

# Fibonacci and $k$ Lucas Sequences as Series of Fractions

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**Abstract** In this paper, we defined new relationship between  $k$  Fibonacci and  $k$  Lucas sequences using continued fractions and series of fractions, this approach is different and never tried in  $k$  Fibonacci sequence literature.

**Keywords:**  $k$ -Fibonacci sequence,  $k$ -Lucas sequence, Recurrence relation

**Mathematics Subject Classification:** 11B39, 11B83

## 1 INTRODUCTION

The Fibonacci sequence is a source of many nice and interesting identities. Many identities have been documented in [2], [3], [10], [11], [12], [13], [17]. A similar interpretation exists for  $k$  Fibonacci and  $k$  Lucas numbers. Many of these identities have been documented in the work of Falcon and Plaza[1], [4], [5], [7], [8], [9], where they are proved by algebraic means. In this paper, we obtained some new properties for  $k$  Fibonacci and  $k$  Lucas sequences using series of fraction.

## 2 PRELIMINARY

**Definition 2.1.** The  $k$ -Fibonacci sequence  $\{F_{k,n}\}_{n=1}^{\infty}$  is defined as,  $F_{k,n+1} = k \cdot F_{k,n} + F_{k,n-1}$ , with  $F_{k,0} = 0, F_{k,1} = 1$ , for  $n \geq 1$

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**Definition 2.2.** The  $k$ -Lucas sequence  $\{L_{k,n}\}_{n=1}^{\infty}$  is defined as,  $L_{k,n+1} = k \cdot L_{k,n} + L_{k,n-1}$ , with  $L_{k,0} = 2, L_{k,1} = k$ , for  $n \geq 1$

Characteristic equation of the initial recurrence relation is,

$$r^2 - k \cdot r - 1 = 0 \quad (1)$$

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Characteristic roots are

$$r_1 = \frac{k + \sqrt{k^2 + 4}}{2} \quad (2)$$

and

$$r_2 = \frac{k - \sqrt{k^2 + 4}}{2} \quad (3)$$

Characteristic roots verify the properties

$$r_1 - r_2 = \sqrt{k^2 + 4} = \sqrt{\Delta} = \delta \quad (4)$$

$$r_1 + r_2 = k \quad (5)$$

$$r_1 \cdot r_2 = -1 \quad (6)$$

Binet forms for  $F_{k,n}$  and  $L_{k,n}$  are

$$F_{k,n} = \frac{r_1^n - r_2^n}{r_1 - r_2} \quad (7)$$

and

$$L_{k,n} = r_1^n + r_2^n \quad (8)$$

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## 2.1 The first few members of this $k$ Fibonacci family are

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$$1,$$

$$k,$$

$$1 + k^2,$$

$$2k + k^3,$$

$$1 + 3k^2 + k^4,$$

$$3k + 4k^3 + k^5,$$

$$1 + 6k^2 + 5k^4 + k^6,$$

$$4k + 10k^3 + 6k^5 + k^7,$$

$$1 + 10k^2 + 15k^4 + 7k^6 + k^8,$$

$$5k + 20k^3 + 21k^5 + 8k^7 + k^9,$$

$$1 + 15k^2 + 35k^4 + 28k^6 + 9k^8 + k^{10},$$

$$6k + 35k^3 + 56k^5 + 36k^7 + 10k^9 + k^{11},$$

$$1 + 21k^2 + 70k^4 + 84k^6 + 45k^8 + 11k^{10} + k^{12},$$

$$7k + 56k^3 + 126k^5 + 120k^7 + 55k^9 + 12k^{11} + k^{13},$$

$$1 + 28k^2 + 126k^4 + 210k^6 + 165k^8 + 66k^{10} + 13k^{12} + k^{14},$$

$$8k + 84k^3 + 252k^5 + 330k^7 + 220k^9 + 78k^{11} + 14k^{13} + k^{15},$$

$$1 + 36k^2 + 210k^4 + 462k^6 + 495k^8 + 286k^{10} + 91k^{12} + 15k^{14} + k^{16},$$

$$9k + 120k^3 + 462k^5 + 792k^7 + 715k^9 + 364k^{11} + 105k^{13} + 16k^{15} + k^{17},$$

$$1 + 45k^2 + 330k^4 + 924k^6 + 1287k^8 + 1001k^{10} + 455k^{12} + 120k^{14} + 17k^{16} + k^{18},$$

$$10k + 165k^3 + 792k^5 + 1716k^7 + 2002k^9 + 1365k^{11} + 560k^{13} + 136k^{15} + 18k^{17} + k^{19}$$

