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Stochastic Reliability Analysis of Two Identical Cold Standby Units with Geometric Failure & Repair Rates

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Abstract The present paper is based on the analysis of an two identical unit system wherein initially one unit is in operative state and other is in cold standby and total failure of the unit is via partial failure mode. Single repairman is always available with the system for any repair of failed unit (partially/completely). Failure/repair time is considered to be a Geometric distribution. Measures of system effectiveness had also been calculated for the system.

Keyword: Geometric distribution, redundant system, discrete random variable.

1. INTRODUCTION

In some situations, discrete failure distributions is appropriate for lifetime model [1,2,4] e.g an discrete distribution is accurate for a equipment operating in phases and the number of phases observed prior to any failure.

Discrete failure case arises in several situations, e.g.: a) A device is monitored only once in any period of time (e.g., hourly, daily, weekly, monthly etc.), and the observation is considered to be the number of time periods completed successfully, prior to device failure. b) Piece of equipment performs in phases and the experimenter successfully observes the number of phases, completed prior to failure.

When the observed values are assumed to be very large i.e. in thousands of phases, a continuous distribution is proved to be an adequate model for the discrete random variable. However, for small observed values, the continuous distribution will not be describing a discrete random variable.

We see that many practical situations of importance are represented with the help of discrete life time models. Similarly, the repair time may be considered as discrete random variable as dividing the whole time interval into

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various small parts of time. Discrete time models are considered by several researchers [1, 2].

Keeping in view these concept of discrete time modeling, we analyze redundant system models [5] having two identical units with geometric failure and repair time distribution. In initial stage, one unit is in operative state and second one has to be in cold standby model. The two types of failure are total failure and partial failure of the unit [3].

2. SYSTEM DESCRIPTION AND ASSUMPTION

System is analyzed under the following assumptions:

(i) A system is consisting of two identical units i.e. operative and standby. Each unit posses three modes of states: normal (N), partial failure (PF) and total failure (F). The standby unit cannot fail.
(ii) System is considered to be in failed state, whether the cause of failure is partial or total.
(iii) Failures are self-announcing.
(iv) The failure and repair time distribution are independent having geometric distribution with parameter p and r respectively.
(v) The failure time of all units will be taken as independent random variable.
(vi) The system may go to partially failed or totally failed states with probability ‘b’ or ‘a’ respectively.

3. NOTATIONS/SYMBOLS FOR STATES OF THE SYSTEMS

a	:	Probability of system going to failed state.
b	:	Probability of system having partially failed state.
N_0	:	Unit having operative/normal mode.
N_s	:	Unit having standby/normal mode.
F_r/F_{wr}	:	Unit having failure mode and is repairing/waiting for repair.
PF_r/PF_{wr}	:	Unit having partial failure mode and is repairing/waiting for repair.

Up States : $S_0 \equiv (N_0, N_s)$, $S1 \equiv (F_r, N_0)$, $S_2 \equiv (PF_r, N_0)$

Down State : $S_3 \equiv (F_r, F_{wr})$, $S_4 \equiv (PF_r, PF_{wr})$, $S_5 \equiv (F_r, PF_{wr})$, $S_6 \equiv (PF_r, F_{wr})$

