DOI: 10.15415/mjis.2016.51004

A Note on Game Theoretic Approach to Detect Arbitrage Strategy: Application in the Foreign Exchange Market

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Received: June 12, 2016| Revised: July 19, 2016| Accepted: August 24, 2016

Published online: September 05, 2016

Abstract A game theoretic approach to detect arbitrage strategy in a foreign exchange market is proposed. Five propositions are given and then the optimal arbitrage path is derived.

Keywords: Arbitrage strategy; Exchange rate matrix, Foreign exchange market; Game theory

1. INTRODUCTION

Consider a foreign exchange market with n currencies. Let a_{ij} be the exchange rate of i-th currency with respect to j-th currency. Denote the exchange rate matrix, by $A=\left(a_{ij}\right)$. Under no arbitrage assumption, then the direct, triangular, and any other types of arbitrage don't exist. Ma (2008) detected arbitrage opportunities in a foreign exchange market. He used maximum eigenvalue of the exchange rates matrix. Ma (2008) argued that Λ is an arbitrage indicator. When there is an arbitrage, then $a_{ij}=\frac{w_i}{w_j}\varepsilon_{ij}$ for some i, j, where w_i is the intrinsic value of i-th currency. Hao (2009) detected the applied the method of Ma's foreign exchange market considering the bid-ask transactions. The following propositions are useful.

Proposition 1. If direct arbitrage doesn't exist, then $a_{ij}=\frac{1}{a_{ji}}, b_{ij}+b_{ji}=0$, where $b_{ij}=\log\left(a_{ij}\right)$. Considering b_{ij} as a game, then it is a sum zero game.

Proposition 2. When there is no triangular arbitrage, then $a_{ik}a_{kj}=a_{ij}$ then

$$A_{ij}^2=\sum_{k=1}^{n}a_{ik}a_{kj}=na_{ij}=nA_{ij}.$$

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Therefore, $A^3 = n^2 A, \dots A^k = n^{k-1} A.$

Remark 1. Under the no direct and triangular arbitrage opportunities, then

$$\sum_{k=1}^{n}\left(a_{ik}a_{kj} - a_{ij}\right) = 0 \text{ and } \sum_{g=1}^{n}\sum_{k=1}^{n}\left(a_{ig}a_{gk}a_{kj} - a_{ij}\right) = 0.$$

Remark 2. The arbitrage path is obtained for some m such that $A^m \neq n^{m-1}A.$ Hence, some relations like them exist in Remark 1 exist and show the arbitrage path (strategy).

Proposition 3. Under the no arbitrage assumption, $A^k = n^{k-1}A$ then for some $h \in \left(-\delta, \delta\right)$, the moment generating function of matrix A.

$$M_A\left(h\right) = M\left(h\right) = e^{hA} = \sum_{j=0}^{\infty}\frac{\left(hA\right)^j}{j!} = I + \frac{e^{nh}-1}{n}A$$

where I is n*n identity matrix and

$$\frac{\partial^k M\left(h\right)}{\partial h^k}\Big|_{n=0} = A^k, \quad k = 1,2,\dots$$

Proposition 4. If there is triangular arbitrage, an optimal weighted arbitrage path with weights $\pi_k, k = 1,2,\dots$ is obtained by

$$\max\sum_{k=1}^{n}\pi_k\left(a_{ik}a_{kj} - a_{ij}\right),$$

such that $\sum_{k=1}^{n}\pi_k = 1$ where $\pi_k = 1$ for $k = \arg\max_{k^*}(a_{ik^*}a_{k^*j} - a_{ij})$ and zero otherwise.

2 Game theoretic approach. It is well-known that there is an equivalence between linear programming (LP) problems and zero-sum games in the sense that any two-person zero-sum game can convert to an LP problem and vice-versa (see, Raghavan,1994). A natural question arises is which row of A should be selected? Here, based on game theory solution is proposed. That is, a randomized strategy $\left\{q_j\right\}_{j=1}^{n}$ (probability measure) is designed such that for each row the expectation of investor's profit is maximized. The necessary condition to this end is $\sum_{j=1}^{n} q_j a_{1j} = \cdots \sum_{j=1}^{n} q_j a_{nj}$, where a matrix equivalent form is $Aq = a^{*}1$ for some arbitrary a^{*} where q is the vector of probabilities. When there is an arbitrage opportunity, then the largest eigenvalue is n and others are negative.

If all of them are non-zero, then the inverse of matrix A exists and

$$q = a^{**}A^{-1}$$

Proposition 5. If there is triangular arbitrage, an optimal weighted arbitrage path with weights $\pi_k, k = 1,2,\ldots$ is obtained by

$$\max_{\pi_k} \sum_{k=1}^{n} \pi_k \left(\varepsilon_{ik}\varepsilon_{kj} - \varepsilon_{ij}\right)$$

Such that $\sum_{k=1}^{n} \pi_k = 1$, where $\pi_k = 1$ for $k = \arg\max_{k^*}\left(\varepsilon_{ik^*}\varepsilon_{k^*j} - \varepsilon_{ij}\right)$ and zero otherwise.

Remark 3. The algorithm for finding the arbitrage path is given as follows

1. Find i_1 as described ib above.
2. Find $k_1^* = \arg\max_{k^*}\left(\varepsilon_{ik^*}\varepsilon_{k^*j} - \varepsilon_{ij}\right)$ and let $i_2 = k_1^*$.
3. Continue to find time points $i_3, \ldots, i_m$.
4. Compute $a_{i_1 i_2} a_{i_2 i_3} \cdots a_{i(m-1) i_m} a_{i_m i_1}$.
5. Set $i_1 = 2$ and repeat steps 2,3,4.
6. Find the maximum of $a_{i_1 i_2} a_{i_2 i_3} \cdots a_{i(m-1) i_m} a_{i_m i_1}$. It is the optimal arbitrage path.

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