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Effect of Non-uniform Temperature Gradient on Marangoni Convection in a Relatively Hotter or Cooler Layer of Liquid

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Abstract The effect of non-uniform temperature gradient on the onset of convection driven by surface tension gradients in a relatively hotter or cooler layer of liquid is studied by means of linear stability analysis. The upper boundary is considered to be free and insulating where surface tension gradients arise on account of variation in temperature and the lower boundary is rigid. The single-term Galerkin technique is used to obtain the eigenvalue equation. Eigenvalues are obtained and presented for both thermally conducting and insulating cases of the lower boundary. This analysis predicts that in either case the critical eigenvalues for different non-uniform temperature gradients are greater in a relatively hotter layer of liquid than the cooler one under identical conditions otherwise. This qualitative effect is quite significant quantitatively as well.

Keywords: Convection; Conducting; Insulating; Linear stability; Surface tension.

1. INTRODUCTION

The phenomenon of the problem of thermal convection in a thin horizontal liquid layer heated from below observed experimentally [3-4] was mathematically explained [16] in terms of buoyancy and [15] in terms of surface tension. In general, convection appears when a certain dimensionless parameter exceeds its critical value. This parameter is a Rayleigh number when the convection is induced by buoyancy effects due to variations in density and is a Marangoni number when surface tension variations induce the convection. In [12], Nield

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accounted for both the surface tension and buoyancy effects and established that the two effects causing instability reinforce each other and that as the depth of the liquid layer decreases the surface tension effects become more dominant. Thus, for thin layers used in the experimental work of Bénard the convective motion is due to mainly surface tension effects. In fact, as reported in [2] that if the fluid has a free surface then convection can still be induced by surface tension effects even if buoyancy forces are absent (zero gravity), and hence it is of importance to calculate the critical Marangoni number below which convection cannot occur. The copious literature on this phenomenon has been reviewed both experimentally and theoretically [5-6, 10, 14, 18].

Since the process of controlling convection in a fluid, has recently assumed importance in material processing in space because of its application extending from producing large crystals of uniform properties to manufacturing new materials with unique properties. The effect of non-uniform temperature gradient including various other effects such as rotation or/and magnetic field on buoyancy driven as well as surface tension driven convection is now well established [7, 11, 13, 17]. In [9], Gupta and Shandil have examined the surface tension driven problem in a relatively hotter or cooler layer of liquid, and established that the hotter layer with its heat diffusivity apparently increased as a consequent of actual decrease in its specific heat at constant volume, must exhibit convection at a higher temperature difference and hence at a greater Marangoni number than a cooler layer under identical conditions otherwise. Nevertheless, investigation of the effect of non-uniform temperature gradient in a liquid layer which is relatively hotter or cooler has not been given attention in the literature despite its importance in understanding convective instability encountered in many scientific, engineering and technological fields.

In the present paper, therefore, we attempt to investigate the effect of non-uniform temperature gradient on the onset of surface tension driven convection in a relatively hotter or cooler layer of liquid which is heated from below. The Galerkin method is useful for the present problem to find the eigenvalue equation with a minimum of mathematical calculations. This analysis predicts that the different non-uniform temperature gradients suppress the phenomenon more effectively in a relatively hotter layer of liquid than the cooler one under identical conditions otherwise.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider an infinite horizontal layer of homogeneous viscous fluid which is of uniform thickness d and heated from below. The lower rigid boundary of the layer is kept at a constant temperature T0 and the upper free surface is open to the atmosphere at temperature T1 subject to constant heat flux. The lower boundary is at temperature ΔT higher than that of the upper boundary. We choose a Cartesian coordinate system of axes with the x and y axes in the plane of the lower surface of the layer and z axis along the vertically upward direction so that the fluid layer is confined between the planes z = 0 and z = d as shown in Figure 1.

