

Similarity Solution of Forced Convection Flow of Powell-Eyring & Prandtl-Eyring Fluids by Group-Theoretic Method

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Abstract Generalized one parameter group theoretical method is applied to study Powell-Eyring and Prandtl-Eyring fluid models for heat transfer in forced convection boundary layer flow. The velocity and the temperature variations for two dimensional steady incompressible, laminar forced convection flow of both fluid models past a flat plate is considered. Velocity and temperature variation for different values of fluid index and physical parameter A, B, α, β and Pr are presented graphically. Also, comparison for both fluid models is done graphically.

Keywords: Non-Newtonian fluid, Generalized group theoretic method, Powell-Eyring model, Prandtl-Eyring model, Forced convection.

NOMENCLATURE

A-group parameter

G-group

U-main stream velocities in x direction

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Shukla, H.	a -thermal diffusivity
Patel, J	f -similarity function
Surati, HC	n -flow behaviour indices
Patel, M	u -velocity component in X direction
Timol, MG	v -velocity component in Y direction
	τ - stress component
<hr/>	θ - Temperature of fluid
	ψ - stream function
	η - independent similarity variable
	$\alpha, \beta,$ and $\alpha_1, \dots, \alpha_7, \beta_1, \dots, \beta_7$ - real constants
	τ_{yx} -stress component in the direction of x, perpendicular to y
	C_p -specific heat at constant pressure
	θ_w - wall temperature of fluid
	P^S, Q^S are real valued and differentiable in their real argument 'A'.

1. INTRODUCTION

The process of heat transfer through a fluid in the presence of bulk fluid motion is called convection. Generally, convection is classified in two ways depending on how the fluid motion is started. One is natural convection or free convection which is initiated because of buoyancy effect. In natural convection, warmer fluid is rising and cooler fluid is falling. Second type is forced convection in which fluid is forced to flow by using fan or pump. To study heat transfer problem is very important because its wide applications in different industrial instrument and some household equipment.

Observation of literature survey is that most of flow analysis is done for Newtonian fluids and limited analysis done for non-Newtonian fluids almost up to power law fluid. Kapur [6], Hansen and Na[1], M. Pakdemirli [11], Patel *et al* [8,9,14] worked on Non -Newtonian fluids almost up to power law fluid. Heat transfer in natural convection and forced convection are studied by Lin and Lin [5]. He used a similarity solution technique to study different cases regarding heat convection problem. Using similarity analysis and finite-difference method, Na [15], studied non-Newtonian fluid model namely Reiner-Philippoff model. Numerical analysis done by Timol and Surati [3] for different non-Newtonian fluid model using assumed group method.

Prasad [7] investigated the problem of heat transfer of an incompressible, viscous non-Newtonian fluid over a non-isothermal stretching sheet in the presence of viscous dissipation and internal heat generation/absorption. Munir [12] analysed the Sisko fluid model for two different cases for nonlinear

stretching sheet. He is considered forced convection heat transfer problem for the sheet with variable temperature and the sheet with variable heat flux. Hayat [2] investigate the Powell Eyring model and analysed the effects of convective heat and mass transfer.

We observed that most of Similarity analysis done for different non-Newtonian fluid model by either assuming similarity variable or by assumed group of transformation in area of heat transfer. Sutterby fluid models considered for similarity analysis by applying general group theoretic method by Jain [13]. Boundary layer flow under the influence of transverse magnetic field is examined by applying deductive group theoretic method by author. Recently, Similarity solution is derived for Sisko fluid using dimensional analysis method by Hema [4].

Different non-Newtonian fluid models are defined by arbitrary functional relationship between shear-stress and rate of the strain [14,16]. From these defined models, most of researcher worked on power law model because of simple mathematical relationship between shear-stress and rate of the strain. Also, relationship between shear-stress and rate of the strain is derived empirically for power law model. Other fluid models are mathematically more complex than power law model and governing equations of models are non-linear partial differential equations. So, it is not easy to solve analytically or numerically. Our interest to solve these two more mathematically complex models because of two main aspects, first is that the relationship of fluid model is derived from kinetic theory of liquids and second these models converts to Newtonian fluid model for high and low shear rates [1].

