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Analysis of HIV Model by KTADM

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Abstract This manuscript presents a procedure in the direction of get the emulsion of dynamic pattern in place of HIV infection of CD4+T cells. Intended for methodical mix of non linear differential equation, we are by Kamal Transform Adomian Decomposition Method (KTADM). This procedure gives consistent as a consequence effectual suspension of HIV model.

Keywords: Kamal Transform, Kamal Transform Adomian Decomposition Method (KTADM), HIV infection, $CD4^+$ T cells.

1. INTRODUCTION:

HIV (human immune deficiency virus) is an uncommon key up of retrovirus called lent virus to facilitate causes acquired immunodeficiency syndrome (AIDS)[1].HIV destroys the CD4+T cells lymphocytes of the immune system, which comfort the immune coordination dispute rancid infections. In the cell, HIV produces virus particles. These HIV viruses convert viral RNA into DNA and then making many RNA. With no treatments, HIV advances stylish stages along with three stages of HIV infections are: Acute HIV infection, Clinical latency and AIDS (acquired immune deficiency syndrome). Toward answer the typical in support of HIV infection of CD4+ T cells [2,6].we are fretful in the direction of open out the treatment of the reasoned KTADM.

2. PRELIMINARIES & DEFINITIONS

2.1 Kamal Transform

The Kamal transform is denoted by operator K (.) and Kamal transform of $f(t)$ is defined by the integral equation [7]:

$$\mathrm{K}\left(f\left(t\right)\right)=\mathrm{G}\left(v\right)=\int_{0}^{\infty}f\left(t\right)e^{\frac{-t}{v}}dt,\mathrm{t}\geq 0, and\ k_1\leq v\leq k_2 \qquad (1)$$

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in a set A the function is defined in the form

$$A = \left\{ f(t) : \exists M, k_1, k_2 > 0. |f(t)| < Me^{\frac{|t|}{k_j}}, if\ t \in (-1)^j \times [0, \infty) \right\},$$

where k_1 and k_2 may be finite or infinite and the constant M must be finite number. For existence of Kamal transform are that *f*(*t*) for $t \geq 0$ be piece wise continuous and of exponential order, else it will not be exist.

2.2 Derivative of Kamal Transform

Let function *f*(*t*) then derivative of *f*(*t*) with respect to t and the n^{th} order derivative of the same with respect to t are respectively. Then kamal transform of derivative given by[7],

$$K[f^n(t)] = \frac{1}{v^n} G(v) - \Sigma_{k=0}^{n-1} v^{k-n+1} f^k(0)$$

$n = 1, 2, 3...$ in kamal transform of derivative which gives first and second derivative of *f*(*t*) with respect to $'t'$.

$$K[f'(t)] = \frac{1}{v} G(v) - f(0)$$

$$K[f''(t)] = \frac{1}{v^2} G(v) - \frac{1}{v} f(0) - f'(0)$$

3. COUPLING OF KAMAL TRANSFORM AND ADOMIAN DECOMPOSITION METHOD [8]

Let the ordinary differential equation with initial condition given below:

$$Lf(t) + Rf(t) + Nf(t) = y(t), \tag{2}$$

and the initial condition as followed

$$f(0) = a \tag{3}$$

Here, L is invertible and first order derivative, R is linear differential operator, N is the non- linear term and $y(t)$ is the source term. Applying kamal transform on both sides of the equation (2), we get

$$K\left[Lf(t)+Rf(t)+Nf(t)\right]=K\left[y(t)\right]. \tag{4}$$

Now, the differentiation property of kamal transform and the initial condition from equation (3), is used. Hence, we obtain

$$\frac{1}{v}K\left[f(t)\right]-\left[f(0)\right]=K\left[y(t)\right]-K\left[Rf(t)+Nf(t)\right] \tag{5}$$

$$\frac{1}{v}K\left[f(t)\right]=\left[f(0)\right]+K\left[y(t)\right]-K\left[Rf(t)+Nf(t)\right] \tag{6}$$

$$K\left[f(t)\right]=av+vk\left[y(t)\right]-vK\left[Rf(t)+Nf(t)\right]. \tag{7}$$

Now, we operate inverse of kamal transform on both sides

$$\left[f(t)\right]=a+K^{-1}\{vk\left[y(t)\right]-\left[Rf(t)+Nf(t)\right]\}, \tag{8}$$

