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Effect of Nonthermal Ions on Dust Acoustic Waves in Magnetized Plasma

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ABSTRACT

A fluid model is considered to study the nonlinear dust- acoustic waves (DAW) in magnetized dusty plasma. The model consists of dust parti-cles having negative charge, nonthermal ions, and Boltzmann electrons. Sagdeev Potential equation is derived in the form of an energy integral by applying nonperturbative approach. The pseudopotential (Sagdeev potential) profile is analyzed to study the characteristic of the solitary waves. The study has been made related to the transition of the DAW and the corresponding characters by observing the variation of amplitudes and width of the solitons with Mach number, temperature ratio, density ratio, concentration of nonthermal ions, and the direction cosine. The parametric ranges are estimated numerically to confirm the existence of solitary waves of arbitrary amplitude.

1. Introduction

Nonlinear phenomena in plasma dynamics are most rapid growing branch in both theoretical and experimental research in plasma. This explains different observations of plasma process in space science, planetary ring, comentary tails etc. Plasma is a dispersive medium and due to the strong nonlinearity they may emerge out and move along a nonlinear path. This nonlinear path maintains its shape and size for a long time. Theoretically, this nonlinearity is studied by means of some solitary and shock waves, whose formation and characteristics depend upon the components present in the plasma model. Many researchers have done on solitary waves in multicomponent plasma.

Now-a day, dusty plasma is most rapidly growing branch of plasma research. As the dust particles are heavier compared to other charged particles (ions, electrons, positrons etc.), dust particles become charged positively or negatively due to the interaction with the charged particles. First theoretical investigation on dusty plasma was done by Rao et. al., in 1990. Later, his findings were experimentally verified by Barkan et. al., Till now several theoretical investigations have been done on dusty plasma to study the nonlinearity in terms of dust acoustic solitary waves (DASW). (Mamun and Shukla) had studied DASW in unmagnetized plasma containing of negatively charged dust grains, isothermal electrons and ions.

Recently, the effect of dust temperature on DASW has been studied by many researchers. The effect of ion temperature in dust acoustic waves was numerically analyzed by Rahman et. al., The conditions of dusty plasma have been found in most of the space as well as laboratory plasma, which helps in growing the branch very fast. This leads to the study of various features of nonlinear behavior of plasma, viz. solitary waves, shock waves, double layers, formation of sheath etc.

Since plasma is the ionized state of matter consisting of charged particles, a magnetic is created within the plasma. Plasma waves can propagate both perpendicularly and obliquely to the magnetic field. The magnetic field intensity depends upon the charge, temperatures etc. of ion, electron and other charged particles. The magnetic field is one of important factors to be considered in the propagation of plasma acoustic waves. Quantum effect is also one of the important factors, which can change the characteristic behaviour of the solitons in plasma. Mamun et. al., Hass et. al., Mahmood, Misra and Chowdhury had investigated the characteristics properties of solitary waves in magnetized as well as in unmagnetized plasma to study the influence of quantum effect.

In this paper, we have studied the properties of dust acoustic solitary waves in magnetized plasma in presence of dust grains having negative charge, nonthermal ions,

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and Boltzmann distributed electrons. The study has been made related to the transition of subsonic rarefactive and compressive solitary waves and its corresponding characteristics that vary with the parameters of the pseudopotential derived from the basic equations by nonperturbative approach.

2. Basic equations

In order to study the dust-acoustic waves in magnetized plasma propagating obliquely to the magnetic field, we consider a fluid model consisting of the equations of continuity and momentum of dust having negative charge, nonthermal ions and Maxwellian electrons as follows:

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d v_{dx})+\frac{\partial}{\partial z}(n_d v_{dz})=0 \tag{1}$$

$$\frac{\partial v_{dx}}{\partial t}+\left(v_{dx}\frac{\partial}{\partial x}+v_{dz}\frac{\partial}{\partial z}\right)v_{dx}=\beta_d\frac{\partial\phi}{\partial x}-v_{dy} \tag{2}$$

$$\frac{\partial v_{dy}}{\partial t}+\left(v_{dx}\frac{\partial}{\partial x}+v_{dz}\frac{\partial}{\partial z}\right)v_{dy}=v_{dx} \tag{3}$$

$$\frac{\partial v_{dz}}{\partial t}+\left(v_{dx}\frac{\partial}{\partial x}+v_{dz}\frac{\partial}{\partial z}\right)v_{dz}=\beta_d\frac{\partial\phi}{\partial z} \tag{4}$$

$$n_e=(1-\mu)\exp(\sigma\phi) \tag{5}$$

$$n_i=\left(1+\beta\phi+\beta\phi^2\right)\exp(\phi) \tag{6}$$

where, $\beta_d=\frac{T_i}{B_0 e},\mu=\frac{z_d n_{d0}}{n_{i0}},\sigma=\frac{T_i}{T_e}$