In the present paper, generalized one parameter group theoretical method is applied on Powell-Eyring and Prandtl-Eyring fluid models to study heat transfer in forced convection boundary layer flow. The velocity and the temperature variations for two dimensional steady incompressible, laminar forced convection flow of both fluid models past a flat plate is considered. Velocity and Temperature variations for different values of fluid index and physical parameter A , B , α , β and Pr are presented graphically. Also, comparison for both fluid models is presented by graph.

2. GOVERNING EQUATION([3])

Present research is based on the following assumption.

1. The fluid is assumed incompressible and viscoelastic.
2. The flow is two-dimensional and steady.
3. The constant pressure specific heat C_p is assumed to be constant with respect to temperature changes.

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4. The fluids under consideration are of the form in which shearing stress τ_{yx} is related to rate of strain by the arbitrary function of the type

$$F\left(\tau_{yx}, \frac{\partial u}{\partial y}\right) = 0 \quad (1)$$

The governing equations of two dimensional, steady incompressible, laminar forced convection flow over a flat plate with a cartesian coordinate system in usual notations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \frac{1}{\rho} \frac{\partial}{\partial y} (\tau_{yx}) \quad (3)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = a \frac{\partial^2 \theta}{\partial y^2} \quad (4)$$

(Where is thermal diffusivity)

With the boundary conditions:

$$u(0) = 0, v(0) = 0, \theta(0) = 0, u(\infty) = U(x), \theta(\infty) = \theta_w \quad (5)$$

Using stream function $\psi(x, y)$, to reduce one dependent variable which satisfies equation (2).

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (6)$$

The dimensionless basic partial differential equations for low velocity forced convection flow of non-Newtonian fluids with stream function ψ can be derived as follows:

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \Psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \Psi}{\partial y^2} = U \frac{dU}{dx} + \frac{1}{\rho} \frac{\partial}{\partial y} (\tau_{yx}) \quad (7)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} = \frac{1}{3pr} \frac{\partial^2 \theta}{\partial y^2} \quad (8)$$

Subject to the boundary conditions;

$$\begin{aligned}
 y = 0 &\Rightarrow \frac{\partial \psi}{\partial y} = -\frac{\partial \psi}{\partial x} = 0, \theta = 0 \\
 y = \infty &\Rightarrow \frac{\partial \psi}{\partial y} = U(x), \theta \Rightarrow \theta_w
 \end{aligned}
 \tag{9}$$

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With the stress-strain relation

$$F\left(\tau_{yx}, \frac{\partial^2 \psi}{\partial y^2}\right) = 0
 \tag{10}$$

Where following non-dimensional quantities used.

$$\begin{aligned}
 u' &= \frac{u}{U_0}, v' = \frac{v}{U_0} \sqrt{\frac{Re}{3}}, x' = \frac{x}{L}, y' = \frac{y}{L} \sqrt{\frac{Re}{3}}, \\
 U' &= \frac{U}{U_0}, \tau'_{yx'} = \frac{\tau_{yx}}{\rho U_0^2} \sqrt{\frac{Re}{3}}, Re = \frac{\rho U_0 L}{\mu}, pr = \frac{U_0 L}{a Re}
 \end{aligned}
 \tag{11}$$

(Des are dropped for simplicity)

3. GENERALIZED GROUP THEORETIC METHOD

The method used in this paper is Generalized group theoretic method. Under this General group of transformation, the two independent variables will be reduced by one and the boundary value type partial differential equations (7)-(10) which has two independent variables and y transform into boundary value type ordinary differential equations in only one-independent variable, which is called similarity equation.

First, introduced a one-parameter group transformation of the form

$$G : \begin{cases} \bar{x} = P^x(A)x + Q^x(A) \\ \bar{y} = P^y(A)y + Q^y(A) \\ \bar{\Psi} = P^\Psi(A)\Psi + Q^\Psi(A) \\ \bar{\tau} = P^\tau(A)\tau + Q^\tau(A) \\ \bar{\theta} = P^\theta(A)\theta + Q^\theta(A) \\ \bar{\theta}_w = P^\theta w(A)\theta_w + Q^\theta w(A) \\ \bar{U} = P^U(A)U + Q^U(A) \end{cases}
 \tag{12}$$

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Where, A is the parameter of the transformation. P^S And Q^S are real valued and at least differentiable in their real argument 'A'.