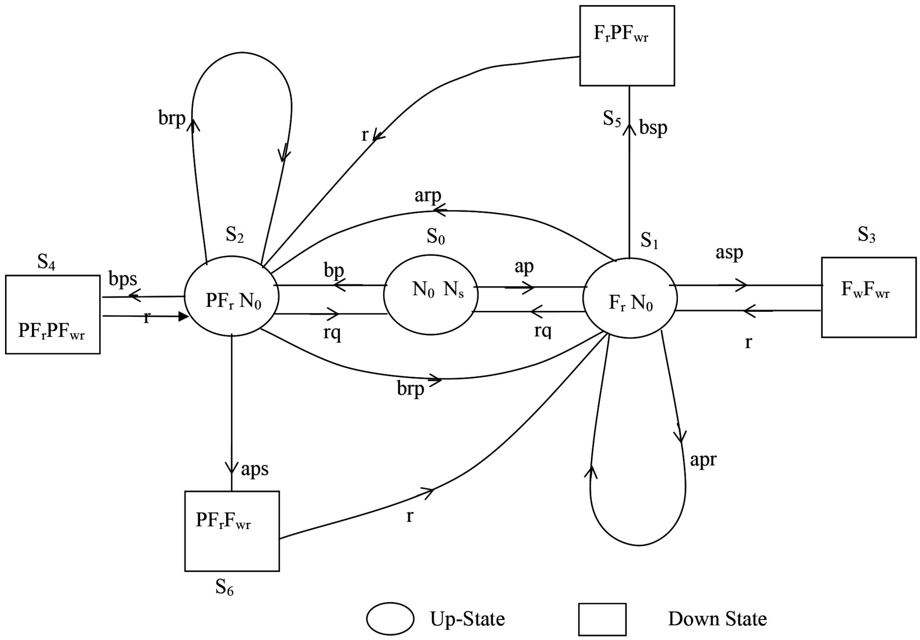


Figure 1.1: Transition Diagram

4. TRANSITION PROBABILITIES AND SOJOURN TIMES

$$Q_{01}(t) = ap\frac{1-q^{(t+1)}}{1-q} \qquad Q_{02}(t) = bp\ \frac{1-q^{(t+1)}}{1-q} \qquad Q_{11}(t) = arp\frac{1-(qs)^{(t+1)}}{1-qs}$$

$$Q_{13}(t) = aps\frac{1-(qs)^{(t+1)}}{1-qs} \qquad Q_{10}(t) = rq\frac{1-(qs)^{t+1}}{1-qs} \qquad Q_{15}(t) = bps\frac{1-(qs)^{(t+1)}}{1-qs}$$

$$Q_{12}(t) = brp\frac{1-(qs)^{(t+1)}}{1-qs} \qquad Q_{20}(t) = rq\frac{1-(qs)^{(t+1)}}{1-qs} \qquad Q_{22}(t) = brp\frac{1-(qs)^{(t+1)}}{1-qs}$$

$$Q_{21}(t) = arp\frac{1-(qs)^{(t+1)}}{1-qs} \qquad Q_{24}(t) = bps\ \frac{1-(qs)^{(t+1)}}{1-qs} \qquad Q_{26}(t) = aps\ \frac{1-(qs)^{(t+1)}}{1-qs}$$

$$Q_{31}(t) = Q_{42}(t) = Q_{52}(t) = Q_{61}(t) = r\frac{1-s^{(t+1)}}{1-s} \qquad (1.1\ \text{-}1.13)$$

The probability of steady-state transition from ‘S_i’ to ‘S_j’ is calculated by using

$$P_{ij} = \lim_{t\to\infty} Q_{ij}$$

and it is verified that

$P_{01} + P_{02} = 1,\quad P_{11} + P_{10} + P_{12} + P_{15} + P_{13} = 1, P_{22} + P_{20} + P_{21} + P_{24} + P_{26} = 1$

$P_{31} = P_{42} = P_{52} = P_{61} = 1$

5. MEAN SOJOURN TIMES

Let T_i be assumed to be sojourn time in state $S_i(i = 0, 1, 2)$ and then, the mean sojourn time of S_i is calculated by

$$\mu_i = \sum_{t=0}^{\infty} P(T_i > t)$$

so that, $\mu_0 = \dfrac{1}{1-q}\quad \mu_1 = \mu_2 = \dfrac{1}{1-qs}\quad \mu_3 = \mu_4 = \mu_5 = \mu_6 = \dfrac{1}{1-s}$ (1.14-1.16)

By defining m_{ij} in state S_i, when system transits into state S_j i.e.

$$m_{ij} = \sum_{t=0}^{\infty} t\, q_{ij}(t)$$

we obtained

$m_{01} + m_{02} = q\mu_0 \qquad m_{11} + m_{10} + m_{12} + m_{13} + m_{15} = (qs)\mu_1$

$m_{22} + m_{20} + m_{21} + m_{24} + m_{26} = (qs)\mu_2 \quad m_{31} = m_{52} = m_{42} = m_{61}$ (1.17-1.20)

6. MEAN TIME TO SYSTEM FAILURE

Letbe the system probability to work for at least epochs, when initial it starts from state