The symbols n_d, n_e, n_i denote respectively the densities of dust, electrons and ions, v_d represents the velocity of dust particles.

We have normalized the densities by n_{i0} (the unperturbed density of ions), the velocity components by $C_d=\left(\frac{kz_d}{m_d}\right)^{\frac{1}{2}}$, plasma potential ϕ by $\frac{kT_i}{e}$, time variable by $\Omega_d^{-1}=\frac{m_d}{B_0 z_d e}$, space variable by $C_d\Omega_d^{-1}$ where k is the Boltzmann constant, Z_d is the number of dust charged particles, m_d is the mass of dust particles, T_i, T_e are the temperatures of ions and electrons respectively, B_0 is the intensity of the magnetic field and e is charge of electron.

3. Derivation of Energy Integral

To derive the energy integral from the above equations, we consider the following transformation, through which the independent variables are related, as

$$\xi=xk_x+zk_z-Mt \tag{7}$$

where M is the wave Mach number, k_x and k_z are the direction cosines.

Then, $\frac{\partial}{\partial x}\equiv k_x\frac{d}{d\xi},\frac{\partial}{\partial z}\equiv k_z\frac{d}{d\xi}$ and $\frac{\partial}{\partial t}\equiv -M\frac{d}{d\xi}$

Applying (7) in the equations (1)-(4), we get

$$k_x v_{dx}+k_z v_{dz}=M\left(1-\frac{\mu}{z_d n_d}\right) \tag{8}$$

$$-\frac{M\mu}{z_d n_d}\frac{dv_{dx}}{d\xi}=\beta_d k_x\frac{d\phi}{d\xi}-v_{dy} \tag{9}$$

$$-\frac{M\mu}{z_d n_d}\frac{dv_{dy}}{d\xi}=v_{dx} \tag{10}$$

$$-\frac{M\mu}{z_d n_d}\frac{dv_{dz}}{d\xi}=\beta_d k_z\frac{d\phi}{d\xi} \tag{11}$$

From (8), (9) and (11), we get

$$-\frac{M^2\mu^2}{z_d^{\,2}}\frac{1}{n_d^{\,3}}\frac{dn_d}{d\xi}-\beta_d\frac{d\phi}{d\xi}=-k_x v_{dy} \tag{12}$$

From Charge neutrality equation, $z_d n_d+n_e-n_i=0$, we get

$$n_d=\frac{1}{z_d}\left\{\left(1+\beta\phi+\beta\phi^2\right)\exp(\phi)-(1-\mu)\exp(\sigma\phi)\right\} \tag{13}$$

Using (13) in (11), we get

$$v_{dz}=-\frac{\beta_d k_z}{M\mu}\Big[\left(1+\beta-\beta\phi+\beta\phi^2\right)\exp(\phi) -\frac{1-\mu}{\sigma}\exp(\sigma\phi)-(1+\beta)+\frac{1-\mu}{\sigma}\Big] \tag{14}$$

From (8), we get

$$k_x v_{dx}=M\left\{1-\frac{\mu}{\left(1+\beta\phi+\beta\phi^2\right)\exp(\phi)-(1-\mu)\exp(\sigma\phi)}\right\} +\frac{\beta_d k_z^{\,2}}{M\mu}\left\{\left(1+\beta-\beta\phi+\beta\phi^2\right)\exp(\phi) -\frac{1-\mu}{\sigma}\exp(\sigma\phi)-(1+\beta)+\frac{1-\mu}{\sigma}\right\} \tag{15}$$

From (10) we get,

$$-\frac{M\mu}{z_d n_d}\frac{d\left(k_x v_{dy}\right)}{d\xi}=k_x v_{dx} \tag{16}$$