Equations (7) and (8) remain invariant under group of transformations defined by G in equation (12)

$$\begin{aligned} & \frac{\partial \bar{\Psi}}{\partial \bar{y}} \frac{\partial^2 \bar{\Psi}}{\partial \bar{y} \partial \bar{x}} - \frac{\partial \bar{\Psi}}{\partial \bar{x}} \frac{\partial^2 \bar{\Psi}}{\partial \bar{y}^2} - j \frac{d\bar{U}}{d\bar{x}} - \frac{\partial}{\partial \bar{y}} \{ \bar{\tau}_{yx} \} \\ & = \lambda(A) \left\{ \frac{\partial \psi}{\partial y} \frac{\partial^2 \Psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \Psi}{\partial y^2} \right. \\ & \quad \left. - U \frac{dU}{dx} - \frac{\partial}{\partial y} (\tau_{yx}) \right\} \end{aligned} \quad (13)$$

$$\begin{aligned} & \frac{\partial \bar{\Psi}}{\partial \bar{y}} \frac{\partial \bar{\theta}}{\partial \bar{x}} - \frac{\partial \bar{\Psi}}{\partial \bar{x}} \frac{\partial \bar{\theta}}{\partial \bar{y}} - \frac{1}{3pr} \frac{\partial^2 \bar{\theta}}{\partial \bar{y}^2} \\ & = h(A) \left\{ \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} - \frac{1}{3pr} \frac{\partial^2 \theta}{\partial y^2} \right\} \end{aligned} \quad (14)$$

$$F \left(P^\tau \tau + Q^\tau, \frac{P^\Psi}{(P^y)^2} \frac{\partial^2 \Psi}{\partial y^2} \right) = 0 \quad (15)$$

The invariance of equations(13),(14), (15) gives:

$$\left(\frac{(P^\Psi)^2}{(P^y)^2 P^x} \right) = \frac{(P^U)^2}{P^x} = \frac{P^\tau}{P^y} = \lambda(A) \quad (16)$$

$$\frac{P^\Psi P^\theta}{P^x P^y} = \frac{P^\theta}{(P^y)^2} = h(A) \quad (17)$$

$$P^\tau = \frac{P^\Psi}{(P^y)^2} = 1 \quad (18)$$

The invariance of boundary conditions gives:

$$Q^y(A) = Q^\theta = Q^{\theta_w} = Q^U = Q^\tau = 0 \quad (19)$$

From solving above equations we get

$$P^{\bar{}} = (P^y)^2, P^x = (P^y)^3, P^U = P^y, P^\theta = P^{\theta_w}, P^\tau = 1 \quad (20)$$

Here, we get the one-parameter group G, which transforms the differential equation (7) -(10) with the auxiliary conditions invariantly.

The group G is of the following form

$$G: \begin{cases} \bar{x} = (P^y)^3(A)x + Q^x(A) \\ \bar{y} = P^y(A)y \\ \bar{\Psi} = (P^y)^2(A)\Psi + Q^\Psi(A) \\ \bar{\tau} = \tau \\ \bar{\theta} = P^\theta(A)\theta \\ \bar{\theta}_w = P^\theta(A)\theta_w \\ \bar{U} = P^y(A)U \end{cases} \quad (21)$$

4. THE SET OF ABSOLUTE INVARIANTS FOR DEPENDENT AND INDEPENDENT VARIABLES

Now, we obtain a set of absolute invariants for dependent and independent variables. Using these new variables original problem will convert into an ordinary differential equation in new similarity variable via one parameter general group theoretic method. The statement of a basic theorem in group theory [10]; is that: "A function is an absolute invariant of a one-parameter group if it satisfies the following first-order linear differential equation":

$$\sum_{i=1}^7 (\alpha_i S_i + \beta_i) \frac{\partial g}{\partial S_i} = 0 \quad (22)$$

