



## Duplicating a Vertex with an Edge in Divided Square Difference Cordial Graphs

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## ABSTRACT

In this present work, we discuss divided square difference (DSD) cordial labeling in the context of duplicating a vertex with an edge in DSD cordial graphs such as path graph, cycle graph, star graph, wheel graph, helm graph, crown graph, comb graph and snake graph.

## Keywords:

Duplication of a vertex by an edge, path, cycle graph, star graph, wheel graph, helm graph, crown graph, comb graph, snake graph.

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### 1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges. For basic definitions we refer Harary [8]. In 1967, Rosa [10] introduced a labeling of  $G$  called  $\beta$ -valuation. A dynamic survey on different graph labeling was found in Gallian [7]. Cordial labeling was introduced by Cahit [5]. R.Varatharajan, *et. al* [11] have introduced the notion of divisor cordial labeling. A. Alfred Leo *et.al* [1] introduced divided square difference cordial labeling graphs. V.J. Kaneria *et. al* [9] introduced balanced cordial labeling. The motivation behind the divided square difference cordial labeling is due to R. Dhavaseelan *et.al* on their work even sum cordial labeling graphs [6]. The motivation behind this article is due to S.K. Vaidya *et.al* on their work [12]. In this present work, we discuss divided square difference (DSD) cordial labeling in the context of duplication of a vertex by an edge in DSD cordial graphs such as path graph, cycle graph, star graph, wheel graph, helm graph, bistar graph, crown graph, comb graph and snake graph.

### 2. Preliminaries

#### Definition 2.1 [7]

Graph labeling is an assignment of numbers to the edges or vertices or both subject to certain condition(s).

#### Definition 2.2 [5]

A binary vertex labeling  $f$  of a graph  $G$  is called a *Cordial labeling* if  $|v_f(0) - v_f(1)| \leq 1$  and  $|e_f(0) - e_f(1)| \leq 1$ .

A graph  $G$  is cordial if it admits cordial labeling.

#### Definition 2.3 [9]

A cordial graph  $G$  with a cordial labeling  $f$  is called a *balanced cordial graph* if

$$|e_f(0) - e_f(1)| = |v_f(0) - v_f(1)| = 0.$$

It is said to be *edge balanced cordial graph* if

$$|e_f(0) - e_f(1)| = 0 \text{ and } |v_f(0) - v_f(1)| = 1.$$

Similarly it is said to be *vertex balanced cordial graph* if

$$|e_f(0) - e_f(1)| = 1 \text{ and } |v_f(0) - v_f(1)| = 0.$$

A cordial graph  $G$  is said to be *unbalanced cordial graph* if

$$|e_f(0) - e_f(1)| = |v_f(0) - v_f(1)| = 1.$$

**Definition 2.4 [1]**

Let  $G=(V,E)$  be a simple graph and  $f : V \rightarrow \{1,2,3,\dots,|V|\}$  be a bijection. For each edge, assign the label 1 if  $\left| \frac{(f(u))^2 - (f(v))^2}{f(u) - f(v)} \right|$  is odd and the label 0 otherwise.  $f$  is called divided square difference cordial labeling if  $|e_f(0) - e_f(1)| \leq 1$ , where  $e_f(1)$  is number of edges labeled with 1 and  $e_f(0)$  is number of edges labeled with 0.

A graph  $G$  is called DSD cordial if it admits DSD cordial labeling.

**Definition 2.5 [4]**

A divided square difference cordial graph  $G$  is called a *balanced DSD graph* if  $|e_f(0) - e_f(1)| = 0$ .

A divided square difference cordial graph  $G$  is called a *unbalanced DSD cordial graph* if  $|e_f(0) - e_f(1)| = 1$ .

**Proposition 2.6 [1]**

1. Any path  $P_n$  is a DSD cordial graph.
2. Any cycle  $C_n$  is a DSD cordial graph except  $n \equiv 2(mod 4)$ .
3. The star graph  $K_{1,n}$  is a DSD cordial.

