DOI: 10.15415/mjis.2017.61002

Analysis and Modelling of Annamalai Computing Geometric Series and Summability

CHINNARAJI ANNAMALAI

Indian Institute of Technology Kharagpur, Kharagpur, West Bengal

Email: anna@iitkgp.ac.in

Received: August 12, 2017 | Revised: August 19, 2017 | Accepted: August 29, 2017

Published Online: September 01, 2017

Abstract: This paper presents a mathematical model for the formation as well as computation of geometric series in a novel way. Using Annamalai computing methoda simple mathematical model is established for analysis and manipulation of geometric series and summability.This new modelcould be used in the research fields of physics, engineering, biology, economics, computer science, queueing theory, and finance.In this paper, a novel computational model hadalso been developed such that $a\sum_{i=k}^{\infty} y^i = \frac{ay^k}{1-y}$ and $\sum_{i=0}^{\infty}\sum_{j=i}^{\infty} ay^j = \frac{a}{(1-y)^2}$, $(0 < y < 1)$. This could be very interesting and informative for current students and researchers.

Keywords: Annamalai computing method, Computational geometric series

1. INTRODUCTION

Geometric series played a vital role in differential and integral calculusat the earlier stage of developmentand still continues as an important part of the study in mathematics, science,management, and technology.The finite and infinite geometric sequence, series, andsummabilityhave important applications in engineering, economics, computer science, medicine, biology, physics, queueing theory, and finance [3, 4, 5, 8].

It is eventually understoodthat the summations offinite geometric series were

$$\sum_{i\backslash=0}^{n-1} ax^i = \frac{a(x^n-1)}{(x-1)} \text{ and } \sum_{i=1}^{n-1} ax^i = \frac{a(x^n-x)}{x-1},\ x \neq 1$$

and the summations of infinite geometric series were

Mathematical Journal of Interdisciplinary Sciences
Vol-6, No-1,
September 2017
pp. 11–15

$$\sum_{i=0}^{\infty} ax^i = \frac{a}{1-x} \text{ and } \sum_{i=1}^{\infty} ax^i = \frac{ax}{1-x} \quad (0 < x < 1)$$

Geometric series can be used to convert the decimal to a fraction.

For examples,

(i) $0.9999999...... = \frac{9}{10} + \frac{9}{10^2} + \frac{9}{10^3} + ... = \frac{ax}{1-x} = 1$ where $a = 9$ and $x = \frac{1}{10}$

(ii) $9.9999999...... = 9 + \frac{9}{10} + \frac{9}{10^2} + \frac{9}{10^3} + ... = \frac{a}{1-x} = 10$ where $a = 9$ and $x = \frac{1}{10}$

(iii) $0.777777......... = \frac{7}{10} + \frac{7}{10^2} + \frac{7}{10^3} + ... = \frac{ax}{1-x} = \frac{7}{9}$ where $a = 7$ and $x = \frac{1}{10}$

2. Annamalai ComputingGeometric Series

Annamalai computing method [1] provided a novel approach for computation of geometric series in a new way.

$$\sum_{i=-m}^{n-1} ax^i = \frac{a\left(x^n - x^{-m}\right)}{x-1} \Leftrightarrow ax^n = ax^n, \left(x \neq 1\right)$$

Proof

RHS $\Rightarrow ax^n = ax^n$

$$\Rightarrow ax^n = a(x-1)x^{n-1} + ax^{n-1}$$

$$\Rightarrow ax^n = a(x-1)x^{n-1} + a(x-1)x^{n-2} + ... + a(x-1)x^i + ...$$
$$+ a(x-1)x^{-m} + ax^{-m}$$

$$ax^n = ax^n \Rightarrow \sum_{i=-m}^{n-1} ax^i = \frac{a\left(x^n - x^{-m}\right)}{x-1} \quad (1)$$

LHS $\Rightarrow \sum_{i=-m}^{n-1} ar^i = \dfrac{a\left(x^n - x^{-m}\right)}{x-1}$

$$\Rightarrow ax^n = a(x-1)x^{n-1} + a(x-1)x^{n-2} + \ldots + a(x-1)x^i + \ldots. + a(x-1)x^{-m} + ax^{-m}$$

$$\Rightarrow ax^n = ax^n$$

$$\sum_{i=-m}^{n-1} ax^i = \frac{a\left(x^n - x^{-m}\right)}{x-1} \Rightarrow ax^n = ax^n \qquad (2)$$

From (1) and (2) we get:

$$\sum_{i=-m}^{n-1} ax^i = \frac{a\left(x^n - x^{-m}\right)}{x-1} \Leftrightarrow ax^n = ax^n, (x \neq 1)$$

From the above result we can further find the following summability:

(i) $a\sum_{i=k}^{n-1} x^i = \dfrac{a\left(x^n - 1 + 1 - x^k\right)}{x-1} = \dfrac{a\left(x^n - 1\right)}{x-1} - \dfrac{a\left(x^k - 1\right)}{x-1} = a\left(\sum_{i=0}^{n-1} x^i - \sum_{i=0}^{k-1} x^i\right)$