Where, $S_i = x, y, \psi, \tau, U, \theta, \theta_w$

$$\alpha_i = \frac{\partial P^{S_i}}{\partial A}(A_0), \beta_i = \frac{\partial Q^{S_i}}{\partial A}(A_0), i = 1, 2, \dots, 7 \quad (23)$$

Where, A_0 denotes the value of parameter which yield the identity element of the group. By considering

$x_1 = x, x_2 = y, y_1 = \psi, y_2 = \tau, y_3 = U, y_4 = \theta, y_5 = \theta_w$ equation (22) becomes

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$$\begin{aligned}
 & (\alpha_1 x_1 + \beta_1) \frac{\partial g}{\partial x_1} + (\alpha_2 x_2 + \beta_2) \frac{\partial g}{\partial x_2} \\
 & + (\alpha_3 y_1 + \beta_3) \frac{\partial g}{\partial y_1} + (\alpha_4 y_2 + \beta_4) \frac{\partial g}{\partial y_2} \\
 & + (\alpha_5 y_3 + \beta_5) \frac{\partial g}{\partial y_3} + (\alpha_6 y_4 + \beta_6) \frac{\partial g}{\partial y_4} \\
 & + (\alpha_7 y_5 + \beta_7) \frac{\partial g}{\partial y_5} = 0
 \end{aligned} \tag{24}$$

Using the definitions of $\alpha_i, \beta_i; (i = 1, 2, \dots, 7.)$ from Equations (19) and (20), we obtain and the relations between α_i 's & β_i as follows

$$\alpha_1 = 3\alpha_2 = \frac{3}{2}\alpha_3 = \alpha_4, \alpha_5 = 0, \alpha_6 = \alpha_7, \beta_2 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = 0 \tag{25}$$

Now solving this equation (23) and using above relations of α_i, β_i , we obtain independent and dependent similarity variables as follows

$$\begin{aligned}
 \eta &= \frac{y}{(x + \lambda)^{\frac{1}{3}}} \left(\text{where, } \lambda = \frac{\beta_1}{\alpha_1} \right) \\
 F_1(\eta) &= \frac{\Psi + \gamma}{(x + \lambda)^{\frac{2}{3}}} \left(\text{where, } \gamma = \frac{\beta_3}{\alpha_1} \right) \\
 F_2(\eta) &= \frac{U}{(x + \lambda)^{\frac{1}{3}}}, F_3(\eta) = \tau_{yx}, F_4(\eta) = \frac{\theta}{(x + \lambda)^m} \left(\text{where, } m = \frac{\alpha_6}{\alpha_1} \right) \\
 F_5(\eta) &= \frac{\theta_w}{(x + \lambda)^m} \left(\text{By taking } U = (x + \lambda)^{\frac{1}{3}} \right)
 \end{aligned} \tag{26}$$

5. THE REDUCTION TO AN ORDINARY DIFFERENTIAL EQUATION

Independent and dependent absolute invariants are used to convert equations (13) -(15) into the following non-linear ordinary differential equations

$$(F_1')^2(\eta) - 2F_1(\eta)F_1''(\eta) = 1 + 3F_3'(\eta) \tag{27}$$

$$F_4''(\eta) - pr(3mF_4(\eta)F_1'(\eta) - 2F_1(\eta)F_4'(\eta)) = 0 \tag{28}$$

$$F(F_3, F_1'') = 0 \quad (29)$$

Subject to the boundary conditions

$$\eta = 0 \Rightarrow F_1(\eta) = 0, F_1'(\eta) = 0, F_4(\eta) = 0 \quad (30)$$

$$\eta = \infty \Rightarrow F_1(\eta) = 1, F_4(\eta) = 1 \quad (31)$$

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5. POWELL EYRING MODEL

The stress-strain relationship for Powell-Eyring model is:

$$\tau_{yx} = \mu \frac{\partial u}{\partial y} + \frac{1}{B} \sinh^{-1} \left(\frac{1}{C} \frac{\partial u}{\partial y} \right) \quad (32)$$