The physical quantities that are assumed to vary within the fluid are the temperature and the surface tension only. The surface tension on the upper free surface of fluid is regarded as a function of temperature only which is given by the simple linear law

$$\tau = \tau_1 - \sigma\left(T - T_1\right) \tag{1}$$

where the constant τ_1 is the unperturbed value of τ at the unperturbed surface temperature $T = T_1$ and $-\sigma = \left(\partial\tau/\partial T\right)_{T=T_1}$ represents the rate of change of surface tension with temperature T_1, evaluated at temperature , and surface tension being a monotonically decreasing function of temperature, σ is positive. The governing equations for this configuration are well known and given in [6]. Following [1], the modified linearized and dimensionless equations governing the system in the present context can be written as

$$\left(D^2 - a^2\right)\left(D^2 - a^2 - p\right)W = 0 \tag{2}$$

$$\left[D^2 - a^2 - \left(1 - \alpha_2 T_0\right) p P_r\right]T = -\left(1 - \alpha_2 T_0\right) f\left(z\right) W \tag{3}$$

where $D = d/dz$; $W(z)$ and $T(z)$ represent the amplitude of the z-component of the velocity and temperature distribution respectively; a the horizontal wave

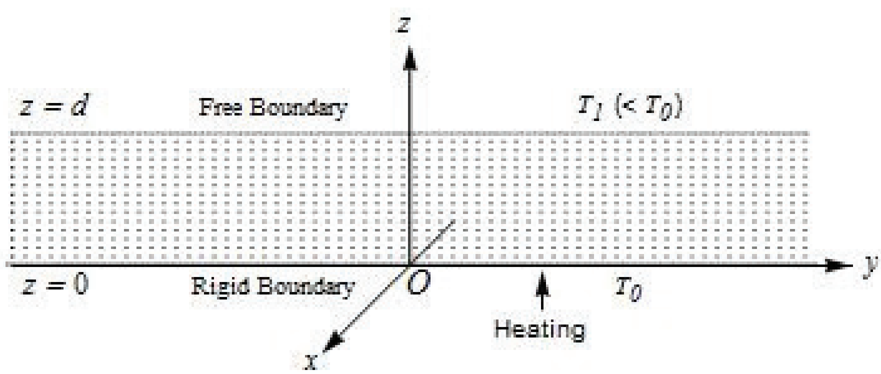

Figure 1: Schematic representation of a liquid layer heated from below.

number, P_r is the Prandtl number and p is the time growth rate of disturbances (which can be complex). The coefficient α_2 (due to variation in specific heat at constant volume on account of variation in the temperature) lies in the range from 0 to 10^{-4}, and range of the dimensionless parameter a_2T_0 covering the usual laboratory conditions is $0 \le a_2T_0 < 1$ for liquids with which we are mostly concerned. In this range, any given value of α_2T_0 ($\neq 0$) corresponds to the layer of liquid which is relatively hotter compared to that associated with its value less than (including $a_2T_0 = 0$) the given one. Further, $f(z)$ represents the dimensionless basic non-uniform temperature gradient ([13]) which must satisfy the condition that $\int_0^1 f(z)\,dz = 1$, the linear temperature profile $f(z)$ = 1 is the basic uniform temperature gradient. The various non-uniform basic temperature profiles including the linear temperature profile considered in this paper are presented in Figure 2.

(a) Linear: $f(z) = 1$,

(b) Piecewise linear (heated from below): $f(z) = \varepsilon^{-1}$ for $0 \le z < \varepsilon$, and $f(z) = 0$ for $\varepsilon \le z \le 1$,