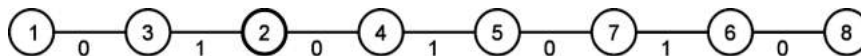
**Proposition 2.7 [2]**

1. The wheel graph  $W_n, (n \equiv 0,1(mod 4))$  is DSD cordial
2. The helm graph  $H_n, (n \equiv 0,1(mod 4))$  is DSD cordial.

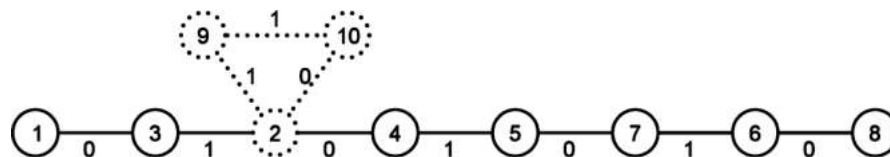
**Proposition 2.8 [3]**

1. The crown graph  $C_n \odot K_1$  is DSD cordial.
2. The comb graph  $P_n \odot K_1$  is DSD cordial.

**Example 3.4**



**Figure 1(a)** Unbalanced DSD cordial graph  $G(P_8)$



**Figure 1(b)** Balanced DSD cordial graph  $G'$  (by duplicating).

**Proposition 2.9 [4]**

The triangular snake graph  $T_n$  (except  $n \equiv 3(mod 4)$ ) is a balanced DSD cordial when  $n$  is odd.

**3. Main Results**

**Proposition 3.1**

A graph got by duplicating a vertex  $v_k$  with an edge  $e' (= u'v')$  in a DSD cordial path  $P_n$  (except  $n \equiv 2(mod 4)$ ) is DSD cordial.

**Proof**

Let  $G$  be a path graph  $P_n$  (except  $n \equiv 2(mod 4)$ ). By Proposition 2.6, we draw a DSD cordial path  $P_n$ . Now, we duplicate any of the vertex  $v_k$  in  $G$  with an edge  $e' = u'v'$  and construct a new graph  $G'$ . In this graph  $G', |V(G')| = n + 2$  and  $|E(G')| = n + 2$ . For DSD cordial labeling pattern, let the vertex labels are  $\{1, 2, \dots, n + 2\}$ . Then, by labeling  $u'$  and  $v'$  by  $f(u') = n + 1$  and  $f(v') = n + 2$ , we get  $|e_f(0) - e_f(1)| \leq 1$ .

Hence  $G'$  is also a DSD cordial.

**Remark 3.2**

From Proposition 3.1, in particular we get  $|e_f(0) - e_f(1)| = 0$  when  $n$  is even.

Hence, we can conclude that  $G'$  is a balanced DSD cordial graph when  $n$  is even and unbalanced DSD cordial when  $n$  is odd.

**Note 3.3**

For  $n \equiv 2(mod 4)$ , the path graph  $P_n$  is DSD cordial whereas  $G'$  obtained by duplicating any of the vertex with an edge in  $P_n$  is not DSD cordial.

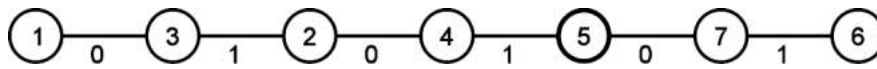


Figure 2(a) Balanced DSD cordial graph  $G(P_7)$

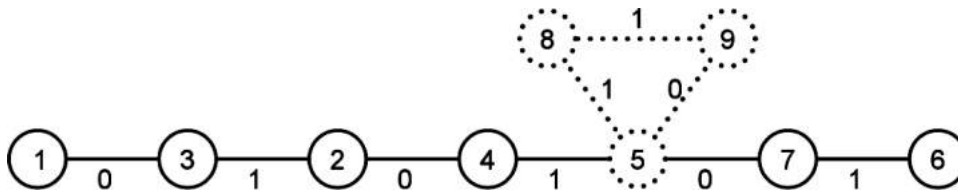


Figure 2(b) Unbalanced DSD cordial graph  $G'$  (by duplicating ).

