




## Determination of Exponential Congestion Factor of Road Traffic Flow Caused by Irregular Occurrences

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### ABSTRACT

The present paper deals exponential congestion model of road traffic flow caused by irregular occurrences. Congestion that is happened by unpredictable events, for example, auto collisions, handicapped vehicles, climate conditions, over burdens and unsafe materials of vehicles. On account of these sorts of sudden occasions, the travel times taken on the roadways are questionable. We established the steady state conditions based on number of vehicles on road links. The large  $c$  values of those links, M/M/1 queues model under the batch service interruptions may be used. The formulation and assumptions of the proposed models have been developed. The exponential congestion factor (ECF) models based on M/MSP/C queuing have been presented. Finally, the numerical examples have also been discussed.



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## 1. Introduction

Queuing theory is the mathematical theory. In everyday life, individuals will experience all sort of queuing issues, for example, remaining at transport quits, going to medical clinic, and heading off to the ticket office to purchase the tickets, etc. With the improvement of economy, vehicles keep up a generous increment in volume of China, waiting theory is so prevalent in street traffic. In the twentieth century, queuing theory presented from Erlang while investigation of phone trade proficiency of correspondence framework. After the Second World War, development of information technology, communication, waiting line concepts got much more attention and also it became important part of operations research.

In applied mathematics, civil engineering traffic flow is the study of interactions between vehicles, drivers, and infrastructure (main road, signage, and traffic control automated devices), with the aim of research and developing an optimal road network with efficient movement of traffic and to minimize traffic congestion problems. Road traffic jams remain a major problem in most of the cities around the world. Rapid increasing crude oil prices, growth of population, workplace change etc., and the demand for transportation

has increased exponentially. Because of the poorly planned road networks, a general outcome in many of the developing regions is the presence of small areas for congestion; poorly traffic planning around these hotspots leads traffic jams. Expanded traffic stream on existing roadways results in an inescapable ascent in congestion. Congestion results delays, diminishing traffic stream, high fuel utilization, and has awful ecological impacts. Number of vehicles increased in the road traffic, therefore, Congestions occurs. Jain and smith (1996) explained that when customer driving from any given starting point to particular given destinations have tendency to select the road which is shortest travel time, this approaches results the underutilization of certain link. Because of Congestion, there are delays, diminishing traffic stream, more fuel utilization and different impacts. Different analysts from different orders have likewise been giving increasingly more consideration to modelling the vehicular travel to improve the productivity of the present express way frameworks. The landing procedure in roadway traffic is displayed as separately arriving Poisson process. Babicheva (2015) showed that methods of queuing theory helped to obtain detailed solutions of the problem of minimise delays at signal-controlled road intersection Mala and Verma (2016) found that Traffic congestion is a basic

phenomenon of increased disruption of the traffic movement. In India, with rapid increasing vehicles on the road, traffic congestion is quickly increasing.

In the literature, much has been studied about the queuing theory technique and its powerful applications in various fields. Yang Shuguo and Yang Xiaoyan (2014) pointed out that it is for all intents and purposes huge to play out the examination to the traffic flow of crossing point in light of the fact that the limit of intersection influences the productivity of main road organize straightforwardly. They further explained that the outcome demonstrates that queuing theory is connected in the investigation of crossing point traffic flow and it can give references to the next comparative plans. Cheah and Smith (1994) discussed the usefulness of state-dependent M/G/c/c queuing models for modelling pedestrian traffic flows. Jain and Smith (1997) shows that how the M/G/C/C state-dependent queuing models for analysing vehicular traffic flow on a roadway segment which can accommodate a finite number of vehicles.

Heidemann (2001) studied about the transient behaviour of M/M/1 queues to analyse the traffic flow based on non-stationary. Recently, Pandey and Gangeshwer (2018) studied about a Markovian queuing model with heterogeneous server based on specific services on health sector. Some references are made such as Heidemann (1996), Daganzo (1997), Wang and Ruskin (2002), Vandaele et al. (2000), Jian et al. (2002), Pal and Sinha (2002), Sheu (2004), Arasan and Koshy (2005) Woensel and Vandaele (2007), Ferreira et al. (2011), and Rajadurai (2018).

In this paper, determination of exponential congestion model of road traffic flow caused by irregular occurrences has been developed. The steady-state conditions settled and exhibited a plan to acquire the stationary number of vehicles on a connection. For those connections with huge c values, the closed-form arrangement of M/M/1 queues batch service interruptions can be utilized as an estimate. The effect of congestion on the traffic flow has been considered and estimated the travel times using Exponential Congestion Model. Section two explains the formulation and Assumptions of the Models. In section three, development of Indicators and Exponential Congestion Factor (ECF) Model based on M/MSP/C [Markovian Poisson arrival process (M), Markovian service process (MSP), the number of servers and roadway capacity (C)] queuing model have been presented. Further, section four Illustration and discussion of the ECF have presented.