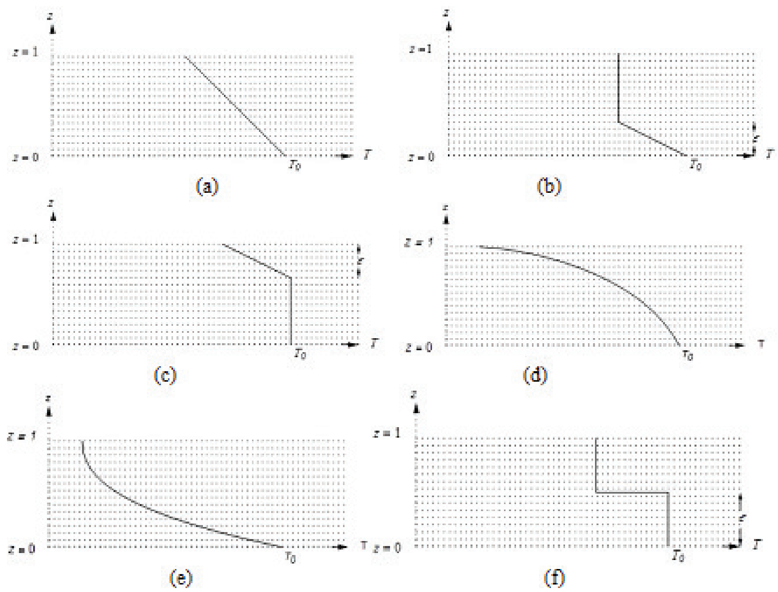

Figure 2:

(c) Piecewise linear (cooled from above): $f(z) = 0$ for $0 \le z < 1-\varepsilon$ and $f(z) = \varepsilon^{-1}$ for $1-\varepsilon \le z \le 1$,
(d) Parabolic: $f(z) = 2z$,
(e) Inverted parabolic: $f(z) = 2(1-z)$,
(f) Step function: $f(z) = \delta(z-\varepsilon)$.

When the principle of exchange of stabilities is valid that is $p = 0$, equations (2)-(3) governing the neutral state are considerably simplified and reduce to the form

$$\left(D^2 - a^2\right)^2 W = 0 \tag{4}$$

$$\left(D^2 - a^2\right)T = -\left(1 - \alpha_2 T_0\right) f(z) W \tag{5}$$

The boundary conditions for thermally insulating free surface at $z = 1$ with temperature dependent surface tension as given in [15] are

$$W(1) = 0, \quad D^2 W(1) + a^2 M T(1) = 0, \quad DT(1) = 0 \qquad \text{(6a, b, c)}$$

where $M = \sigma \Delta T d / \rho \kappa \nu$ represents the Marangoni number in which ρ is the density, ν the kinematic viscosity and κ the thermal diffusivity of the liquid. For the case of thermally conducting rigid bottom surface at $z = 0$, the boundary conditions are

$$W(0) = 0, \qquad DW(0) = 0, \qquad T(0) = 0 \qquad \text{(7a, b, c)}$$

while for the case of thermally insulating rigid bottom surface at $z = 0$, these are given by

$$W(0) = 0, \qquad DW(0) = 0, \qquad DT(0) = 0 \qquad \text{(8a, b, c)}$$

Equations (4)-(5) together with boundary conditions (6a, b, c) and either (7a, b, c) or (8a, b, c) constitutes an eigenvalue problem of order six with M as an eigenvalue.

3. SOLUTION OF THE PROBLEM

The single term Galerkin technique as described in [8] is convenient for solving the present problem. Accordingly, the unknown variables W and T are written as

$$W = AW_1 \ \ and \ \ T = BT_1 \tag{9}$$

where A, B are constants, and W_1 and T_1 are the trial functions which are chosen suitably satisfying the boundary conditions (6a, b, c) and either (7a, b, c) or (8a, b, c).

