
March 2016
pp. 135–139

multi points are proposed. We also introduce the concepts of cluster fuzzy soft multi points of sequences and their basic properties are investigated.

2. PRELIMINARY NOTES

Definition 2.1 [7]. Let X be a non empty set. Then a fuzzy set A is a set having the form A={(x, μ_A(x)): x∈X}, where the functions μ_A: X→[0, 1] represents the degree of membership of each element x∈X.

Definition 2.2 [5]. Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and A⊆E. Then the pair (F, A) is called a soft set over U, where F is a mapping given by F: A→ P(U).

Definition 2.3 [3]. Let U be an initial universe and E be a set of parameters. Let F(U) be the set of all fuzzy subsets of U and A⊆E. Then the pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by F: A→F(U).

Definition 2.4 [1]. Let $\{U_i : i\in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_{U_i} : i\in I\}$ be a collection of sets of parameters. Let $U=\prod_{i\in I} P(U_i)$ where $P(U_i)$ denotes the power set of U_i $E=\prod_{i\in I} E_{U_i}$, and $A\subseteq E$. A pair (F, A) is called a soft multiset over U, where F is a mapping given by F: A → U .

Definition 2.5 [2]. Let $\{U_i : i\in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \phi$ and let $\{E_{U_i} : i\in I\}$ be a collection of sets of parameters. Let $U=\prod_{i\in I} FS(U_i)$ where $FS(U_i)$ denotes the set of all fuzzy subsets of U_i, $E=\prod_{i\in I} E_{U_i}$ and $A\subseteq E$. A pair (F, A) is called a fuzzy soft multiset over U, where F is a mapping given by F: A → U .

Definition 2.6 [2]. A fuzzy soft multiset (F, A) over U is called fuzzy soft multi subset of fuzzy soft multiset (G, B) if *(a)* $A\subseteq B$ and *(b)* $\forall e_{U_i,j}\in a_k$, $(e_{U_i,j}, F_{e_{U_i,j}})$ is a fuzzy subset of $(e_{U_i,j}, G_{e_{U_i,j}})$, where $a_k\in A$, k ∈{1,2,...,n} , $i\in$ {1,2,...,m} and $j\in$ {1,2,...,r} . This relation is denoted by (F, A) $\tilde{\subseteq}$ (G, B).

Definition 2.7 [2]. A fuzzy soft multiset (F, A) over U is called an absolute fuzzy soft multiset, denoted by $(F, A)_U$, if $(e_{U_i,j}, F_{e_{U_i,j}}) = U_i,\ \forall i$.

We consider an absolute fuzzy soft multiset $(\mathfrak{F}, A)$ over U and $FSMS_A(\mathfrak{F}, A)$ denote the family of all fuzzy soft multi sub sets of $(\mathfrak{F}, A)$ in which all the parameter set A are same.

Definition 2.8 [6]. A sub family τ of $FSMS_A(\mathfrak{F},A)$ is called fuzzy soft multi topology on $(\mathfrak{F},A)$, if the following axioms are satisfied:

$[O_1]$ Φ_A, $(\mathfrak{F},A)\in\tau$, (where Φ_A has been defined in [6])

$[O_2]$ $\{(F_i,A):i\in I\}\subseteq\tau\Rightarrow\tilde{\cup}_{i\in I}(F_i,A)\in\tau,$

$[O_3]$ If $(F,A),(G,A)\in\tau$, then $(F,A)\tilde{\cap}(G,A)\in\tau$.

The pair $((\mathfrak{F},A),\tau)$ is called fuzzy soft multi topological space and the members of τ are called fuzzy soft multi open sets.

Definition 2.9 [6]. $(F,A)\in FSMS_A(\mathfrak{F},A)$ is a neighbourhood (in short nbd) of $(G,A)\in FSMS_A(\mathfrak{F},A)$ if there exists $(H,A)\in\tau$ such that $(G,A)\tilde{\subseteq}(H,A)\tilde{\subseteq}(F,A)$.

Theorem 2.10 [6]. $(F,A)\in FSMS_A(\mathfrak{F},A)$ is an open if and only if (F,A) is a neighbourhood of each fuzzy soft multi set (G,A) contained in (F,A).