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**2.2 k Fibonacci sequences in Encyclopaedia of Integer Sequences**

$F_{k,n}$	Classification
$F_{1,n}$	A000045
$F_{2,n}$	A000129
$F_{3,n}$	A006190
$F_{4,n}$	A001076
$F_{5,n}$	A052918
$F_{6,n}$	A005668
$F_{7,n}$	A054413
$F_{8,n}$	A041025
$F_{9,n}$	A099371
$F_{10,n}$	A041041
$F_{11,n}$	A049666

**3 RELATIONSHIP OF THE SEQUENCES  $F_{K,N}$  AND  $L_{K,N}$  AS CONTINUED FRACTIONS:**

In general, a (simple) continued fraction is an expression of the form

$$[a_0, a_1, \dots, a_n] = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \frac{1}{a_4 + \frac{1}{a_5 + \frac{1}{a_6 + \dots}}}}}} \quad (9)$$

The letters  $a_1, a_2, \dots$  denote positive integers. The letter  $a_0$  denotes an integer.

The expansion  $\frac{F_{k,n+1}}{F_{k,n}}$  in continued fraction is written as

$$\frac{F_{k,n+1}}{F_{k,n}} = k + \frac{1}{k + \frac{1}{k + \frac{1}{k + \frac{1}{k + \frac{1}{k + \dots}}}}} \quad (10)$$

Here  $n$  denotes the number of quantities equal to  $k$ .

We knew that

$$F_{k,n}^2 - F_{k,n-1}F_{k,n+1} = (-1)^{n-1} \quad (11)$$

Moreover, in general we have

$$\frac{F_{k,n+1}}{F_{k,n}} = r_1 \frac{1 - \left(\frac{r_2}{r_1}\right)^{n+1}}{1 - \left(\frac{r_2}{r_1}\right)^n}$$

Let,  $r_1$  denote the larger of the root, we have

$$\lim_{n \rightarrow \infty} \frac{F_{k,n+1}}{F_{k,n}} = r_1$$

More generally formula (9) is written as

$$\frac{F_{k,(n+1)t}}{F_{k,nt}} = L_{k,t} - \frac{(-1)^t}{L_{k,t} - \frac{(-1)^t}{L_{k,t} - \frac{(-1)^t}{L_{k,t} - \frac{(-1)^t}{L_{k,t} - \frac{(-1)^t}{L_{k,t} - \frac{(-1)^t}{L_{k,t} - \dots}}}}}} \quad (12)$$

Here,  $n$  denotes the number of  $L_{k,t}$ 's  
When  $n$  increases indefinitely, we have

$$\lim_{n \rightarrow \infty} \frac{F_{k,(n+1)t}}{F_{k,nt}} = (r_1)^t$$

As equation (9), we have relation for  $L_{k,n}$

$$\frac{L_{k,(n)t}}{F_{k,(n-1)t}} = L_{k,t} - \frac{(-1)^t}{L_{k,t} - \frac{(-1)^t}{L_{k,t} - \frac{(-1)^t}{L_{k,t} - \frac{(-1)^t}{L_{k,t} - \frac{(-1)^t}{L_{k,t} - \frac{(-1)^t}{L_{k,t} - \dots}}}}}} \quad (13)$$

Here  $n$  denotes the number of quantities equal to  $L_{k,t}$ .  
We knew that

$$L_{k,n}^2 - L_{k,n-1}L_{k,n+1} = (-1)^n \Delta \quad (14)$$

More generally, equations (10) and (13) are modified as

$$F_{k,nt}^2 - F_{k,(n-1)t}F_{k,(n+1)t} = (-1)^{(n-1)t} (F_{k,t})^2 \quad (15)$$

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$$L_{k,nt}^2 - L_{k,(n-1)t}L_{k,(n+1)t} = -(-1)^{(n-1)t} \Delta(F_{k,t})^2 \quad (16)$$