**Proposition 3.5**

A graph got by duplicating a vertex  $v_k$  with an edge  $e' = u'v'$  in a DSD cordial cycle  $C_n (n \equiv 0,1(mod 4))$  is DSD cordial.

**Proof**

Let  $G$  be a cycle graph  $C_n (n \equiv 0,1(mod 4))$ . By Proposition 2.6, we draw a DSD cordial cycle graph  $C_n$ . Now, we duplicate any of the vertex  $v_k$  in  $G$  with an edge  $e' = u'v'$  and construct a new graph  $G'$ . In this graph  $G', |V(G')| = n + 2$  and  $|E(G')| = n + 3$ . For DSD cordial labeling pattern, let the vertex labels are  $\{1, 2, \dots, n + 2\}$ . Then, by labeling  $u'$  and  $v'$  by  $f(u') = n + 1$  and  $f(v') = n + 2$ , we get  $|e_f(0) - e_f(1)| \leq 1$ .

Hence  $G'$  is also a DSD cordial.

**Remark 3.6**

From Proposition 3.5, in particular we get  $|e_f(0) - e_f(1)| = 0$  when  $n$  is odd.

Hence, we can conclude that  $G'$  is a balanced DSD cordial graph when  $n$  is odd and unbalanced DSD cordial graph when  $n$  is even.

**Note 3.7**

For  $n \equiv 3(mod 4)$ , the cycle graph  $C_n$  is DSD cordial whereas  $G'$  obtained by duplicating any of the vertex with an edge in  $C_n$  is not DSD cordial.

**Example 3.8**

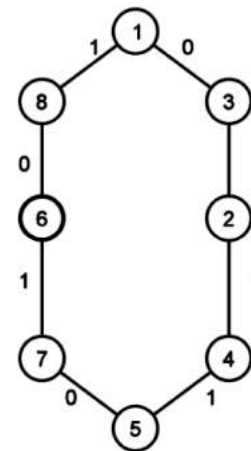


Figure 3(a) Balanced DSD cordial graph  $G(C_8)$

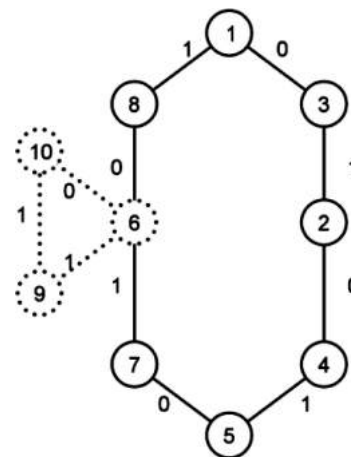


Figure 3(b) Unbalanced DSD cordial graph  $G'$  (by duplicating  $v_6$ )

**Proposition 3.9**

A graph got by duplicating a vertex  $v_k (1 \leq k \leq n)$  with an edge  $e' = u'v'$  in a DSD cordial graph  $K_{1,n}$  ( $n$  is even) is DSD cordial.

**Proof**

Let  $G$  be a star graph  $K_{1,n}$  ( $n$  is even). By Proposition 2.6, we draw a DSD cordial star graph  $K_{1,n}$ . Now, we duplicate any of the vertex  $v_k$  in  $G$  with an edge  $e' = u'v'$  and construct a new graph  $G'$ . In this graph  $G', |V(G')| = n + 3$  and  $|E(G')| = n + 3$ . For DSD cordial labeling pattern, let the vertex labels are  $\{1, 2, \dots, n + 3\}$ . Then, by labeling  $u'$  and  $v'$  by  $f(u') = n + 2$  and  $f(v') = n + 3$ , we get  $|e_f(0) - e_f(1)| \leq 1$ .

Hence  $G'$  is also a DSD cordial.