So, the solution can be represented as an infinite series:

$$f(t)=\sum_{n=0}^{\infty}f_n(t). \tag{9}$$

and the non- linear term can be decomposed as

$$Nf(t)=\sum_{n=0}^{\infty}A_n(f). \tag{10}$$

Where, $A_n(f)$ are Adomian polynomials of $f_{0,}f_1, f_2 \dots$, given by

$$A_n=\frac{1}{n!}\frac{d^n}{d\tau^n}Nf(\tau)at\tau=0,$$

to get the value of A_n, we put a grouping parameter τ.
usinging equation (9) and (10) in equation (8), we have

$$\sum_{n=0}^{\infty}f_n(t)=F(t)+K^{-1}\left\{vK\left[y(t)\right]-\left[R\sum_{n=0}^{\infty}f_n(t)+\sum_{n=0}^{\infty}A_n(f)\right]\right\}, \tag{11}$$

here, $F(t)$ is the term which is arisen from the source term and the given initial condition. On comparing both sides of equation (10) and using standard adomian decomposition method, we get

$$f_0(t) = F(t), \tag{12}$$

$$f_1(t) = -K^{-1}[vK\{Ry_0(t) + A_0\}], \tag{13}$$

$$f_2(t) = -K^{-1}[vK\{Ry_1(t) + A_1\}], \tag{14}$$

And the general solution can be written as

$$f_{n+1}(t) = -K^{-1}[vK\{Ry_n(t) + A_n\}], \ n \geq 0 \tag{15}$$

Again, applying the kamal transform on right hand side of equation (15) and taking inverse of kamal transform, we get $y_0, y_1, y_2 \ldots,$ which is infinite series of form of the desired solution. The system of non – linear differential equation of HIV infection model of $CD4^+T$ [6]cells are given by equation (16).

$$\left.\begin{aligned} \frac{dA}{dt} &= p - \alpha A + rA\left(1 - \frac{A+P}{A_{\max}}\right) - \delta AM \\ \frac{dP}{dt} &= \sigma Am - \beta P \\ \frac{dM}{dt} &= N\beta P - \gamma M - \delta AM \end{aligned}\right\} \tag{16}$$

With the initial conditions

$$A(0) = r_1 0.1, \ \ P(0) = r_2 = 0 \,\&\, M(0) = r_3 = 0.1. \tag{17}$$

Here, A = Healthy cells , P = Infected cells , M = Virus

Here A(t) , P(t) and M(t) gives the concentration of $CD4^+T$ cells , infected $CD4^+T$ cells by HIV and free HIV virus in the blood , respectively. α, β, γ Represent natural turnover rate of healthy $CD4^+T$ cells, infected $CD4^+T$ cells and death rate of HIV virus, respectively. $\delta > 0$ is the rate of infected cell; p is the rate production of $CD4^+T$ cells. N is the number of virus produced by infected $CD4^+T$ cells. A_{max} gives the information about the maximum population of $CD4^+T$ cells, while, $\left(1 - \frac{A+P}{A_{max}}\right)$ and describes the logistic growth of T cells[6] . And σ denotes the rate of infected cells that became active. All the parameters that are chosen as:

$$\alpha = 0.02, \beta = 0.3, \gamma = 2.4, \delta = 0.0027, \mathrm{p} = 0.1, \mathrm{N} = 10,$$
$$A_{max} = 1500, \sigma = 0.00, \mathrm{r} = 3.$$

Equation (16) is rewritten as,

$$\left.\begin{aligned} \frac{dA}{dt} &= p - \alpha A + rA\left(1 - \frac{A+P}{A_{\max}}\right) - \delta AM \\ \frac{dP}{dt} &= \sigma AM - \beta P \\ \frac{dM}{dt} &= N\beta P - \gamma M - \delta AM \end{aligned}\right\}$$

applying kamal transform on both sides in the above equations

$$K\left\{\frac{dA}{dt}\right\} = K\left\{p - \alpha A + r\,A\left(1 - \frac{A+P}{A_{max}}\right) - \delta MA\right\}$$

$$K\left\{\frac{dA}{dt}\right\} = \left\{K(p) - K(\alpha A) + K\left\{r\,A\left(1 - \frac{A+P}{A_{max}}\right)\right\} - K(\delta MA)\right\}$$

$$\begin{aligned} K\left\{\frac{dA}{dt}\right\} &= K\{p\} - K\{\alpha A\} + K\{rA\} \\ &\quad - K\left\{\frac{rA^2}{A_{max}}\right\} - K\left\{\frac{rAP}{A_{max}}\right\} - K\{\delta MA\}, \end{aligned} \tag{19}$$

$$K\left\{\frac{dP}{dt}\right\} = K\{\sigma MA - \beta P\}$$

$$K\left\{\frac{dP}{dt}\right\} = K\{\sigma MA\} - K\{\beta P\}, \tag{20}$$

$$K\left\{\frac{dM}{dt}\right\} = K\{M\beta P - \gamma M - \delta MA\}$$

$$K\left\{\frac{dM}{dt}\right\} = K\{M\beta P\} - K\{\gamma M\} - K\{\delta MA\}, \tag{21}$$