$R_0(t) = Z_0(t) + q_{01}(t)©R_1(t-1) + q_{02}(t-1)©R_2(t-1)$

$R_1(t) = Z_1(t) + q_{10}(t-1)©R_0(t-1) + q_{11}(t-1)©R_1(t-1) + q_{12}(t-1)©R_2(t-1)$

$R_2(t) = Z_2(t) + q_{20}(t-1)©R_0(t-1) + q_{22}(t-1)©R_2(t-1) + q_{21}(t-1)©R_1(t-1)$

(1.21– 1.23)

Taking Geometric Transform on both sides, we get

$$R_0{}^*(h) = \frac{N_1(h)}{D_1(h)}$$

$$D_1(h)=[1-q_{11}^*(h)][1-hq_{22}^*(h)]-h^2q_{12}^*(h)q_{21}^*(h)-h^2q_{01}^*(h)q_{10}^*(h)\times[1-hq_{22}^*(h)]$$
$$-h^2q_{12}^*(h)q_{20}^*(h)q_{01}^*(h)-h^3q_{02}^*(h)q_{10}^*(h)q_{21}^*(h)-h^2q_{02}^*(h)q_{20}^*(h)(1-hq_{11}^*(h))$$

$$N_1(h)= Z_0^*(h)[1-hq_{11}^*(h)][1-hq_{22}^*(h)]-h^2q_{21}^*(h)q_{12}^*(h)Z_0^*(h)+hq_{01}^*(h)$$
$$(1-hq_{22}^*(h))Z_1^*(h)+h^2q_{01}^*(h)q_{12}^*(h)Z_2^*(h)h^2q_{02}^*(h)q_{21}^*(h)Z_1^*(h)$$
$$+h\,q_{02}^*(h)(1-hq_{11}^*(h))Z_2^*(h)$$

Then,

$$\text{MTSF} = N_1/D_1 \tag{1.24}$$

where $D_1 = (1-P_{11})(1-P_{22}) - P_{12}P_{21} - P_{01}P_{10}(1-P_{22}) - P_{12}P_{20}P_{01} - P_{02}P_{10}P_{21} - P_{02}P_{20}(1-P_{11})$

$$N_1 = (1-\mu_0)[P_{12}P_{21}-(1-P_{11})(1-P_{22})] + (\mu_1 + P_{10})(P_{01}(1-P_{22}) + P_{02}P_{21}) + [(\mu_2 + P_{20})P_{02}(1-P_{11}) + P_{12}P_{01}]$$

7. AVAILABILITY ANALYSIS

Let $A_i(t)$ be the system probability having up state at epoch 't', when initial it starts from state S_i.

$$A_0(t) = Z_0(t) + q_{01}(t-1)©\, A_1(t-1) + q_{02}(t-1)©\, A_2(t-1)$$

$$A_1(t) = Z_1(t) + q_{10}(t-1)©\, A_0(t-1) + q_{11}(t-1)©\, A_1(t-1) + q_{13}(t-1)©\, A_3(t-1) + q_{15}(t-1)©\, A_5(t-1) + q_{12}(t-1)©A_2(t-1)$$

$$A_2(t) = Z_2(t) + q_{20}(t-1)©\, A_0(t-1) + q_{21}(t-1)©\, A_1(t-1) + q_{22}(t-1)©\, A_2(t-1) + q_{24}(t-1)©\, A_4(t-1) + q_{26}(t-1)©\, A_6(t-1)$$