Where B and C are constants characteristic of the model. Using the non-dimensional quantities given by equation (11) and stream function ψ and similarity variables into equation (32), the Powell-Eyring model transform into following equation

$$\frac{\rho U_0^2}{\sqrt{Re}} F(\eta) = \frac{\mu U_0}{L} \sqrt{\frac{Re}{3}} F_1''(\eta) + \frac{1}{B} \sinh^{-1} \left(\frac{1}{C} \frac{U_0}{L} \sqrt{Re/3} F_1''(\eta) \right) \quad (33)$$

Differentiating and simplifying it we get

$$F_1'''(\eta) = \frac{3\alpha \sqrt{1 + \beta (F_1''(\eta))^2} F_3'(\eta)}{3 + \alpha \sqrt{1 + \beta (F_1''(\eta))^2}} \quad (34)$$

$$\left(\text{By taking } \alpha = 3\mu BC, \beta = \frac{\rho U_0^3}{3\mu LC^2} \right)$$

Above equation (27)-(29) reduced into

$$\begin{aligned} & \alpha \sqrt{1 + \beta (F_1''(\eta))^2} \left[(F_1')^2(\eta) - 2F_1(\eta)F_1''(\eta) - 1 \right] \\ & = \left[3 + \sqrt{1 + \beta (F_1''(\eta))^2} \right] F_1'''(\eta) \end{aligned} \quad (35)$$

$$F_4''(\eta) - pr(3mF_4(\eta)F_1'(\eta) - 2F_1(\eta)F_4'(\eta)) = 0 \quad (36)$$

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With boundary conditions

$$\eta = 0 \Rightarrow F_1(\eta) = 0, F_1'(\eta) = 0, F_4(\eta) = 0 \quad (37)$$

$$\eta = \infty \Rightarrow F_1'(\eta) = 1, F_4(\eta) = 1 \quad (38)$$

6. PRANDTL-EYRING MODEL

The stress-strain relationship for Prandtl-Eyring model is:

$$\tau_{yx} = B \sinh^{-1} \left(\frac{1}{C} \frac{\partial u}{\partial y} \right) \quad (39)$$

Where B and C are constants characteristic of the model. Using the non-dimensional quantities given by equation (11) and stream function ψ defining by equation (6) into equation (39) and simplifying, the above model becomes

$$\frac{\rho U_0^2}{\sqrt{\frac{Re}{3}}} F_3(\eta) = B \sinh^{-1} \left(\frac{1}{C} \frac{U_0}{L} \sqrt{Re/3} F_1''(\eta) \right) \quad (40)$$

Differentiating it we get

$$F_1'''(\eta) = \frac{\sqrt{(1 + \beta'(F_1''(\eta))^2) F_3'(\eta)}}{\alpha'} \quad (41)$$

$$\left(\beta' = \frac{\rho U_0^3}{3\mu LC^2} \alpha' = \frac{B}{3\mu C} \right)$$

Above equation (27)-(29) reduced into

$$\sqrt{(1 + \beta'(F_1''(\eta))^2)} [(F_1')^2(\eta) - 2F_1(\eta)F_1''(\eta) - 1] = 3\alpha'F_1'''(\eta) \quad (42)$$

$$F_4''(\eta) - pr(3mF_4(\eta)F_1'(\eta) - 2F_1(\eta)F_4'(\eta)) = 0 \quad (43)$$