Moreover using (7) and (8), gives

$$\Delta F_{k,nt}^2 = r_1^{2n+2t} + r_2^{2n+2t} - 2(-1)^{n+t} \quad (17)$$

$$\Delta L_{k,n}^2 = r_1^{2n} + r_2^{2n} - 2(-1)^n \quad (18)$$

Again by subtracting (16) and (17), gives

$$\Delta(F_{k,n+t}^2 - (-1)^t F_{k,n}^2) = (r_1^{2n+t} - r_2^{2n+t})(r_1^t + r_2^t) \quad (19)$$

and

$$F_{k,n+t}^2 - (-1)^t F_{k,n}^2 = F_{k,t}F_{k,2n+t} \quad (20)$$

Similarly, we obtain

$$L_{k,n+t}^2 - (-1)^t L_{k,n}^2 = \Delta F_{k,t}F_{k,2n+t} \quad (21)$$

#### 4 SEQUENCES $F_{K,N}$ AND $L_{K,N}$ AS A SERIES OF FRACTIONS:

**Theorem 4.1.** For  $n, k > 0$ ,

1.

$$\frac{F_{k,n+1}}{F_{k,n}} = \frac{F_{k,2}}{F_{k,1}} - \frac{(-1)}{F_{k,1}F_{k,2}} - \frac{(-1)^2}{F_{k,2}F_{k,3}} - \frac{(-1)^3}{F_{k,3}F_{k,4}} - \dots - \frac{(-1)^{n-1}}{F_{k,n-1}F_{k,n}}$$

2.

$$\frac{L_{k,n+1}}{L_{k,n}} = \frac{L_{k,2}}{L_{k,1}} - \frac{(-1)^2 \Delta}{L_{k,2}L_{k,1}} - \frac{(-1)^3 \Delta}{L_{k,2}L_{k,3}} - \frac{(-1)^4 \Delta}{L_{k,3}L_{k,4}} - \dots - \frac{(-1)^n}{L_{k,n-1}L_{k,n}}$$

*Proof.* We can write expressions of  $\frac{F_{k,n+1}}{F_{k,n}}$  and  $\frac{L_{k,n+1}}{L_{k,n}}$  in series as

$$\begin{aligned} \frac{F_{k,n+1}}{F_{k,n}} &= \frac{F_{k,2}}{F_{k,1}} + \left( \frac{F_{k,3}}{F_{k,2}} - \frac{F_{k,2}}{F_{k,1}} \right) + \left( \frac{F_{k,4}}{F_{k,3}} - \frac{F_{k,3}}{F_{k,2}} \right) + \dots \\ &\quad + \left( \frac{F_{k,n+1}}{F_{k,n}} - \frac{F_{k,n}}{F_{k,n-1}} \right) \\ &= \frac{F_{k,2}}{F_{k,1}} - \frac{(F_{k,2}^2 - F_{k,3}F_{k,1})}{F_{k,1}F_{k,2}} - \frac{(F_{k,3}^2 - F_{k,2}F_{k,4})}{F_{k,2}F_{k,3}} - \dots \\ &\quad - \frac{(F_{k,n}^2 - F_{k,n+1}F_{k,n-1})}{F_{k,n-1}F_{k,n}} \end{aligned}$$

And

$$\begin{aligned} \frac{L_{k,n+1}}{L_{k,n}} &= \frac{L_{k,2}}{L_{k,1}} + \left( \frac{L_{k,3}}{L_{k,2}} - \frac{L_{k,2}}{L_{k,1}} \right) + \left( \frac{L_{k,4}}{L_{k,3}} - \frac{L_{k,3}}{L_{k,2}} \right) + \dots \\ &\quad + \left( \frac{L_{k,n+1}}{L_{k,n}} - \frac{L_{k,n}}{L_{k,n-1}} \right) \\ &= \frac{L_{k,2}}{L_{k,1}} - \frac{(L_{k,2}^2 - L_{k,3}L_{k,1})}{L_{k,1}L_{k,2}} - \frac{(L_{k,3}^2 - L_{k,2}L_{k,4})}{L_{k,2}L_{k,3}} - \dots \\ &\quad - \frac{(L_{k,n}^2 - L_{k,n+1}L_{k,n-1})}{L_{k,n-1}L_{k,n}} \end{aligned}$$