We get from equation (19)

$$\frac{1}{v}\left\{A(v)\right\}-A(0)=pK\left\{1\right\}-\alpha K\left\{A\right\}+rK\left\{A\right\}$$
$$-\frac{r}{A_{max}}K\left\{A^{2}\right\}-\frac{r}{A_{max}}K\left\{AP\right\}-\delta K\left\{MA\right\}$$
$$\frac{1}{v}\left\{A(v)\right\}=r_{1}+pv-\alpha K\left\{A\right\}+rK\left\{A\right\}$$
$$-\frac{r}{A_{max}}K\left\{A^{2}\right\}-\frac{r}{A_{max}}K\left\{AP\right\}-\delta K\left\{MA\right\}$$
$$A(v)=vr_{1}+pv^{2}-v\alpha K\left\{A\right\}+vrK\left\{A\right\}-\frac{rv}{A_{max}}K\left\{A^{2}\right\}$$
$$-\frac{rv}{A_{max}}K\left\{AP\right\}-v\delta K\left\{MA\right\}$$

$$A(v)=r_{1}v+pv^{2}-v\alpha K\left\{A\right\}+vrK\left\{A\right\}-v\frac{r}{A_{max}}K\left\{A^{2}\right\}-v\frac{r}{A_{max}}K\left\{AP\right\}-v\delta K\left\{MA\right\}, \quad (22)$$

now, we applying kamal transform on equation (20), we arrive at

$$K\left\{\frac{dP}{dt}\right\}=K\left\{\sigma MA\right\}-K\{\beta P\}$$
$$\frac{1}{v}\left\{P(v)\right\}-P(0)=\sigma K(MA)-\beta K(P)$$
$$\frac{1}{v}\left\{P(v)\right\}=r_{2}+\sigma K(MA)-\beta K(P)$$
$$\left\{P(v)\right\}=vr_{2}+v\left[\sigma K(MA)-\beta K(P)\right]$$

$$\left\{P(v)\right\}=vr_{2}+v\sigma K(MA)-v\beta K(P), \quad (23)$$

applying kamal transform on equation (21)

$$\frac{1}{v}\left\{M(v)\right\}-M(0)=M\beta K(P)-\delta K(MA)-\gamma K(M)$$
$$\frac{1}{v}\left\{M(v)\right\}=r_{3}+M\beta K(P)-\delta K(MA)-\gamma K(M)$$
$$\left\{M(v)\right\}=vr_{3}+v\left[M\beta K(P)-\delta K(MA)-\gamma K(M)\right]$$

$$\left\{M(v)\right\}=vr_{3}+vM\beta K(P)-v\delta K(MA)-v\gamma K(M) \quad (24)$$

Now, we assume that

$$\mathrm{E} = \sum_{n=0}^{\infty} E_n, \mathrm{H} = \sum_{n=0}^{\infty} A_n, \mathrm{I} = \sum_{0}^{\infty} P_n, \mathrm{V} = \sum_{0}^{\infty} M_n, \mathrm{E} = A^2, \mathrm{F} = \mathrm{AP}, \mathrm{G} = \mathrm{MA}$$

We put the assumed values in equation (22)

$$\left\{K\sum_{n=0}^{\infty} A_n\right\} = r_1 v + pv^2 - v\alpha K\left\{\sum_0^{\infty} A_n\right\} + vrK\left\{\sum_0^{\infty} A_n\right\} - v.\frac{r}{A_{max}}K\left\{\sum_0^{\infty} E_n\right\} - v\frac{r}{A_{max}}K\left\{\sum_0^{\infty} F_n\right\} - v\delta K\left\{\sum_0^{\infty} G_n\right\} \tag{25}$$

We put the assumed values in equation (23), we have

$$\left\{K\sum_{n=0}^{\infty} P_n\right\} = r_2 v + v\sigma K\left\{\sum_0^{\infty} G_n\right\} - v\beta K\left\{\sum_0^{\infty} P_n\right\}, \tag{26}$$

We put the assumed values in equation (24), we obtained

$$\left\{\sum_{n=0}^{\infty} M_n\right\} = r_3 v + M\beta K\left\{\sum_0^{\infty} P_n\right\} - \delta K\left\{\sum_0^{\infty} G_n\right\} - K\left\{\sum_0^{\infty} M_n\right\} \tag{27}$$

Here E_n, F_n, G_n are Adomian polynomials.