(ii) $\sum_{i=-m}^{n-1} ax^i = \sum_{i=-m}^{-1} ax^i + \sum_{i=0}^{n-1} ax^i = \sum_{i=1}^{m} \dfrac{a}{x^i} + \sum_{i=o}^{n-1} ax^i = a\left(\dfrac{\dfrac{1}{x} - \dfrac{1}{x^{m+1}}}{1 - \dfrac{1}{x}} + \dfrac{x^n - 1}{x-1}\right)$

$= \dfrac{a\left(x^n - x^{-m}\right)}{x-1}$

3. Computational Modelling

The equality $ax = ax$ [7] was used to design the computational modelling,

$$ax = ax \Leftrightarrow ax = (x-1)a + a \Leftrightarrow ax = (x-1)\frac{a}{x^0} + (x-1)\frac{a}{x} + (x-1)\frac{a}{x^2} + \ldots. + (x-1)\frac{a}{x^n} + \frac{a}{x^n}$$

$$ax = ax \Leftrightarrow \sum_{1=0}^{n} \frac{a}{x^i} = \frac{\left(ax - \dfrac{a}{x^n}\right)}{x-1} = \frac{ax\left(1 - \dfrac{1}{x^{n+1}}\right)}{x\left(1 - \dfrac{1}{x}\right)} = \frac{a\left(1 - y^{n+1}\right)}{1-y} \text{ where } y = \frac{1}{x}$$

Now $\frac{a}{y}=\frac{a}{y}\Leftrightarrow\sum_{i=0}^{n-1}ay^{i}=\frac{a\left(y^{n}-1\right)}{y-1}$ where it is understood that $\frac{a}{y}=\frac{a}{y}\Rightarrow ay=ay$

We know that if $0<y<1$, then $\sum_{i=0}^{n-1}ay^{i}=\frac{a\left(1-y^{n}\right)}{1-y}$ and $\sum_{i=0}^{\alpha}ay^{i}=\frac{a}{1-y}$

Similarly, using Annamalai computing geometric series

$$a\sum_{i=k}^{n-1}y^{i}=\frac{a\left(y^{n}-y^{k}\right)}{y-1}(y\neq 1),$$

we can derive $a\sum_{i=k}^{n-1}y^{i}=\frac{a\left(y^{k}-y^{n}\right)}{1-y}$ and $a\sum_{i=k}^{\alpha}y^{i}=\frac{ay^{k}}{1-y}(0<y<1)$

where $k>0$ is an integer constant.

From the above result it can further found the following summability:

(i) $\sum_{i=0}^{\propto}ay^{i}-\sum_{i=k}^{\propto}ay^{i}=\frac{a}{1-y}-\frac{ay^{k}}{1-y}=\frac{a\left(1-y^{k}\right)}{1-y}=\sum_{i=0}^{k-1}ay^{i}$

(ii) $\sum_{i=0}^{\propto}\sum_{j=i}^{\propto}ay^{j}=\sum_{j=0}^{\propto}ay^{j}+\sum_{j=1}^{\propto}ay^{j}+\sum_{j=2}^{\propto}ay^{j}+.....=\frac{a}{1-y}+\frac{ay}{1-y}+\frac{ay^{2}}{i-y}+.....=\frac{a}{\left(1-y\right)^{2}}$

CONCLUSION

In the research study, a novel technique has been introduced to form the generalized geometric series and computing it.Also, a novel computational model was also developed such that $a\sum_{i=k}^{\propto}y^{i}=\frac{ay^{k}}{1-y}$ and $\sum_{i=0}^{\propto}\sum_{j=i}^{\propto}ay^{j}=\frac{a}{\left(1-y\right)^{2}},(0<y<1)$. This new model can be used in the research fields of physics, engineering, biology, economics, computer science, queueing theory, and finance.

REFERENCE

[1] Annamalai C 2009 "Computational geometric series model with key applications in informatics", InternationalJournal of Computational Intelligence Research, **Vol5(4),** pp 485–499.

[2] Annamalai C 2009 "A novel computational technique for the geometric progression of powers of two", Journal of Scientific and Mathematical Research, **Vol 3,** pp 16–17.

[3] Annamalai C 2010 "Applications of Exponential Decay and Geometric Series in Effective Medicine Dosage", Journal Advances in Bioscience and Biotechnology, **Vol 1**, pp 51–54.

[4] Annamalai C 2011 "Computational Model to study the Dose Concentration in Bloodstream of Patients", International Journal of Medical and Pharmaceutical Sciences,**Vol 1(5),** pp 1–7.

[5] Annamalai C 2011 "ACM cryptographic key exchange for secure communications", International Journal of Cryptology Research, **Vol. 3(1),** pp 27–33.

[6] Annamalai C 2015 "A Novel Approach to ACM-Geometric Progression", Journal of Basic and Applied Research International, **Vol. 2(1),** pp 39–40.

[7] Annamalai C 2017 "Annamalai Computing Method for Formation of Geometric Series using in Science and Technology, International Journal for Science and Advance Research in Technology, **Vol. 3(8),** pp 287–289.

[8] https://courses.lumenlearning.com/boundless-algebra/chapter/geometric-sequences-and-series/