Using the equations (11) and (14)

$$F_{k,n}^2 - F_{k,n-1}F_{k,n+1} = (-1)^{n-1}$$

$$L_{k,n}^2 - L_{k,n-1}L_{k,n+1} = (-1)^n \Delta$$

Gives

$$\frac{F_{k,n+1}}{F_{k,n}} = \frac{F_{k,2}}{F_{k,1}} - \frac{(-1)}{F_{k,1}F_{k,2}} - \frac{(-1)^2}{F_{k,2}F_{k,3}} - \frac{(-1)^3}{F_{k,3}F_{k,4}} - \dots - \frac{(-1)^{n-1}}{F_{k,n-1}F_{k,n}}$$

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And

$$\frac{L_{k,n+1}}{L_{k,n}} = \frac{L_{k,2}}{L_{k,1}} - \frac{(-1)^2 \Delta}{L_{k,2}L_{k,1}} - \frac{(-1)^3 \Delta}{L_{k,2}L_{k,3}} - \frac{(-1)^4 \Delta}{L_{k,3}L_{k,4}} - \dots - \frac{(-1)^n}{L_{k,n-1}L_{k,n}}$$

□

Taking limit as  $\lim_{n \rightarrow \infty}$ , gives

$$r_1 = \frac{1 + \sqrt{k^2 + 4}}{2} = k + \frac{1}{1.k} - \frac{1}{k.(k^2 + 4)} + \dots$$

For Fibonacci series

$$\frac{1 + \sqrt{5}}{2} = 1 + \frac{1}{1.1} - \frac{1}{1.2} + \frac{1}{2.3} - \frac{1}{3.5} + \frac{1}{5.8} - \frac{1}{8.13} + \dots$$

Now, we obtain more general relation for  $F_{k,n}$  and  $L_{k,n}$  as a series of fractions:-

**Theorem 4.2.** For  $n, k > 0$ ,

1.

$$\begin{aligned} \frac{F_{k,(n+1)t}}{F_{k,nt}} &= \frac{F_{k,2t}}{F_{k,t}} - \frac{(-1)^t F_{k,t}^2}{F_{k,t}F_{k,2t}} - \frac{(-1)^{2t} F_{k,t}^2}{F_{k,2t}F_{k,3t}} - \frac{(-1)^{3t} F_{k,t}^2}{F_{k,3t}F_{k,4t}} - \dots \\ &\quad - \frac{(-1)^{(n-1)t} F_{k,t}^2}{F_{k,(n-1)t}F_{k,nt}} \end{aligned}$$

2.

$$\begin{aligned} \frac{L_{k,(n+1)t}}{L_{k,nt}} &= \frac{L_{k,t}}{L_{k,0}} + \frac{\Delta F_{k,t}^2}{L_{k,0}L_{k,t}} + \frac{(-1)^t F_{k,t}^2}{L_{k,t}L_{k,2t}} + \frac{(-1)^{2t} F_{k,t}^2}{L_{k,2t}L_{k,3t}} + \dots \\ &\quad - \frac{(-1)^{(n-1)t} F_{k,t}^2}{L_{k,(n-1)t}L_{k,nt}} \end{aligned}$$



*Proof.* We can write expressions of  $\frac{F_{k,(n+1)t}}{F_{k,nt}}$  and  $\frac{L_{k,(n+1)t}}{L_{k,nt}}$  in series as

$$\begin{aligned} \frac{F_{k,(n+1)t}}{F_{k,nt}} &= \frac{F_{k,2t}}{F_{k,t}} + \left( \frac{F_{k,3t}}{F_{k,2t}} - \frac{F_{k,2t}}{F_{k,t}} \right) + \left( \frac{F_{k,4t}}{F_{k,3t}} - \frac{F_{k,3t}}{F_{k,2t}} \right) + \dots \\ &\quad + \left( \frac{F_{k,(n+1)t}}{F_{k,nt}} - \frac{F_{k,nt}}{F_{k,(n-1)t}} \right) \\ &= \frac{F_{k,2t}}{F_{k,t}} - \frac{(F_{k,2t}^2 - F_{k,3t}F_{k,t})}{F_{k,t}F_{k,2t}} - \frac{(F_{k,3t}^2 - F_{k,2t}F_{k,4t})}{F_{k,2t}F_{k,3t}} - \dots \\ &\quad - \frac{(F_{k,nt}^2 - F_{k,(n+1)t}F_{k,(n-1)t})}{F_{k,(n-1)t}F_{k,nt}} \end{aligned}$$