Multiplying equation (4) by W and equation (5) by T, integrating the resulting equations with respect to z from 0 to 1 by using the boundary conditions (6a, b, c) and either (7a, b, c) or (8a, b, c), and substituting for W and T from (9), in either case, we obtain the following system of linear homogeneous equations:

$$A\left[\left\langle\left(D^2W_1\right)^2+2a^2\left(DW_1\right)^2+a^4\left(W_1\right)^2\right\rangle\right] +B\left[a^2M\,DW_1(1)T_1(1)\left\langle W_1T_1f(z)\right\rangle\right]=0 \tag{10}$$

$$A\left(1-\alpha_2T_0\right)\left[\left\langle W_1T_1f(z)\right\rangle\right]-B\left[\left\langle\left(DT_1\right)^2+a^2\left(T_1\right)^2\right\rangle\right]=0 \tag{11}$$

The system of equations given by (10)-(11) will have a non-trivial solution if and only if

$$M=-\frac{\left\langle\left(D^2W\right)^2+2a^2\left(DW\right)^2+a^4\left(W\right)^2\right\rangle\left\langle\left(DT\right)^2+a^2\left(T\right)^2\right\rangle}{\left(1-\alpha_2T_0\right)a^2DW(1)T(1)\left\langle WTf(z)\right\rangle} \tag{12}$$

where angular bracket $\langle\ldots\rangle$-$\langle\ldots\rangle$ denotes the integration with respect to z from 0 to 1 and suffixes have been dropped for simplicity while writing the eigenvalue equation (12).

4. RESULTS AND DISCUSSION

We now select the trial functions satisfying the appropriate boundary conditions for use in the single term Galerkin method. We consider the two cases depending upon whether the lower rigid boundary surface is conducting or it is insulating.

4.1 The conducting case

In this case, the velocity must satisfy the three boundary conditions (6a)-(7a, b) namely, $W(1) = 0$, $W(0) = 0$ and $DW(0) = 0$ and the temperature must satisfy the two boundary conditions (6c) and (7c) namely, $DT(1) = 0$ and $T(0) = 0$. The lowest order polynomials satisfying these requirements are

$$W = z^2(1-z) \ and \ T = z\left(1-\frac{z}{2}\right) \quad (13)$$

and the residual from the remaining boundary condition (6b) namely, $D^2W(1)+a^2MT(1)=0$, is included in a residual from differential equations while performing integration by parts. Substitution of trial functions given by (13) into the equation (12), we get

$$M = \frac{4\left(420+28a^2+a^4\right)\left(5+2a^2\right)}{1575\left(1-\alpha_2T_0\right)a^2\left\langle\left\{z^2\left(z^3-3z^2+2z\right)\right\}f(z)\right\rangle} \quad (14)$$

For any given $f(z)$ and α_2T_0, expression given by (14) gives the critical value of M as function of the wave number a. We consider the six different temperature profiles and denote the Marangoni numbers by M_i (i = 1 to 6) and their critical values respectively by M_{ci}.

(a) The linear temperature profile

$$f(z)=1 \quad (15)$$

represents the case of basic uniform temperature gradient. The Marangoni number M_1 corresponding to this case obtained from equation (14) is given by

$$M_1 = \frac{4\left(420+28a^2+a^4\right)\left(5+2a^2\right)}{105a^2\left(1-\alpha_2T_0\right)}.$$

For given α_2T_0 this has minimum value $M_{c1} = \frac{66.85}{1-\alpha_2T_0}$ attained at $a = 2.254$. When $\alpha_2T_0 = 0$, we have $M_{c1} = 66.85$ which is close to the known exact value 79.61 attained at a = 1.993 ([12], [15]).

(b) For the piecewise linear temperature profile due to sudden heating from below the temperature profile is

$$f(z)=\begin{cases}\varepsilon^{-1} & for \quad 0\le z<\varepsilon, \\ 0 & for \quad \varepsilon \le z \le 1\end{cases} \quad (16)$$

where ε is the quasi-time dependent thermal depth ranging from 0 to 1. The corresponding Marangoni number M_2 obtained from equation (14) is given by

$$M_2 = \frac{8\left(420+28a^2+a^4\right)\left(5+2a^2\right)}{105a^2\left(1-\alpha_2 T_0\right)\varepsilon^3\left(15\varepsilon^2-18\varepsilon+15\right)}.$$

For given $\alpha_2 T_0$ and *a* = *2.254*, $\alpha_2 T_0 = 0$ the minimum value $M_{c2} = \frac{64.26}{1-\alpha_2 T_0}$ is attained at $\varepsilon = 0.92$. When $\alpha_2 T_0 = 0$, we note that as ε increases from 0 to 1, the critical Marangoni number M_{c2} decreases from ∞ to a minimum value 64.26 at $\varepsilon = 0.92$ and then increases for increasing values of ε from 0.92 to 1.