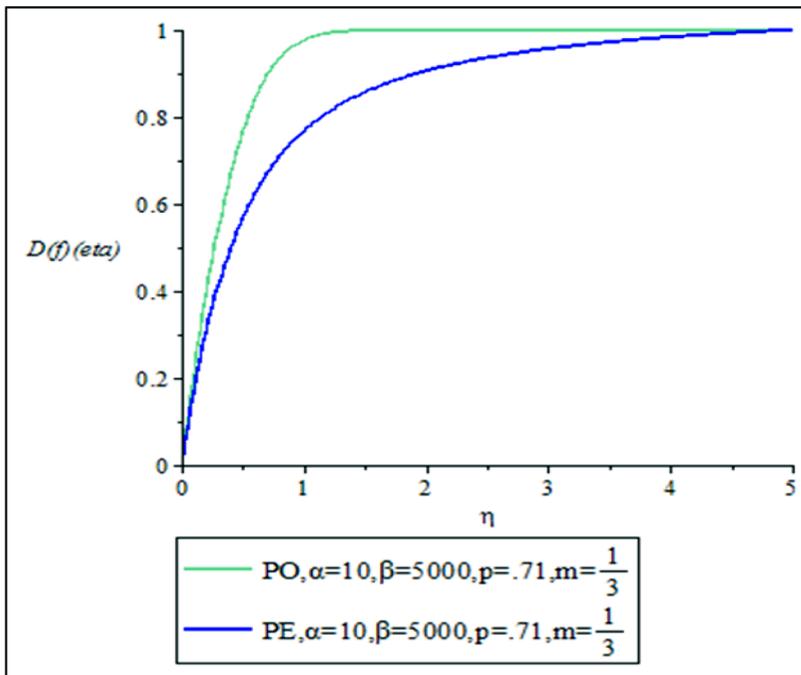


Figure 1: velocity profile for Powell-Eyring and Prandtl-Eyring model.

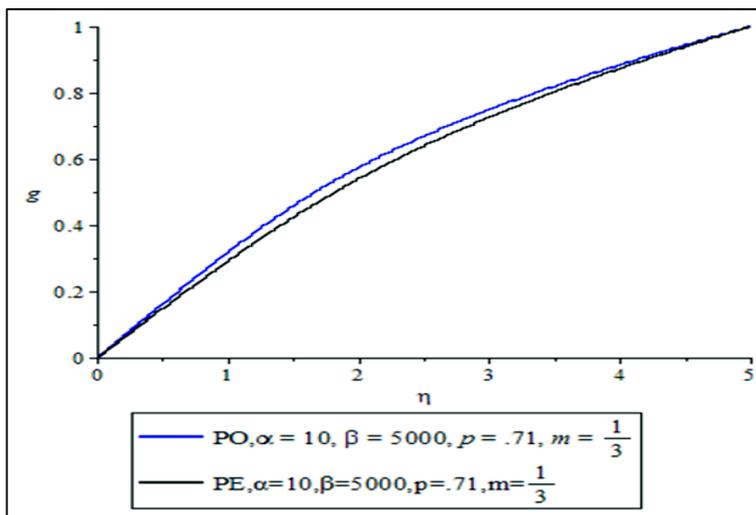


Figure 2: Temperature profile for Powell-Eyring and Prandtl-Eyring model.

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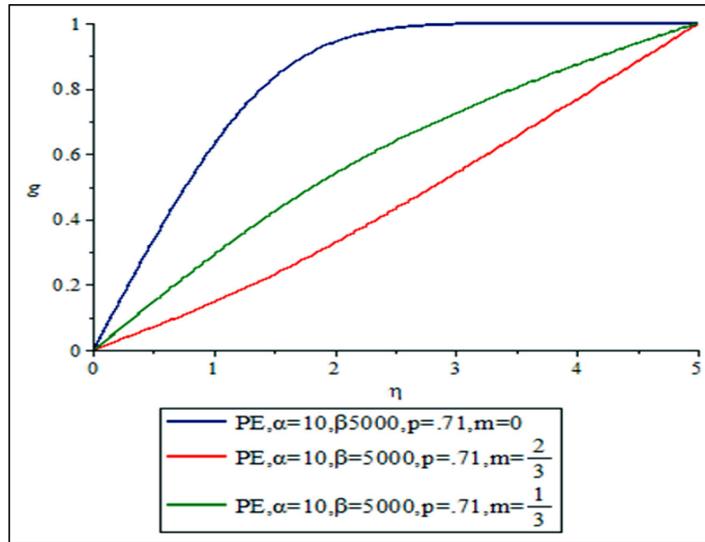


Figure 3: Temperature profile for Powell-Eyring model for different values of m .