Definition 2.11 [6]. $(\mathrm{F,A})\in FSMS_A(\mathfrak{F},A)$ is called a fuzzy soft multi point and denoted by $e_{(\mathrm{F,A})}$, if for $e\in A, F(e)\neq\varphi$ and $\forall e'\in A-\{e\}, F(e')=\varphi$.

Definition 2.12 [6]. A fuzzy soft multi point $e_{(\mathrm{F,A})}$ is said to be in the fuzzy soft multiset (G,A) , denoted by $e_{(F,A)}\tilde{\in}(G,A)$, if $(F,A)\tilde{\subseteq}(G,A)$.

2.2 Figures (subsection 2, italic)

3. MAIN RESULTS

In this section we introduce a new sequence of fuzzy soft multi points in fuzzy soft multi topological spaces and study their basic properties.

Definition 3.1 Let $((\mathfrak{F},A),\tau)$ be the fuzzy soft multi topological space on $(\mathfrak{F},A)$ and N be the set of all natural numbers. A sequence of fuzzy soft multi points in $((\mathfrak{F},A),\tau)$ is a mapping from N to $FSMS_A(\mathfrak{F},A)$ and is denoted by $\{e_{(F_n,A)}\}$ *or* $\{e_{(F_n,A)}:n=1,2,...\}$.

Definition 3.2 A sequence $\{e_{(F_n,A)}\}$ of fuzzy soft multi points is said to be eventually contained in a fuzzy soft multi set (F, A) if and only if there is a positive integer m such that, $n\geq m$ implies $e_{(F_n,A)}\tilde{\in}(F,A)$.

Definition 3.3 A sequence $\{e_{(F_n,A)}\}$ of fuzzy soft multi points is said to be frequently contained in a fuzzy soft multi set (F, A) if and only if for each positive integer m, there is a positive integer m such that, $n\geq m$ implies $e_{(F_n,A)}\tilde{\in}(F,A)$.

Definition 3.4 A sequence $\{e_{(F_n,A)}\}$ of fuzzy soft multi points in a fuzzy soft multi topological space $((\mathfrak{F},A),\tau)$ is said to be convergence and converge to a fuzzy soft multi point $e_{(F,A)}$ if it is eventually contained in each nbd of the fuzzy soft multi set (F, A).

Definition 3.5 A fuzzy soft multi point $e_{(F,A)}$ in a soft topological space $((\mathfrak{F},A),\tau)$is a cluster fuzzy soft multi point of a sequence $\{e_{(F_n,A)}\}$ if the sequence $\{e_{(F_n,A)}\}$ is frequently contained in every nbd of (F, A).

Theorem 3.6 If the nbd system of each fuzzy soft multi set in a soft topological space $((\mathfrak{F},A),\tau)$ is countable, then a fuzzy soft multi set (*F, A*) is open if and only if each sequence $\{e_{(F_n,A)}\}$ of fuzzy soft multi points which converges to a fuzzy soft multi point $e_{(G,A)}$ contained in (*F, A*) is eventually contained in (*F, A*).

Proof: Since (*F, A*) is open, (*F, A*) is a nbd of (*G, A*). Hence, $\{e_{(F_n,A)}\}$ is eventually contained in (*F, A*).

Conversely, for each $e_{(G,A)}\tilde{\in}(F,A)$, let $e_{(G_1,A)}, e_{(G_2,A)}, \dots, e_{(G_n,A)},\dots$ be the fuzzy soft multi points and $(G_1, A), (G_2, A),\dots, (G_n, A),\dots$ be the nbd system of (G, A). Let $(H_n,A)=\tilde{\cap}_{i=1}^{n}\{(G_i,A)\}$. Then $\{e_{(H_n,A)} : n=1,2,\dots\}$ is a sequence of fuzzy soft multi points which is eventually contained in each nbd of (*G, A*), i.e., $\{e_{(H_n,A)} : n=1,2,\dots\}$ converges to $e_{(G,A)}$. Hence, there is an m such that for $n\geq m$, $(H_n,A)\tilde{\subseteq}(F,A)$. The (H_n,A) are nbd's of (*G, A*). This implies (*F, A*) is a nbd of (*G, A*) and by theorem 2.10, (*F, A*) is fuzzy soft multi open set.