$$A_3(t) = q_{31}(t-1)©\, A_1(t-1)$$

$$A_4(t) = q_{42}(t-1)©\, A_2(t-1)$$

$$A_5(t) = q_{52}(t-1)©\, A_2(t-1)$$

$$A_6(t) = q_{61}(t-1)©\, A_1(t-1) \tag{1.25-1.31}$$

By taking geometric transformation, and solving above equations we get

$$A_0{}^*(h) = \frac{N_2(h)}{D_2(h)}$$

where

$$N_2(h) = Z_0^*(h)[(1-hq_{11}^*(h))(1-hq^*{}_{22}(h))] - h^2q_{24}^*(h)(1-hq_{11}^*(h))q_{42}^*(h)Z_0^*(h) -h^2q_{21}^*(h)q_{12}^*(h)Z_0^*(h) - h^3q_{26}^*(h)q_{12}^*(h)q_{61}^*(h)Z_0^*(h) - h^2q_{31}^*(h)q_{13}^*(h) (1-hq_{22}^*(h)).Z_0^*(h) + h^4q_{42}^*(h)q_{31}^*(h)q_{24}^*(h)q_{13}^*(h) - h^3q_{21}^*(h) q_{15}^*(h)q_{52}^*(h)Z_0^*(h) - h^4q_{26}^*(h)q_{15}^*(h)_{61}^*(h)q_{52}^*(h) + h(1-hq_{22}^*(h)) q_{01}^*(h)Z_1^*(h) - h^3q_{24}^*(h)q_{42}^*(h)q_{01}^*(h)Z_1^*(h) + h^2q_{01}^*(h)q_{12}^*(h)Z_2^*(h) +h^3q_{01}^*(h)q_{15}^*(h)q_{52}^*(h)Z_2^*(h) + h^2q_{21}^*(h)q_{02}^*(h)Z_1^*(h) + h^3q_{26}^*(h) q_{02}^*(h)q_{61}^*(h)Z_1^*(h) + hq_{02}^*(h)(1-hq_{11}^*(h))Z_2^*(h) - h^3q_{13}^*(h)q_{31}^*(h) q_{02}^*(h)Z_2^*(h)$$

$$D_2(h) = (1-h\, q_{11}^*(h))(1-hq_{22}^*(h)) - h^2q_{24}^*(h)q_{42}^*(h)(1-hq_{11}^*(h)) - h^2q_{12}^*(h) q_{21}^*(h) - h^3q_{26}^*(h)q_{12}^*(h)q_{61}^*(h) - h^2q_{13}^*(h)q_{31}^*(h)(1-hq_{22}^*(h)) + h^4 q_{24}^*(h)q_{13}^*(h)q_{31}^*(h) - h^3q_{15}^*(h)q_{21}^*(h)q_{52}^*(h) - h^4q_{15}^*(h)q_{26}^*(h) q_{52}^*(h) - h^2q_{01}^*(h)q_{10}^*(h)(1-hq_{22}^*(h)) + h^4q_{01}^*(h)q_{10}^*(h)q_{24}^*(h) q_{42}^*(h) - h^3q_{12}^*(h)q_{01}^*(h)q_{20}^*(h) - h^4q_{15}^*(h)q_{01}^*(h)q_{20}^*(h)q_{42}^*(h) - h^3 q_{02}^*(h)q_{10}^*(h).q_{21}^*(h) - h^4q_{02}^*(h)q_{10}^*(h)q_{26}^*(h)q_{61}^*(h) - h^2q_{02}^*(h)q_{20}^*(h) (1-hq_{11}^*(h)) + h^4q_{31}^*(h)q_{02}^*(h)q_{13}^*(h)q_{20}^*(h)$$

where,

$$Z_0(t) = q^t, \quad Z_2(t) = Z_1(t) = (qs)^t, \quad Z_3(t) = Z_4(t) = Z_5(t) = Z_6(t) = s^t$$

Hence,

$$Z_i^*(1) = \mu_i$$

The availability of steady state system is calculated by

$$A_0 = \lim_{t\to\infty} A_0(t)$$

Hence, using 'L' Hospital Rule, we get

$$A_0 = -\frac{N_2(1)}{D_2'(1)} \tag{1.32}$$

where

$$N_2(1) = (1 - p_{22} - p_{24})(\mu_0(1-p_{11}) + \mu_1 p_{01})$$
$$+ (p_{21} + p_{26})(\mu_1 p_{02} - \mu_0 p_{12}) - p_{15} p_{21}\mu_0 - p_{13}(1-p_{22})\mu_0$$
$$+ \mu_2 p_{01}(p_{12} + p_{15}) + \mu_2 p_{02}(1-p_{11}-p_{13}) + p_{13}p_{24} - p_{26} p_{15}$$

and

$$D_2'(1) = -[qs\mu_1(1-p_{22} - p_{24} - p_{20}p_{02}) + q\,\mu_0(p_{20}(p_{12} + p_{15}) - p_{10}(p_{24} - p_{22}))$$
$$+ qs\mu_2(1-p_{11} - p_{13} - p_{01}p_{10}) + s\mu_3[p_{24}(1-p_{11} - p_{01}p_{10}) + p_{15}(p_{01}p_{20}$$
$$+ (p_{21}+p_{26})) + p_{13}(1-p_{22}-p_{24} - p_{02}p_{20}) + p_{26}(p_{12} + p_{02}p_{10})]$$