**Remark 3.10**

From Proposition 3.9, in particular we get  $|e_f(0) - e_f(1)| = 1$ . Hence, we can conclude that  $G'$  is a unbalanced DSD cordial.

**Example 3.11**

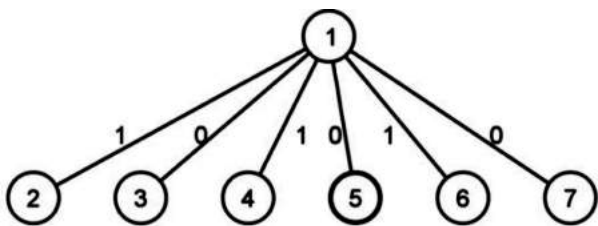


Figure 4(a) Balanced DSD cordial graph  $G(K_{1,6})$ .

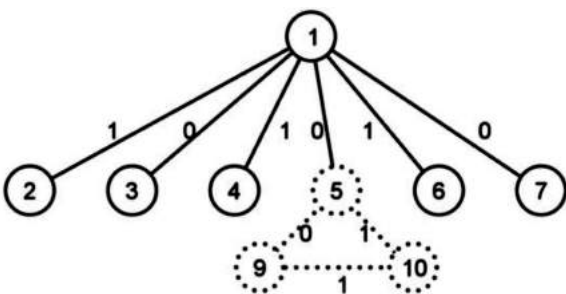


Figure 4(b) Balanced DSD cordial graph  $G'$  (by duplicating  $v_4$ ).

**Proposition 3.12**

A graph got by duplicating a vertex  $v_k (1 \leq k \leq n)$  with an edge  $e' = u'v'$  in a DSD cordial wheel graph  $W_n (n \equiv 0, 1 \pmod{4})$  is DSD cordial.

**Proof**

Let  $G$  be a DSD wheel graph  $W_n$ . By Proposition 2.7, we draw a DSD cordial wheel graph  $W_n$ . Now, we duplicate any of the vertex  $v_k$  in  $G$  with an edge  $e' = u'v'$  and construct a new graph  $G'$ . In this graph  $G', |V(G')| = n + 3$  and  $|E(G')| = 2n + 3$ . For DSD cordial labeling pattern, let the vertex labels are  $\{1, 2, \dots, n + 3\}$ . Then, by labeling  $u'$  and  $v'$  by  $f(u') = n + 2$  and  $f(v') = n + 3$ , we get  $|e_f(0) - e_f(1)| \leq 1$ .

Hence  $G'$  is also a DSD cordial.

**Remark 3.13**

From Proposition 3.12, in particular we get  $|e_f(0) - e_f(1)| = 1$ . Hence, we can conclude that  $G'$  is a unbalanced DSD cordial graph.

**Example 3.14**

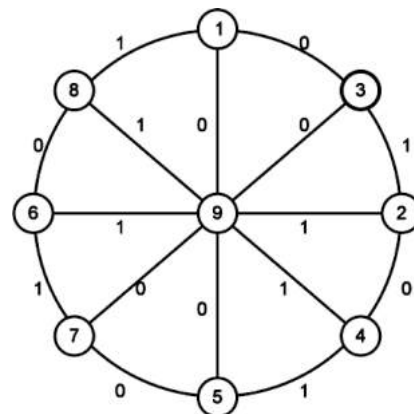


Figure 5(a) Balanced DSD cordial graph  $G(W_8)$ .

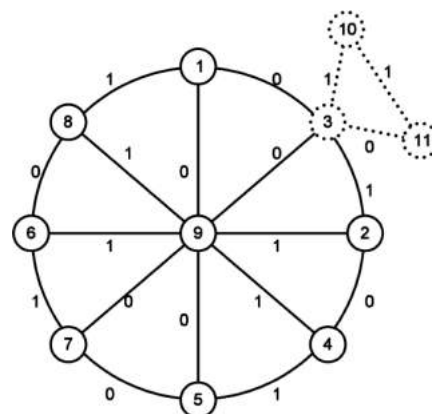


Figure 5(b) Unbalanced DSD cordial graph  $G'$  (by duplicating  $v_2$ ).