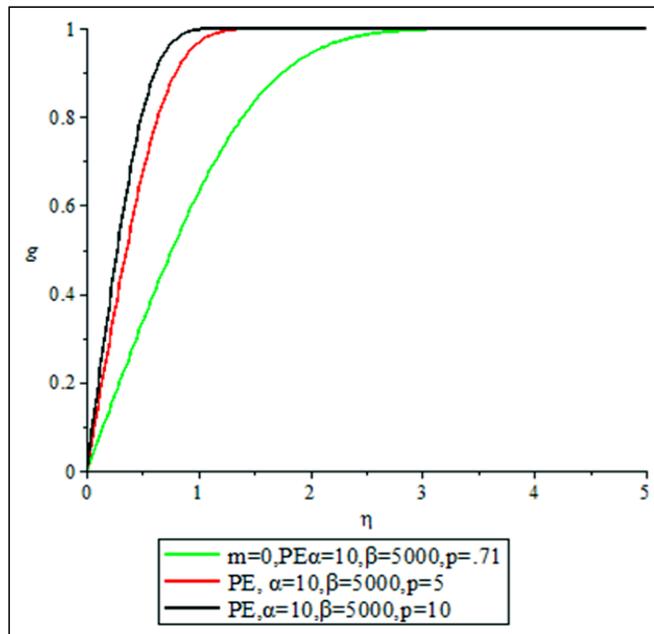


Figure 4: Temperature profile for Prandtl-Eyring model for different Prandtl number.

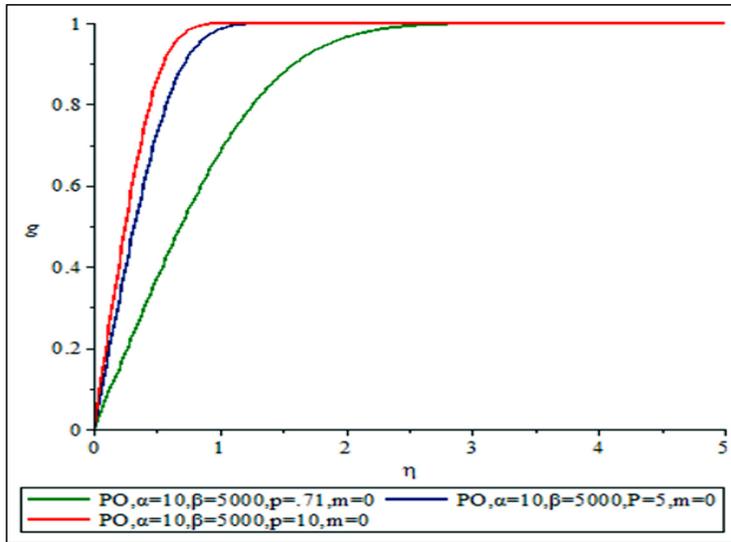


Figure 5: Temperature profile for Powell-Eyring model for different Prandtl number.

$$\eta = 0 \Rightarrow F_1(\eta) = 0, F_1'(\eta) = 0, F_4(\eta) = 0 \quad (44)$$

$$\eta = \infty \Rightarrow F_1'(\eta) = 1, F_4(\eta) = 1 \quad (45)$$

($F_1 = f$ and $F_4 = g$, $pr = p$, PO = Powell-Eyring, PE=Prandtl-Eyring in graph)

CONCLUSION

In the present investigation, the generalized group theoretic method is applied to the governing equations of forced convection flow over a flat plate for a Powell Eyring model and Prandtl Eyring model to derive the proper similarity transformation. The obtained non-linear ordinary differential equations (35),(36),(42) and (43) with the boundary conditions (37),(38), (44) and (45) are solved numerically by Maple ode solver. Figures 1 and 2 gives the influence of parameter α on Temperature profile and velocity profile. Figures 3 and 4 gives the influence of parameter β on velocity profile and temperature profile. Figure 5 gives the temperature profile for Powell-Eyring model for different Prandtl number. As the Prandtl number is increased there is a rapid increase in temperature in the initial stage.

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The main advantage of present analysis is that any non linear boundary value type partial differential equation satisfying Group invariance condition can be transform into boundary value type ordinary differential equation. The model considered for analysis is Powell – Eyring. It is hoped that this model represent a wide cross section of fluids encountered in many process industries. The stress-strain relationship for different type of visco-elastic fluids and similarity equations using these relationships will be helpful to many researchers and engineers for further research.

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