## 2. Problem Formulation and Assumptions of the Models

The following are the main consideration of problem formulation:

- (i) Suppose that vehicles going on a roadway and in between an occurrence of traffic disintegrates, thus number of servers and the entire service rate decline.
- (ii) Once an incident occurs, the police patrolling van arrived, and clear the incident. After some time, considerable numbers of servers are come back to their ordinary dimension.
- (iii) Now, due to incident involves congestion and as well as decrease of road capacity. In the present investigation, a low service rate  $\mu' \geq 0$ , influencing each server will be utilized to represent to the significant effect of congestion.

For this proposed model below assumptions have been considered and suppose that interruptions arrives based on a Poisson distribution, and the clearance time or repair time is exponentially distributed:

- i)  $C$  = servers functioning at free speed
- ii)  $\mu$  = service rate
- iii)  $\mu' \geq 0$  = during the mishaps the service rates of these  $C$  servers drop from  $\mu$  to  $\mu' \geq 0$ . After some time the leeway of the intrusion, the service rates are re-established to  $\mu$ .
- iv)  $\lambda = t$  vehicles entries are as per a Poisson procedure
- v) The service times are viewed as free and exponentially distributed.

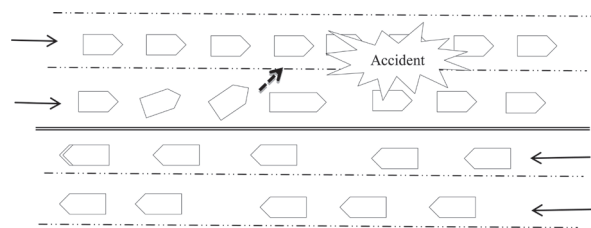


Figure 1: Four-lane Road Traffic Flow.

## 3. Exponential Congestion Factor (ECF) Model based on M/MSP/C Queuing

In the particular situation, when the density of vehicular increases, then the average travel speed of vehicles are deteriorated. There are numerous elements, for example, number of paths, vehicle types, level of substantial vehicles, clearing design and so on. The vehicular density veh/mi-lan has been considered. Some researchers have paid attention to Congestion Model such as Greenshields (1935), Edie (1961), underwood (1961), Drake et al. (1967), Essa and May (1980), and Yuhaski and Smith (1989). In the traffic flow, incidents occur randomly, independent of service completion.

According to decomposition of stochastic principle the number of vehicles on the road link,  $X$  is has form the following equation:

$$X = X_{\infty} + Y$$

Whereas  $X_{\infty}$  = stationary number of vehicles which are arriving at Poisson distribution on a link in interrupted traffic

Y = some extra vehicles or traffic on the link due to traffic incidents.

Further, Baykal and Xiao (2004), and Baykal and Duan (2007) proposed that when some partial failure occurs the Average value and standard deviation of X is obtained, that is Var (X) greater than E(X), on the other hand for uninterrupted traffic, standard deviation (and variance) is equal to its estimated value. Mathematically Var(X) = E(X). Baykal-Gursory et al. (2009, page 5) explained that if at particular time t, the traffic system is faced interruptions, then U (t) = F (failure), otherwise U (t) = F (Normal). The traffic system is called to be in failure state (i, f), suppose if there are number of vehicles i, which is interrupted, whereas the system is called to be normal state (i, N), if there are I vehicles in the system.

The congestion factor A is represented by  $A_i = V_n / V_{free}$  Whereas  $V_n$ = vehicle speed based  $V_{free}$  on when there are n vehicles.

Therefore, vehicle speeds when there are total n vehicles on the road is as follows:

$$V_n = V_{free} \exp \left[ - \left( \frac{n-1}{\beta} \right) \right]^{\gamma}$$

**Table 1:** Estimation of Speed and ECF concerning Number of vehicles [When A= 60,  $\beta = 5373$ ,  $\gamma = 0.2913$ ].

n	n-1	n-1/ $\beta$	$[(n-1)/\beta]^{\gamma}$	$\exp [(n-1)/\beta]^{\gamma}$	$V_n = A \times \exp [(n-1)/\beta]^{\gamma}$	ECF
50	49	0.0091	0.2545	0.7753	47	0.7753
60	59	0.0110	0.2687	0.7644	46	0.7644
70	69	0.0128	0.2812	0.7549	45	0.7549
100	99	0.0184	0.3124	0.7317	44	0.7317
130	129	0.0240	0.3374	0.7136	43	0.7136
160	159	0.0296	0.3586	0.6986	42	0.6986
190	189	0.0352	0.3772	0.6858	41	0.6858
220	219	0.0408	0.3937	0.6746	40	0.6746
250	249	0.0463	0.4087	0.6645	40	0.6645
300	299	0.0556	0.4311	0.6498	39	0.6498
350	349	0.0650	0.4509	0.6370	38	0.6370

Source: Author's own calculation.