(c) For the piecewise linear temperature profile due to sudden cooling from top, the temperature gradient is of the form

$$f(z) = \begin{cases} 0 & \text{for} \quad 0 \le z < 1-\varepsilon, \\ \varepsilon^{-1} & \text{for} \quad 1-\varepsilon \le z \le 1. \end{cases} \tag{17}$$

The corresponding Marangoni number M_3 obtained from equation (14) is given by $M_3 = \frac{8\left(420+28a^2+a^4\right)\left(5+2a^2\right)}{105a^2\left(1-\alpha_2 T_0\right)\varepsilon\left(-5\varepsilon^4+18\varepsilon^3-20\varepsilon+15\right)}$.

For given $\alpha_2 T_0$ and *a* = *2.254*, $\alpha_2 T_0 = 0$, the minimum value $M_{c3} = \frac{43.09}{\left(1-\alpha_2 T_0\right)}$ is attained at $\varepsilon = 0.47$. When $\alpha_2 T_0 = 0$, we note that as ε increases from 0 to 1, M_{c3} decreases from ∞ to a minimum of 43.09 at $\varepsilon = 0.47$ and then increases for increasing values of ε from 0.47 to 1.

(d) For the parabolic temperature profile in which the basic temperature gradient is zero at the lower boundary is of the form

$$f(z) = 2z \tag{18}$$

The corresponding Marangoni number M_4 obtained from the equation (14) is given by $M_4 = \frac{4\left(420+28a^2+a^4\right)\left(5+2a^2\right)}{135a^2\left(1-\alpha_2 T_0\right)}$.

For given $\alpha_2 T_0$, this has minimum value $M_{c4} = \dfrac{52.00}{1-\alpha_2 T_0}$ attained at a = *2.254*.

(e) For the inverted parabolic temperature profile generated in conducting fluid layer through the Joule heating with alternating current the basic temperature gradient is zero at the upper boundary is of the form

$$f(z) = 2(1-z) \tag{19}$$

The corresponding Marangoni number M_5 obtained from equation (14) is given by $M_5 = \dfrac{4\left(420+28a^2+a^4\right)\left(5+2a^2\right)}{75a^2\left(1-\alpha_2 T_0\right)}$.

For given $\alpha_2 T_0$, this has minimum value $M_{c5} = \dfrac{93.59}{1-\alpha_2 T_0}$ attained at a = *2.254*.

(f) For the step function temperature profile in which the basic temperature drops suddenly at $z = \varepsilon$ but is otherwise uniform, the temperature gradient is of the form

$$f(z) = \delta(z-\varepsilon) \tag{20}$$

The corresponding Marangoni number M_6 obtained from equation (14) is given by $M_6 = \dfrac{4\left(420+28a^2+a^4\right)\left(5+2a^2\right)}{1575a^2\left(1-\alpha_2 T_0\right)\varepsilon^3(\varepsilon-1)(\varepsilon-2)}$.

For given $\alpha_2 T_0$ and a = 2.254, the minimum value $M_{c6} = \dfrac{33.29}{\left(1-\alpha_2 T_0\right)}$ attained at $\varepsilon = 0.71$. When $\alpha_2 T_0 = 0$, we note that as ε increases from 0 to 1, M_{c6}

Table 1: Values of critical Marangoni numbers M_{ci} for various values of $\alpha_2 T_0$.