Definition 3.7 Let f be a mapping over the set of positive integers. Then the sequence $\{e_{(G_n,A)}\}$ is a subsequence of a sequence $\{e_{(F_n,A)}\}$ if and only if there is a map f such that $e_{(G_i,A)}=e_{(F_{f(i)},A)}$ and for each integer m, there is an integer n_o such that $f(i)\geq m$ whenever $i\geq n_o$.

Theorem 3.8 If nbd system of each fuzzy soft multi set in a fuzzy soft multi topological space $((\mathfrak{F},A),\tau)$ is countable, then for $e_{(F,A)}$ is a cluster fuzzy soft multi point of a sequence $\{e_{(F_n,A)}\}$ there is a subsequence converging to $e_{(F,A)}$.

Proof: Let $e_{(K_1,A)}, e_{(K_2,A)}, \dots, e_{(K_n,A)},\dots$ be fuzzy soft multi points and $(K_1, A), (K_2, A),\dots, (K_n, A),\dots$ be nbd system of (*F, A*) and let $(L_n,A)=\tilde{\cap}_{i=1}^{n}\{(K_i,A)\}$. Then $\{e_{(F_n,A)} : n=1,2,\dots\}$ is a sequence such that $(F_{n+1},A)\tilde{\subseteq}(F_n,A)$ for each n and is eventually contained in each nbd of (*F, A*), i.e., $\{e_{(F_n,A)} : n=1,2,\dots\}$ converges to $e_{(F,A)}$. For every positive integer i, choose $f(i)$ such that $f(i) \geq i$ *and* $(F_{f(i)}, A) \tilde{\subseteq} (L_i, A)$. $f(i) \geq i$ *and* $(F_{f(i)}, A) \tilde{\subseteq} (L_i, A)$. Then $\{e_{(F_{f(i)}, A)} : i = 1,2,...\}$ is a subsequence of the sequence $\{e_{(F_n, A)} : n = 1,2,...\}$ and which converges to $e_{(F, A)}$.

CONCLUSION

Fuzzy sets, soft sets, soft multisets, fuzzy soft sets and fuzzy soft multisets are all mathematical tools for dealing with uncertainties and vagueness. In this research work, we have introduced a new sequence of fuzzy soft multi points in fuzzy soft multi topological spaces together with some basic properties over a fixed parameter set. The concepts of subsequence and convergence sequence of fuzzy soft multi points are proposed. We also introduce the concepts of cluster fuzzy soft multi points of sequences and study their basic properties.

REFERENCES

[1] Alkhazaleh, S., Salleh, A.R., N. Hassan, N., Soft Multisets Theory, Applied Mathematical Sciences, **5**, 3561–3573 (2011).

[2] Alkhazaleh, S., Salleh, A. R., Fuzzy Soft Multisets Theory, Hindawi Publishing Corporation Abstract and Applied Analysis, Vol. 2012, Article ID 350603, 20 pages, http://dx.doi.org/10.1155/2012/350603.

[3] Maji, P. K., Roy, A.R., Biswas, R., Fuzzy soft sets, Journal of Fuzzy Mathematics, **9**, 589-602 (2001).

[4] Maji, P. K., Roy, A.R. Biswas, R., Soft set theory, Computers and Mathematics with Applications, **45**, 555-562 (2003). http://dx.doi.org/10.1016/S0898-1221(03)00016-6

[5] Molodtsov, D., Soft set theory-first results, Computers and Mathematics with Applications, **37**, 19–31 (1999). http://dx.doi.org/10.1016/S0898-1221(99)00056-5

[6] Mukherjee, A., Das, A.K., Topological structure formed by fuzzy soft multi sets, Bulletin of Calcutta Mathematical Society, **106**, 112-128 (2013).

[7] Zadeh, L. A., Fuzzy sets, Inform. Control. **8**, 338-353 (1965). http://dx.doi.org/10.1016/S0019-9958(65)90241-X