Now, the system expected uptime at epoch 't' is calculated by

$$\mu_{up}(t) = \sum_{x=0}^{t} A_0(x)$$

so that

$$\mu_{up}^*(h) = \frac{A_0{}^*(h)}{1-h}$$

8. BUSY PERIOD ANALYSIS

Let be the repair probability i.e busy repair time of any failed unit, when initial the system starts from .

$$B_0(t) = q_{01}(t-1) © B_1(t-1) + q_{02}(t-1) © B_2(t-1)$$

$$B_1(t) = Z_1(t) + q_{10}(t-1) © A_0(t) + q_{11}(t-1) © B_1(t-1) + q_{12}(t-1) © A_2(t-1)$$
$$+ q_{13}(t-1) © B_3(t-1) + q_{15}(t-1) © A_5(t-1)$$

$$B_2(t) = Z_2(t) + q_{20}(t-1) © B_0(t-1) + q_{22}(t-1) © B_2(t-1) + q_{21}(t-1) © B_1(t-1)$$
$$+ q_{24}(t-1) © B_4(t-1) + q_{26}(t-1) © B_6(t-1)$$

$$B_3(t) = Z_3(t) + q_{31}(t-1) © B_1(t-1)$$

$$B_4(t) = Z_4(t) + q_{42}(t-1) © B_2(t-1)$$

$$B_5(t) = Z_5(t) + q_{52}(t-1) © B_2(t-1)$$

$$B_6(t) = Z_6(t) + q_{61}(t-1) © B_1(t-1) \qquad (1.33 – 1.39)$$

By taking geometric transformation and solving above equations we get

$$B_0^*(h) = \frac{N_3(h)}{D_2(h)}$$

where,

$$N_3(h) = hq_{01}^*(h)(1-hq_{22}^*(h)) - h^3 q_{01}^*(h)q_{24}^*(h)q_{42}^*(h)Z_1^*(h) + h^2 q_{01}^*(h)q_{12}^*(h)$$
$$Z_2^*(h) + h^3 q_{01}^*(h)q_{12}^*(h)q_{24}^*(h)Z_4^*(h) + h^3 q_{01}^*(h)q_{12}^*(h)q_{26}^*(h)Z_6^*(h) + h^2$$
$$q_{01}^*(h)q_{13}^*(h)(1-hq_{22}^*(h))Z_3^*(h) - h^4 q_{01}^*(h)q_{13}^*(h)q_{24}^*(h)q_{42}^*(h)Z_3^*(h) + h^2$$
$$q_{02}^*(h)q_{21}^*(h)Z_1^*(h) + h^3 q_{02}^*(h)q_{26}^*(h)q_{61}^*(h)Z_1^*(h) + hq_{02}^*(h)(1-hq_{11}^*(h))Z_2^*(h)$$
$$+h^2 q_{02}^*(h)q_{24}^*(h)(1-hq_{11}^*(h))Z_4^*(h) + h^2 q_{26}^*(h)q_{02}^*(h)(1-hq_{11}^*(h))Z_6^*(h) - h^3 q_{02}^*(h)$$
$$q_{13}^*(h)q_{31}^*(h)Z_2^*(h) + h^3 q_{02}^*(h)q_{13}^*(h)q_{21}^*(h)Z_3^*(h) + h^4 q_{02}^*(h)q_{13}^*(h)q_{26}^*(h)q_{31}^*(h)Z_4^*(h)$$
$$+h^4 q_{02}^*(h)q_{13}^*(h)q_{26}^*(h)q_{61}^*(h)Z_3^*(h) - h^4 q_{02}^*(h)q_{13}^*(h)q_{26}^*(h)q_{31}^*(h)Z_6^*(h) + h^3 q_{02}^*(h)$$
$$q_{15}^*(h)q_{21}^*(h)Z_3^*(h) + h^4 q_{02}^*(h)q_{15}^*(h)q_{26}^*(h)q_{61}^*(h)Z_5^*(h)$$