$\alpha_2 T_0$	M_{c1}	M_{c2}	M_{c3}	M_{c4}	M_{c5}	M_{c6}
0.0	66.85	64.26	43.09	52.00	93.59	33.29
0.1	74.28	71.40	47.88	57.78	103.99	36.99
0.2	95.50	91.80	61.56	74.29	133.70	47.56
0.3	133.71	128.52	86.18	104.00	187.18	66.58
0.4	222.84	214.20	143.63	173.33	311.97	110.97
0.5	668.53	642.60	430.90	520.00	935.90	332.90

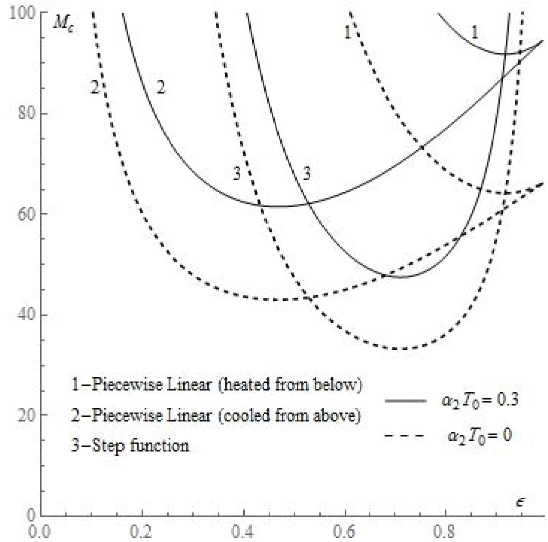

Figure 3: Critical Marangoni number as a function of thermal depth ε when $\alpha_2 T_0 = 0, 3$.

decreases from ∞ to a minimum value 33.29 at $\varepsilon = 0.71$ and then increases for increasing values of ε from 0.71 to 1.

Values of M_{ci} (i = 1 to 6) computed for various values of $\alpha_2 T_0$ are tabulated in Table 1. The critical Marangoni number as a function of thermal depth ε are plotted in Figure 3 for fixed values of $\alpha_2 T_0$ (when a = 2.254). Figure 3 shows that as ε increases from 0 to 1, the critical Marangoni number first decreases, attains a minimum and then increases.

4.2 The insulating case

In this case, we select the trial functions satisfying the boundary conditions namely, (6a, c) and (8a, b, c) as

$$W = z^2 (1 - z) \tag{21}$$

and the residual from the remaining boundary condition (6b) namely, $D^2 W(1) + a^2 M T(1) = 0$, is included in a residual from differential equations while performing integration by parts. Substitution of trial functions given by (21) into the equation (12), we get

$$M = \frac{4\left(1+\frac{1}{15}a^2+\frac{1}{420}a^4\right)}{\left(1-\alpha_2 T_0\right) < \left(z^2-z^3\right) f(z) >} \tag{22}$$

Since for the fluid layer heated from below the non-uniform temperature gradient $f(z) \geq 0$ for all values of z from 0 to 1. Thus, the expression (22) for M shows that its minimum exists for $a = 0$ and given by

$$M_c = \frac{4}{\left(1-\alpha_2 T_0\right) < \left(z^2-z^3\right) f(z) >} \tag{23}$$

(a) For the linear temperature profile $f(z) = 1$, the critical Marangoni number M_{c1} corresponding to this case, obtained from equation (23) is given by $M_{c1} = \frac{48}{\left(1-\alpha_2 T_0\right)}$. When $\alpha_2 T_0 = 0$ we have $M_{c1} = 48$ which is exactly the same value as that given in [13, 15].

(b) For the piecewise linear temperature profile due to sudden heating from below given by (16). The corresponding critical Marangoni number M_{c2} obtained from equation (23) is given by $M_{c2} = \frac{48}{\left(1-\alpha_2 T_0\right)\left(4\varepsilon^2 - 3\varepsilon^3\right)}$. For given $\alpha_2 T_0$ the minimum value $M_{c2} = \frac{45.56}{\left(1-\alpha_2 T_0\right)}$ is attained at $\varepsilon = 0.89$.