These are given by

$$E_n = \begin{array}{c} E_{0=} A_0^{\ 2} \\ E_{1=} 2A_0A_1 \\ E_{2=} 2A_0A_2 + A_1^{\ 2} \\ E_{3=} 2A_0A_3 + 2A_1A_2 \\ E_{4=} 2A_0A_4 + 2A_1A_3 + A_2^{\ 2} \\ E_{5=} 2A_0A_5 + 2A_1A_4 + 2A_2A_3 \end{array}$$

$$F_n = \begin{array}{c} F_{0=} A_0P_1 \\ F_{1=} A_1P_0 + A_0P_1 \\ F_{2=} A_2P_0 + A_1P_1 + A_0P_2 \\ F_3 = A_3P_0 + A_2P_1 + A_1P_2 + A_0P_3 \\ F_{4=} A_4P_0 + A_3P_1 + A_2P_2 + A_1P_3 + A_0P_4 \\ F_{5=} A_5P_0 + A_4P_1 + A_3P_2 + A_2P_3 + A_1P_4 + A_0P_5 \end{array}$$

$$G_n = \begin{cases} G_{0=} M_0 A_0 \\ G_{1=} M_1 A_0 + M_0 A_1 \\ G_{2=} M_2 A_0 + M_1 A_1 + M_0 A_2 \\ G_3 = M_3 A_0 + M_2 P_1 + M_1 A_2 + M_0 A_3 \\ G_4 = M_4 A_0 + M_3 A_1 + M_2 A_2 + M_1 A_3 + M_0 A_4 \\ G_5 = M_5 A_0 + M_4 A_1 + M_3 A_2 + M_2 A_3 + M_1 A_4 + M_0 A_5 \end{cases}$$

Using the above values in equations (25), (26), (27) respectively and we have following solution from equation (25)

$$K\{A_0\} = r_1 v + p v^2$$
$$K\{A_1\} = -v\alpha K\{A_0\} + vrK\{A_0\} - v\frac{r}{A_{max}}K\{E_0\} - v\frac{r}{A_{max}}K\{F_0\} - v\delta K\{G_0\}$$
$$K\{A_2\} = -v\alpha K\{A_1\} + vrK\{A_1\} - v\frac{r}{A_{max}}K\{E_1\} - v\frac{r}{A_{max}}K\{F_1\} - v\delta K\{G_1\}$$

$$K\{A_{n+1}\} = -v\alpha K\{A_n\} + vrK\{A_n\} - v\frac{r}{A_{max}}K\{E_n\} - v\frac{r}{A_{max}}K\{F_n\} - v\delta K\{G_n\}, \quad (28)$$

from equation (26)

$$K\{P_0\} = r_2 v$$
$$K\{P_1\} = v\sigma K\{G_0\} - v\beta K\{P_0\}$$
$$K\{P_2\} = v\sigma K\{G_1\} - v\beta K\{P_1\}$$

$$K\{P_{n+1}\} = v\sigma K\{G_n\} - v\beta K\{P_n\}, \quad (29)$$

from equation (27)

$$\begin{aligned} K\{M_0\} &= r_3 v \\ K\{M_1\} &= vM\beta K\{P_0\} - v\delta K\{G_0\} - v\gamma K\{M_0\} \\ K\{M_{n+1}\} &= vM\beta K\{P_n\} - v\delta K\{G_n\} - v\gamma K\{M_n\}. \end{aligned} \quad (30)$$

Result

Let us take

$$\mathrm{A}(0) = r_1 = 0.1, P(0) = r_2 = 0, M(0) = r_3 = 0.1$$

And we use kamal inverse transform on both the side of the above equation (28) ,(29) ,(30), we get

$$A(t) = 0.1 + 0.3979526t + 0.59289t^2 + 0.5887188t^3 + 0.438295t^4 + 0.2608633t^5 - 0.129195t^6$$

$$P(t) = 0.000027t + 0.000017273t^2 - 0.00000840515t^3 - 0.00000614728t^4 - 0.000002835857t^5 + 0.000001153t^6$$

$$M(t) = 0.1 - 0.2407253t + 0.28792736t^2 + 0.2304149t^3 + 0.138243t^4 - 0.0663528t^5 + 0.0265397t^6$$

A(t) , P(t) and M(t) gives the concentration of $CD4^{+}T$ cells , infected CDT cells by HIV and free HIV virus in the blood.

DISCUSSION AND CONCLUSION

We employed the amalgamation of kamal transform then adomian decay procedure in the direction of gaining a clogged variety of key of classic HIV model. The innovative means is complimentary on or after redundant precise complexities. Even if the difficult painstaking rejects careful solution, the correctness also reliability of the recent system is assured. Eventually the mathematical solution of non linear differential equation ($CD4^{+}T$) systems can be easily obtained. The illustration proves that the KADM is a suitable method to solve non- linear systems easily.

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