**Proposition 3.15**

A graph got by duplicating a vertex  $v_k$  with an edge  $e' = u'v'$  in a DSD cordial helm graph  $H_n$  ( $n \equiv 0,1 \pmod{4}$ ) is DSD cordial.

**Proof**

Let  $G$  be a helm graph  $H_n$ . By Proposition 2.7, we draw a DSD cordial helm graph  $H_n$ . Now, we duplicate any of the vertex  $v_k$  (either rim vertices or apex vertex or pendent vertices) in  $G$  with an edge  $e' = u'v'$  and construct a new graph  $G'$ . In this graph  $G'$ ,  $|V(G')| = 2n + 3$  and  $|E(G')| = 3n + 3$ . For DSD cordial labeling pattern, let the vertex labels are  $\{1, 2, \dots, 2n + 3\}$ . Then, by labeling  $u'$  and  $v'$  by  $f(u') = 2n + 2$  and  $f(v') = 2n + 3$ , we get  $|e_f(0) - e_f(1)| \leq 1$ .

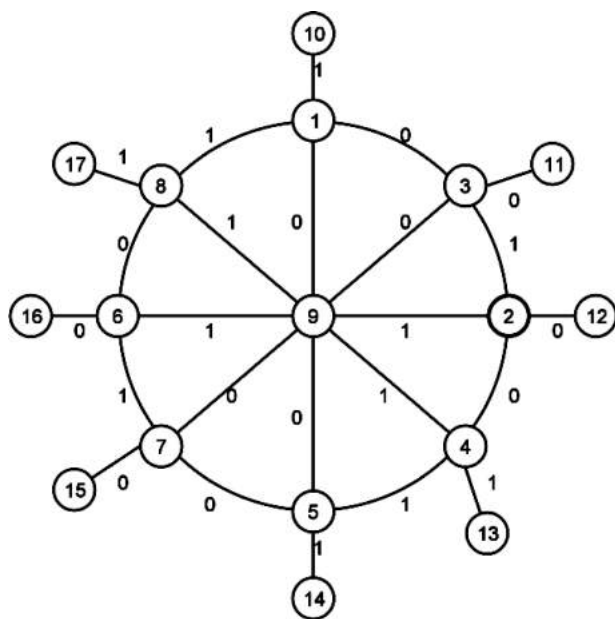
Hence  $G'$  is also a DSD cordial.

**Remark 3.16**

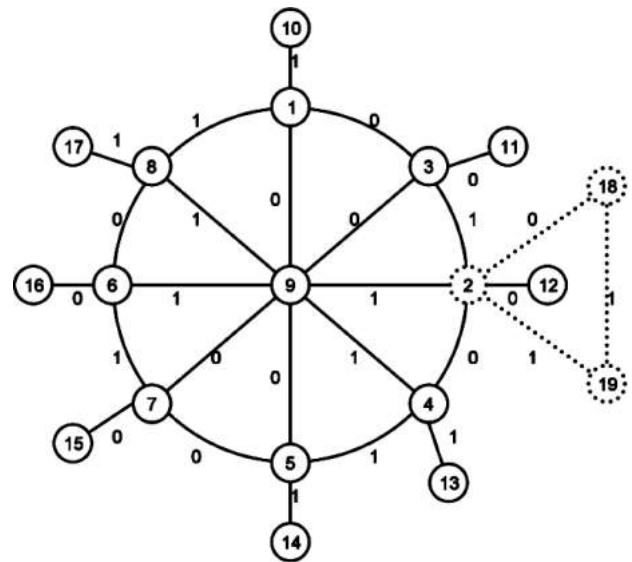
From Proposition 3.15, in particular we get  $|e_f(0) - e_f(1)| = 0$  when  $n$  is odd.

Hence, we can conclude that is a balanced DSD cordial graph when  $G'$  and unbalanced DSD cordial graph when  $n$  is even.