When $\alpha_2 T_0 = 0$, we have $M_{c2} = 45.56$ which is exactly the same value as that obtained in [13].

(c) For the piecewise linear temperature Profile due to sudden cooling from top given by (17), the corresponding critical Marangoni number M_{c3} obtained from equation (23) is given by $M_{c3} = \frac{48}{\left(1-\alpha_2 T_0\right)\left(3\varepsilon^3 - 8\varepsilon^2 + 6\varepsilon\right)}$. For given $\alpha_2 T_0$ the minimum value $M_{c3} = \frac{34.79}{1-\alpha_2 T_0}$ is attained at $\varepsilon = 0.54$.

When $\alpha_2 T_0 = 0$, we have $M_{c3} = 34.79$ which is exactly the same value as that obtained in [13].

(d) For the parabolic temperature profile given by (18), the corresponding critical Marangoni number M_{c4} obtained from equation (23) is given by

$M_{c4} = \dfrac{40}{(1-\alpha_2 T_0)}$. When $\alpha_2 T_0 = 0$, we have $M_{c4} = 40$ which is exactly the same value as that obtained in [13].

(e) For the inverted parabolic temperature profile given by (19), the corresponding critical Marangoni number M_{c5} obtained from equation (23) is given by $M_{c5} = \dfrac{60}{(1-\alpha_2 T_0)}$. When $\alpha_2 T_0 = 0$, we have $M_{c5} = 60$ which is exactly the same value as that obtained in [13].

(f) For the step function temperature profile given by (20), the corresponding critical Marangoni number M_{c6} obtained from equation (23) is given by $M_{c6} = \dfrac{4}{(1-\alpha_2 T_0)(\varepsilon^2 - \varepsilon^3)}$. For given $\alpha_2 T_0$ the minimum value $M_{c6} = \dfrac{27}{1-\alpha_2 T_0}$ is attained at $\varepsilon = 0.67$. When $\alpha_2 T_0 = 0$ we have $M_{c6} = 27$ which is exactly the same value as that obtained in [13].

Table 2: Values of critical Marangoni numbers M_{ci} for various values of $\alpha_2 T_0$.

$\alpha_2 T_0$	M_{c1}	M_{c2}	M_{c3}	M_{c4}	M_{c5}	M_{c6}
0.0	48.00	45.56	34.79	40.00	60.00	27.00
0.1	53.33	50.63	38.66	44.44	66.67	30.00
0.2	60.00	56.95	43.49	50.00	75.00	33.75
0.3	68.57	65.09	49.70	57.14	85.71	38.57
0.4	80.00	75.94	57.99	66.67	100.00	45.00
0.5	96.00	91.13	69.58	80.00	120.00	54.00

In this case, values of M_{ci} ($i = 1$ to 6) computed for various values of $\alpha_2 T_0$ are tabulated in Table 2. The critical Marangoni number as a function of thermal depth ε are plotted in Figure 4 for fixed values of $\alpha_2 T_0$ (when $a = 0$). Figure 4 shows that as ε increases from 0 to 1, the critical Marangoni number first decreases, attains a minimum and then increases.

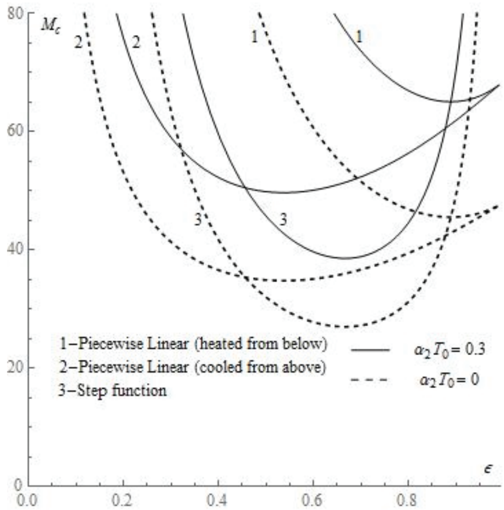

Figure 4: Critical Marangoni number as a function of thermal depth ε when $\alpha_2 T_0 = 0, 3$.