Wherein, Jain and Smith (1997, page 9) proposed the, the shape parameter and scale parameters respectively for the exponential system as

$$\gamma = \ln \left[ \frac{\ln(V_a / A)}{\ln(V_b / A)} \right] / \ln \left( \frac{a-1}{b-1} \right)$$

$$\beta = \frac{a-1}{[\ln(A/V_a)]^{1/\gamma}} = \frac{b-1}{[\ln(A/V_b)]^{1/\gamma}}$$

Where,

- Vn = Average travel speed for n vehicles within a road link
- Va= Average travel speed at vehicular density 20 veh/mi-lane
- Vb = Average travel speed at vehicular density 140 veh/mi-lane
- A= V1= Average travel speed of a lane occupant
- n = number of vehicles utilizing the road link

#### 4. Determination of the Exponential Congestion Factor (ECF)

Suppose that in this proposed model, a vehicles speed is that 60 mph, Average travel speed at vehicular density 20 veh/mi-lane is 50, Average travel speed at vehicular density 140 veh/mi-lane is 16, and length of road link is 100 km, number of once side lanes on the road link. Symbolically, Suppose that A=V1=Vfree= 60 mph,  $V_a = 50$ ,  $V_b=16$ , L=50 km, N =2, therefore  $a= 20 \times L \times N$ , =  $20 \times 50 \times 2 = 2000$ , and  $b= 140 \times L \times N = 140 \times 50 \times 2 = 14,000$ .

Table 1 shows that if the number of vehicles is 50 then the average speed will be 47 kmh and ECF is 0.7753, similarly, the average speed and ECF have been computed for various numbers of vehicles based on the  $\gamma$  and  $\beta$ .

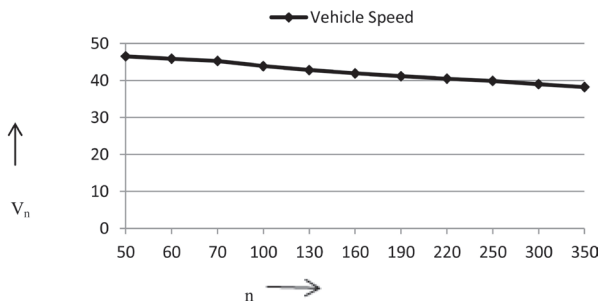


Figure 2: Vehicle Speed.

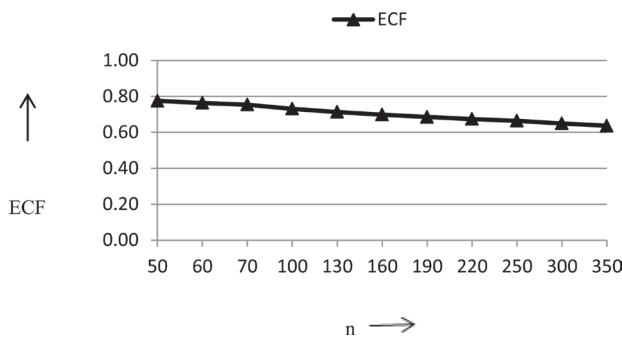


Figure 3: Exponential Congestion Factor (ECF).

Figure 2 shows that when the number of vehicles increases, then the average speed of vehicles is decreases accordingly. Figure 3 indicates that when the number of vehicles increases, the ECF of vehicles decreases accordingly.

### Conclusion

In this paper, exponential congestion factor of road traffic flow caused by irregular occurrences has been determined. It has been observed that when the number of vehicles increases, the average speed is decreases, and on the other side when the number of vehicles increases, the ECF decreases. Numerical examples have been discussed to support the development. The present solution would be useful for researchers, road construction engineers and queuing theory professionals.

### Authorship Contribution

The following contents/ derivations are the contribution of the Authors:

**First Author:** Thaneshwar Lal Verma

- a. Abstract
- b. Section 1: Introduction

- c. Section 3. Exponential Congestion Factor (ECF) Model based on M/MSP/C Queuing
- d. Section 4. Determination of the Exponential Congestion Factor (ECF)
- e. References

**Second Author:** D. K. Gangeshwer

- a. Title
- b. Section 2: Problem Formulation and Assumptions of the Models
- c. Section 4. Determination of the Exponential Congestion Factor (ECF)
- d. Conclusion

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### Conflict of Interest

This is to certify that, there is no conflict of interest of the paper.

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