And

$$\begin{aligned} \frac{L_{k,(n+1)t}}{L_{k,nt}} &= \frac{L_{k,t}}{L_{k,0}} + \left( \frac{L_{k,2t}}{L_{k,t}} - \frac{L_{k,t}}{L_{k,0}} \right) + \left( \frac{L_{k,3t}}{L_{k,2t}} - \frac{L_{k,2t}}{L_{k,t}} \right) + \dots \\ &\quad + \left( \frac{L_{k,(n+1)t}}{L_{k,nt}} - \frac{L_{k,nt}}{L_{k,(n-1)t}} \right) \\ &= \frac{L_{k,t}}{L_{k,0}} - \frac{(L_{k,t}^2 - L_{k,0}L_{k,2t})}{L_{k,0}L_{k,t}} - \frac{(L_{k,2t}^2 - L_{k,t}L_{k,3t})}{L_{k,t}L_{k,2t}} - \dots \\ &\quad - \frac{(L_{k,nt}^2 - L_{k,(n+1)t}L_{k,(n-1)t})}{L_{k,(n-1)t}L_{k,nt}} \end{aligned}$$

Using the equations (15) and (16)

$$F_{k,nt}^2 - F_{k,(n-1)t}F_{k,(n+1)t} = (-1)^{(n-1)t}(F_{k,t})^2 \quad (22)$$

$$L_{k,nt}^2 - L_{k,(n-1)t}L_{k,(n+1)t} = -(-1)^{(n-1)t}\Delta(F_{k,t})^2 \quad (23)$$

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Gives

$$\frac{F_{k,(n+1)t}}{F_{k,nt}} = \frac{F_{k,2t}}{F_{k,t}} - \frac{(-1)^t F_{k,t}^2}{F_{k,t} F_{k,2t}} - \frac{(-1)^{2t} F_{k,t}^2}{F_{k,2t} F_{k,3t}} - \frac{(-1)^{3t} F_{k,t}^2}{F_{k,3t} F_{k,4t}} - \dots$$

$$- \frac{(-1)^{(n-1)t} F_{k,t}^2}{F_{k,(n-1)t} F_{k,nt}}$$

And

$$\frac{L_{k,(n+1)t}}{L_{k,nt}} = \frac{L_{k,t}}{L_{k,0}} + \frac{\Delta F_{k,t}^2}{L_{k,0} L_{k,t}} + \frac{(-1)^t F_{k,t}^2}{L_{k,t} L_{k,2t}} + \frac{(-1)^{2t} F_{k,t}^2}{L_{k,2t} L_{k,3t}} + \dots - \frac{(-1)^{(n-1)t} F_{k,t}^2}{L_{k,(n-1)t} L_{k,nt}}$$

□

**Theorem 4.3.** For  $n, m, k > 0$ ,

1.

$$\frac{F_{k,n+mt}}{L_{k,n+mt}} = \frac{F_{k,n}}{L_{k,n}} + 2(-1)^n F_{k,t} \left[ \frac{1}{L_{k,n} L_{k,n+t}} + \frac{(-1)^t}{l_{k,n+t} L_{k,n+2t}} + \frac{(-1)^{2t}}{L_{k,n+2t} L_{k,n+3t}} \right.$$

$$\left. + \dots + \frac{(-1)^{(m-1)t}}{L_{k,n+(m-1)t} L_{k,n+mt}} \right]$$

2.

$$\frac{F_{k,n+mt}}{L_{k,n+mt}} = \frac{F_{k,n}}{L_{k,n}} + 2(-1)^n F_{k,t} \left[ \frac{1}{L_{k,n} L_{k,n+t}} + \frac{(-1)^t}{l_{k,n+t} L_{k,n+2t}} + \frac{(-1)^{2t}}{L_{k,n+2t} L_{k,n+3t}} \right.$$

$$\left. + \dots + \frac{(-1)^{(m-1)t}}{L_{k,n+(m-1)t} L_{k,n+mt}} \right]$$

*Proof.* We can write expressions of  $\frac{F_{k,n+mt}}{L_{k,n+mt}}$  and  $\frac{L_{k,n+mt}}{L_{k,n+mt}}$  in series as