Probability of a repair facility busy in repairing failed unit is:

$$B_0(t) = \lim_{t\to\infty} B_0(t)$$

Hence, using ‘L’Hospital Rule, we get

$$B_0 = -\frac{N_3(1)}{D_2'(1)} \tag{1.40}$$

where

$$N_3(1) = p_{01}(1-p_{22}) + \mu_1 (p_{02}(p_{21}+p_{26}) - p_{01}p_{24}) + \mu_2(p_{01}p_{12} - p_{02}(p_{11}+p_{13}-1))$$
$$+ \mu_3 \{(p_{26} + p_{24})(p_{01} p_{12} + p_{02}(1-p_{11})) + p_{02}(p_{21} + p_{26})(p_{13} + p_{15})\}$$

9. PROFIT FUNCTION ANALYSIS

The profit expected in steady-state is

$$P = C_0 A_0 - C_1 B_0 \tag{1.41}$$

where
C_0 / C_1: per unit up/down time revenue/expenditure by/on the system.

10. PARTICULAR CASE

$$P_{01} = a, \quad P_{02} = b, \quad P_{11} = P_{21} = \frac{arp}{1-q}, \quad P_{13} = P_{26} = \frac{aps}{1-qs}$$

$$P_{10} = P_{20} = \frac{rq}{1-qs}, \quad P_{12} = P_{22} = \frac{brp}{1-qs}, \quad P_{15} = P_{24} = \frac{bps}{1-qs}$$

On fixing following numerical values as:

P =0.1, r = 0.25, a = 0.45, b = 0.55.

The values of different system effectiveness measures are as:

MTSF = 52.33 time units

Availability (A_0) = 0.23758

Busy period of analysis of repairman (B_0) = 0.754556

Graphical Representation:

Fig. 1.3 reflects the behavior repair rate It is clear that, the increases with increasing repair rate.

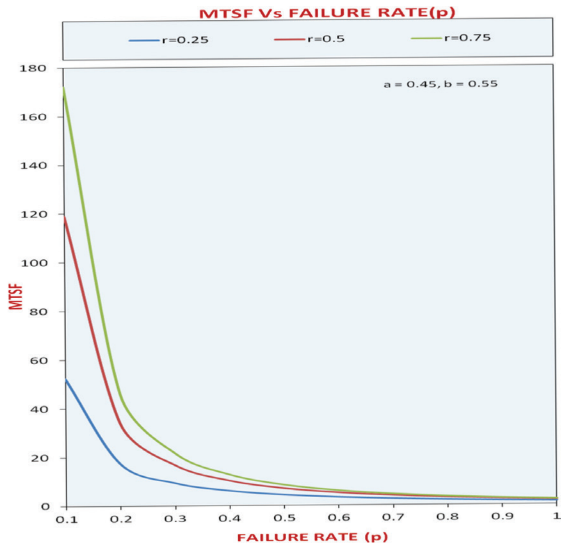
Fig. 1.4 reflects the pattern of the profit failure rate for distinct repair rate Profit decreases with increasing failure rate and is higher for bigger values of repair rate Following observations from the graph are:

(i) For r = 0.5, P > or = or < 0 according as p < or = or > 0.7356. So, system is profitable only if, failure rate is less than 0.7356.

(ii) For r = 0.75, P > or = or < 0 according as p < or = or > 0.8098. Thus the system is not profitable when > 0.8098.

Hence the companies using such systems can be suggested to purchase only those system which do not have failure rates greater than those discussed in points (i) to (iii) above in this particular case.

Fig. 1.5 reflects the pattern of the profit w.r.t repair rate for distinct failure rate Profit increases with increasing repair rate and is lower for higher values of failure rate



Figures 1.2: reflects the behavior of failure rate It is clear that, the decreases with increasing failure rate.