**Example 3.17**



**Figure 6(a)** Balanced DSD cordial graph  $G(H_8)$ .



**Figure 6(b)** Unbalanced DSD cordial graph  $G'$  (by duplicating  $v_3$ ).

**Proposition 3.18**

A graph got by duplicating a vertex  $v_k$  with an edge  $e' = u'v'$  in a DSD cordial crown graph  $C_n \odot K_1$  is divided square difference cordial.

**Proof**

Let  $G$  be a crown graph  $C_n \odot K_1$ . By Proposition 2.8, we draw a DSD cordial crown graph  $C_n \odot K_1$ . Now, we duplicate any of the vertex  $v_k$  (either vertices in the cycle or pendent vertices) in  $G$  with an edge  $e' = u'v'$  and construct a new graph  $G'$ . In this graph  $G'$ ,  $|V(G')| = 2n + 2$  and  $|E(G')| = 2n + 3$ . For DSD cordial labeling pattern, let the vertex labels are  $\{1, 2, \dots, 2n + 2\}$ . Then, by labeling  $u'$  and  $v'$  by  $f(u') = 2n + 1$  and  $f(v') = 2n + 2$ , we get  $|e_f(0) - e_f(1)| \leq 1$ .

Hence  $G'$  is also a DSD cordial.

**Remark 3.19**

From Proposition 3.18, in particular we get  $|e_f(0) - e_f(1)| = 1$ .

Hence, we can conclude that  $G'$  is a unbalanced DSD cordial graph.

Example 3.20

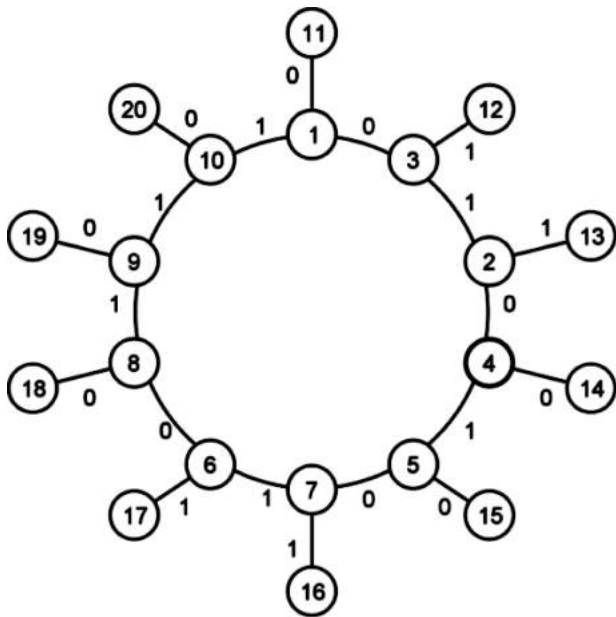


Figure 7(a) Balanced DSD cordial graph  $G (C_{10} \odot K_1)$ .

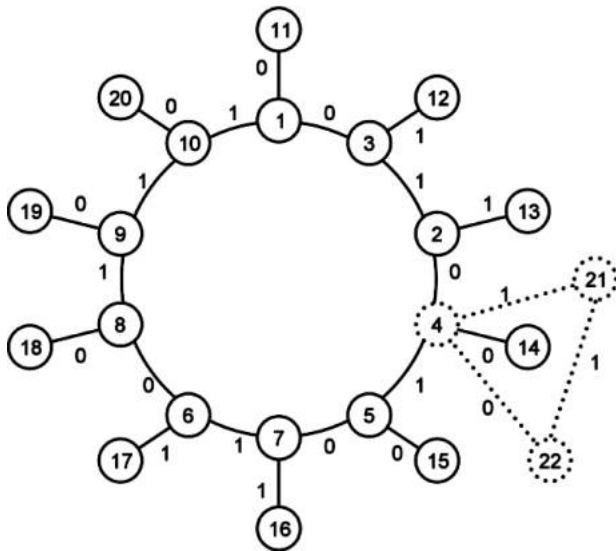


Figure 7(b) Unbalanced DSD cordial graph  $G'$  (by duplicating  $v_4$ ).