$$\begin{aligned} \frac{F_{k,n+mt}}{L_{k,n+mt}} &= \frac{F_{k,n}}{L_{k,n}} + \left( \frac{F_{k,n+t}}{L_{k,n+t}} - \frac{F_{k,n}}{L_{k,n}} \right) + \left( \frac{F_{k,n+2t}}{L_{k,n+2t}} - \frac{F_{k,n+t}}{L_{k,n+t}} \right) + \dots \\ &\quad + \left( \frac{F_{k,n+mt}}{L_{k,n+mt}} - \frac{F_{k,n+(m-1)t}}{L_{k,n+(m-1)t}} \right) \\ &= \frac{F_{k,n}}{L_{k,n}} + \frac{(F_{k,n+t}L_{k,n} - F_{k,n}L_{k,n+t})}{L_{k,n}L_{k,n+t}} + \frac{(F_{k,n+2t}L_{k,n+t} - F_{k,n+t}L_{k,n+2t})}{L_{k,n+t}L_{k,n+2t}} \\ &\quad + \dots + \frac{(F_{k,n+mt}L_{k,n+(m-1)t} - F_{k,n+(m-1)t}L_{k,n+mt})}{L_{k,n}L_{k,n+(m-1)t}} \end{aligned}$$

And

$$\begin{aligned} \frac{L_{k,n+mt}}{F_{k,n+mt}} &= \frac{L_{k,n}}{F_{k,n}} + \left( \frac{L_{k,n+t}}{F_{k,n+t}} - \frac{L_{k,n}}{F_{k,n}} \right) + \left( \frac{L_{k,n+2t}}{F_{k,n+2t}} - \frac{L_{k,n+t}}{F_{k,n+t}} \right) \\ &\quad + \dots + \left( \frac{L_{k,n+mt}}{F_{k,n+mt}} - \frac{L_{k,n+(m-1)t}}{F_{k,n+(m-1)t}} \right) \\ &= \frac{L_{k,n}}{F_{k,n}} - \frac{(F_{k,n+t}L_{k,n} - F_{k,n}L_{k,n+t})}{L_{k,n}L_{k,n+t}} - \frac{(F_{k,n+2t}L_{k,n+t} - F_{k,n+t}L_{k,n+2t})}{L_{k,n+t}L_{k,n+2t}} \\ &\quad + \dots - \frac{(F_{k,n+mt}L_{k,n+(m-1)t} - F_{k,n+(m-1)t}L_{k,n+mt})}{L_{k,n}L_{k,n+(m-1)t}} \end{aligned}$$

Using the equations (15) and (16)

$$\begin{aligned} F_{k,nt}^2 - F_{k,(n-1)t}F_{k,(n+1)t} &= (-1)^{(n-1)t}(F_{k,t})^2 \\ L_{k,nt}^2 - L_{k,(n-1)t}L_{k,(n+1)t} &= -(-1)^{(n-1)t}\Delta(F_{k,t})^2 \end{aligned}$$

Gives

$$\begin{aligned} \frac{F_{k,n+mt}}{L_{k,n+mt}} &= \frac{F_{k,n}}{L_{k,n}} + 2(-1)^n F_{k,t} \left[ \frac{1}{L_{k,n}L_{k,n+t}} + \frac{(-1)^t}{L_{k,n+t}L_{k,n+2t}} + \frac{(-1)^{2t}}{L_{k,n+2t}L_{k,n+3t}} \right. \\ &\quad \left. + \dots + \frac{(-1)^{(m-1)t}}{L_{k,n+(m-1)t}L_{k,n+mt}} \right] \end{aligned}$$

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And

$$\frac{F_{k,n+mt}}{L_{k,n+mt}} = \frac{F_{k,n}}{L_{k,n}} + 2(-1)^n F_{k,t} \left[ \frac{1}{L_{k,n}L_{k,n+t}} + \frac{(-1)^t}{L_{k,n+t}L_{k,n+2t}} + \frac{(-1)^{2t}}{L_{k,n+2t}L_{k,n+3t}} \right. \\ \left. + \dots + \frac{(-1)^{(m-1)t}}{L_{k,n+(m-1)t}L_{k,n+mt}} \right]$$

□

## 5 CONCLUSIONS

Some new relationship between  $k$  Fibonacci and  $k$  Lucas sequences using continued fractions and series of fractions are derived, this approach is different and never tried in  $k$  Fibonacci sequence literature.

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Fibonacci and  $k$   
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