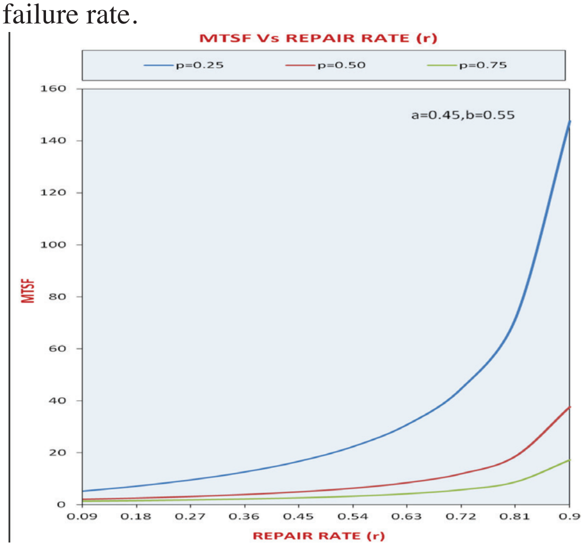


Figure 1.3

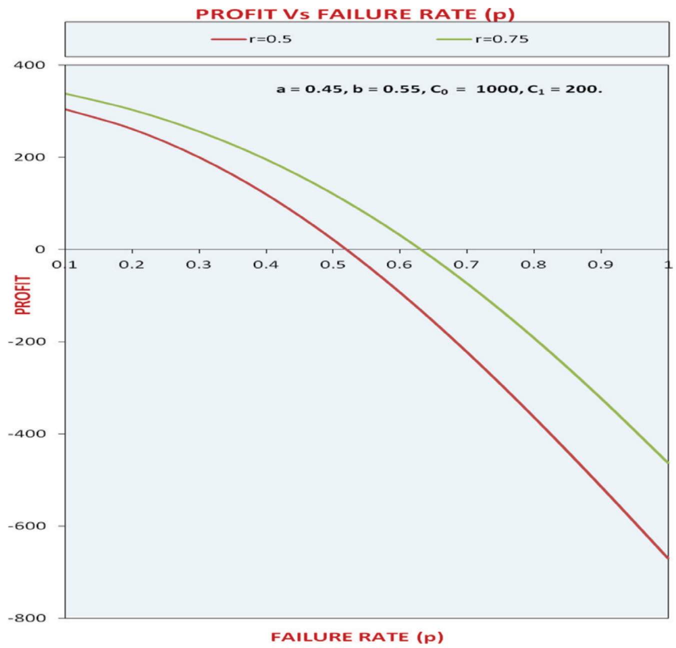


Figure 1.4

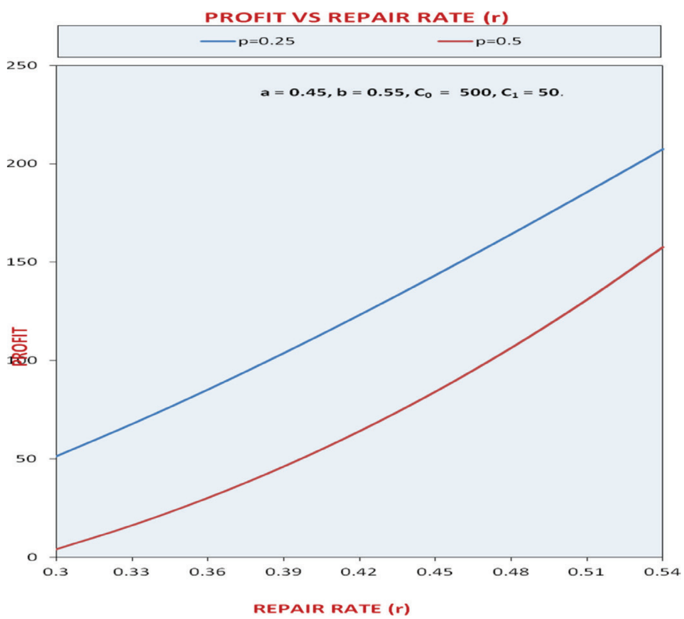


Figure 1.5

CONCLUSIONS

The analysis discussed above shows that the MTSF and the expected up time of used system decreases with increasing the values of rate of partial and total failures. For the profit of the system, the analysis stated various cut of points of the revenue per unit up time and cost per unit repair of the failed unit to enhance the profit of the system.

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