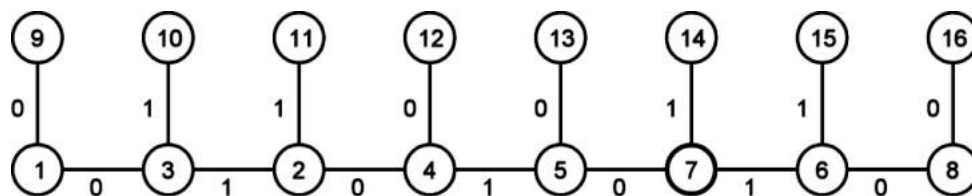


Figure 8(a) Unbalanced DSD cordial graph  $G(P_8 \odot K_1)$ .

Proposition 3.21

A graph got by duplicating a vertex  $v_k$  with an edge  $e' = u'v'$  in a DSD cordial comb graph  $P_n \odot K_1$  (except  $n \equiv 1 \pmod 4$ ) is divided square difference cordial.

Proof

Let  $G$  be a comb graph  $P_n \odot K_1$  (except  $n \equiv 1 \pmod 4$ ). By Proposition 2.8, we draw a DSD cordial comb graph  $P_n \odot K_1$ . Now, we duplicate any of the vertex  $v_k$  (either in path or pendent vertex) in  $G$  with an edge  $e' = u'v'$  and construct a new graph  $G'$ . In this graph  $G'$ ,  $|V(G')| = 2n + 2$  and  $|E(G')| = 2n + 2$ . For DSD cordial labeling pattern, let the vertex labels are  $\{1, 2, \dots, 2n + 2\}$ . Then, by labeling  $u'$  and  $v'$  by  $f(u') = 2n + 1$  and  $f(v') = 2n + 2$ , we get  $|e_f(0) - e_f(1)| \leq 1$ .

Hence  $G'$  is also a DSD cordial.

Remark 3.22

From Proposition 3.21, in particular we get  $|e_f(0) - e_f(1)| = 0$ .

Hence, we can conclude that  $G'$  is a balanced DSD cordial graph.

Note 3.23

For  $n \equiv 1 \pmod 4$ , the comb graph  $P_n \odot K_1$  is DSD cordial whereas  $G'$  obtained by duplicating any of the vertex with an edge in  $P_n \odot K_1$  is not DSD cordial.

Example 3.24

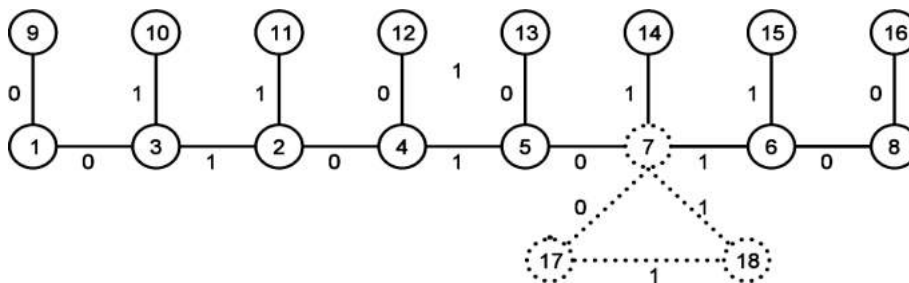


Figure 8(b) Balanced DSD cordial graph  $G'$  (by duplicating  $v_6$ ).

**Proposition 3.25**

A graph got by duplicating a vertex  $v_k$  with an edge  $e' = u'v'$  in a DSD cordial triangular snake graph  $T_n$  (except  $n \equiv 2 \pmod 4$ ) is divided square difference cordial.