CONCLUSION

For any given *f*(*z*) the critical Marangoni number increase with increasing value of $\alpha_2 T_0$, showing that the relatively hotter layer of liquid is more stable than the cooler one irrespective of whether the lower rigid boundary is thermally conducting or insulating. However, the effect of inverted parabolic profile in a relatively hotter layer of liquid makes the system more stable than any other temperature profile.

REFERENCES

[1] Banerjee, M. B., Gupta, J. R., Shandil, R. G., Sharma, K. C. and Katoch, D. C. A modified analysis of thermal and thermohaline instability of a liquid layer heated underside. Jour. Math. Phy. Sci. **17,** 603-629, (1983).

[2] Bannister, T. C., Grodzka, P. G., Spradley, L. W., Bourgeois, S. V., Heddon, R. O. and Facemire, B. R. Apollo 17 heat flow and convection experiments. Final data analysis results. N.A.S.A. Tech. Memo. X-64772, (1973).

[3] Bénard, H. Les tourbillions cellulaires dans une nappe liquide transportant de la chaleur par convection en régime permanent. Revue générale des Sciences pures at appliqués **11,** 1261-1271, (1900).

[4] Bénard, H. Les tourbillions cellulaires dans une nappe liquide transportant de la chaleur par convection en régime permanent. Annales de. Chimie et de Physique **23,** 62-144, (1901).

[5] Block, M. J. Surface tension as the cause of Bénard cells and surface deformation in a liquid film. Nature **178,** 650-51, (1956).

[6] Chandrasekhar, S. (1981) Hydrodynamic and Hydromagnetic Stability. Dover, New York.

[7] Chiang, K. T. Effect of a non-uniform basic temperature gradient on the onset of Bénard-Marangoni convection: Stationary and oscillatory analysis. International Communications in Heat and Mass Transfer **32(12),** 192-203, (2005).

[8] Finlayson, B.A. (1972) The Method of Weighted Residuals and Variational Principles with Applications in Fluid Mechanics, Heat and Mass Transfer. Academic Press Inc. (London) Ltd., New York.

[9] Gupta, A. K. and Shandil, R. G. Marangoni convection in a relatively hotter or cooler liquid layer. Proc. Natl. Acad. Sci.., India, Sect. A Phys. Sci. **82(2),** 103-106, (2012).

[10] Koschmieder, E. L. Bénard cells and Taylor vortices. Cambridge University Press, Cambridge, (1993).

[11] Lebon, G. and Cloot, A. Effects of non-uniform temperature gradient on Bénard-Marangoni instability. Journal of Non-Equilibrium Thermodynamics, **61(1),** 15-30, (1981).

[12] Nield, D. A. Surface-tension and buoyancy effects in cellular convection. J. Fluid Mechanics **19,** 571-574, (1964).

[13] Nield, D. A. The onset of transient convective instability. J. Fluid Mechanics **71,** 441-454, (1975).

[14] Normand, C. Pomeau, Y and Velarde, M. G. Convective instability: A physicist's approach. Rev. Mod. Phys. **49,**581-624, (1977).

[15] Pearson, J. R. A. "On convection cells induced by surface tension", J. Fluid Mech., **4,** 489-500, (1958).

[16] Rayleigh, L. On convection currents in a horizontal layer of fluid, when the higher temperature is underside. Phil. Mag. **32,** 529-546, (1916).

[17] Rudraiah, N and Siddheshwar, P.G. Effect of non-uniform basic-temperature gradient on the onset of Marangoni convection in a fluid with suspended particles. Aerospace Science and Technology **4(8),** 517-523, (2000).

[18] Schatz, M., Vanhook, S., Mccormick, W., Swift, J. and Swinney, H. Onset of surface-tension-driven Bénard convection. Phys. Rev. Lett. **75,** (1938-1941), (1995).