**Proof**

Let  $G$  be a triangular snake graph  $T_n$  (except  $n \equiv 3 \pmod 4$ ). By Proposition 2.9, we draw a DSD cordial triangular snake graph  $T_n$ . Now, we duplicate any of the vertex  $v_k$  (either vertex in path or triangle) in  $G$  with an edge  $e' = u'v'$  and construct a new graph  $G'$ . In this graph  $G'$ ,  $|V(G')| = 2n + 1$  and  $|E(G')| = 3n$ . For DSD cordial labeling pattern, let the vertex labels are  $\{1, 2, \dots, 2n + 1\}$ . Then, by labeling

$u'$  and  $v'$  by  $f(u') = 2n$  and  $f(v') = 2n + 1$ , we get  $|e_f(0) - e_f(1)| \leq 1$ .

Hence  $G'$  is also a DSD cordial.

**Remark 3.26**

From Proposition 3.25, in particular we get  $|e_f(0) - e_f(1)| = 0$  when  $n$  is even.

Hence we can conclude that  $G'$  is a balanced DSD cordial graph.

**Note 3.27**

1. For  $n \equiv 2 \pmod 4$ , the triangular snake graph  $T_n$  is DSD cordial whereas  $G'$  got by duplicating any of the vertex with an edge in  $T_n$  is not DSD cordial.

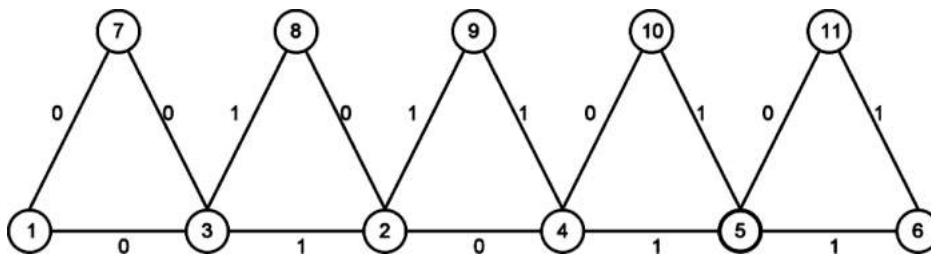


Figure 9(a) Unbalanced DSD cordial graph  $G(T_6)$ .

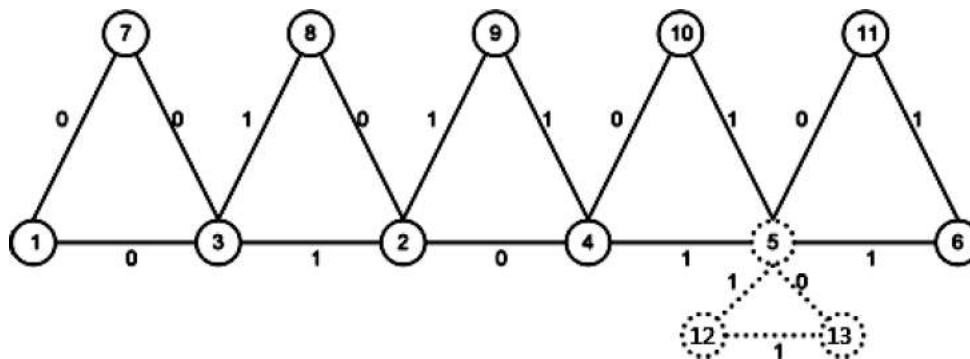


Figure 9(b) Balanced DSD cordial graph  $G'$  (by duplicating  $v_5$ ).

2. For  $n \equiv 3 \pmod{4}$ , the triangular snake graph  $T_n$  is not DSD cordial whereas  $G'$  got by duplicating any of the vertex with an edge in  $T_n$  is DSD cordial.

### Example 3.28

## 4. Conclusion

In this article, we have discussed and proven that the graph got by duplicating a vertex with an edge in divided square difference (DSD) cordial graphs such as path graph, cycle graph, star graph, wheel graph, helm graph, bistar graph, crown graph, comb graph and snake graph were also DSD cordial graphs.

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