From (12), (13), (15) and (16) we get,

$$-\frac{\beta_d k_z^2}{M^2\mu^2}\left\{\left(1+\beta-\beta\phi+\beta\phi^2\right)\exp(\phi)-\left(\frac{1-\mu}{\sigma}\right)\left(\exp(\sigma\phi)-1\right)-(1+\beta)\right\}$$
$$\left\{\left(1+\beta\phi+\beta\phi^2\right)\exp(\phi)-(1-\mu)\exp(\sigma\phi)\right\} \tag{17}$$

Integrating (17) with respect to ξ, wit boundary conditions, $\frac{d\phi}{d\xi}\to 0,\phi\to 0$ as $|\xi|\to\infty$ we get

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2=\left[M^2\ \mu^2\frac{\left(1+\beta+3\beta\phi+\beta\phi^2\right)\ \exp(\phi)-\sigma(1-\mu)\ \exp(\sigma\phi)}{\left\{\left(1+\beta\phi+\beta\phi^2\right)\ \exp(\phi)-(1-\mu)\ \exp(\sigma\phi)\right\}^3}+\beta_d\right]^{-2}$$

$$\left[\frac{\mu M^2}{\left(1+\beta\phi+\beta\phi^2\right)\exp(\phi)-(1-\mu)\exp(\sigma\phi)}-\frac{{}^{\beta}d}{\mu}\left\{\left(1+\beta\phi+\beta\phi^2\right)\exp(\phi)-(\beta+2\beta\phi)\exp(\phi)+2\beta\exp(\phi)-\frac{1-\mu}{\sigma}\exp(\sigma\phi)\right\}\right.$$
$$-\frac{M^2\mu^2}{2}\left\{\left(1+\beta\phi+\beta\phi^2\right)\exp(\phi)-(1-\mu)\exp(\sigma\phi)\right\}^{-2}$$
$$+\beta_d\phi+\beta_d k_z^2\phi\frac{\beta_d k_z^2\left(1+\beta-\beta\phi+\beta\phi^2\right)\exp(\phi)}{\left(1+\beta\phi+\beta\phi^2\right)\exp(\phi)-(1-\mu)\exp(\sigma\phi)}$$
$$-\frac{\beta_d^2 k_z^2}{2M^2\mu^2}\left\{\left(1+\beta-\beta\phi+\beta\phi^2\right)\exp(\phi)\right\}^2+\frac{(1-\mu)\beta_d^2 k_z^2\beta}{M^2\mu^2(1+\sigma)^2}(2-4\phi+\sigma-2\sigma\phi)\exp\left\{(1+\sigma)\phi\right\}$$
$$-\beta_d k_z^2\frac{1-\mu}{\sigma}\frac{\exp(\sigma\phi)}{\left(1+\beta\phi+\beta\phi^2\right)\exp(\phi)-(1-\mu)\exp(\sigma\phi)}-\beta_d k_z^2\left(1+\beta-\frac{1-\mu}{\sigma}\right)\frac{1}{\left(1+\beta\phi+\beta\phi^2\right)\exp(\phi)-(1-\mu)\exp(\sigma\phi)}$$
$$+\frac{\beta_d^2 k_z^2}{M^2\mu^2}\left\{\left(1+\beta-\beta\phi+\beta\phi^2\right)\exp(\phi)-\frac{1-\mu}{\sigma}\exp(\sigma\phi)\right\}\left\{\frac{1-\mu}{\sigma}\exp(\sigma\phi)1+\beta-\frac{1-\mu}{\sigma}\right\}$$
$$-\frac{\beta_d^2 k_z^2(1-\mu)}{M^2\mu^2(1+\sigma)^3}\left\{1+4\beta-\left(\sigma^2+4\sigma+3\right)\beta\phi+\left(\sigma^2+2\sigma+1\right)\beta\phi^2+\sigma^2+\sigma^2\beta+3\sigma\beta+2\sigma\right\}\exp\left\{(1+\sigma)\phi\right\}$$
$$+\frac{\beta_d^2 k_z^2(1-\mu)^2}{2M^2\mu^2\sigma^2}\exp(2\sigma\phi)-\frac{M^2}{2}+\frac{\beta_d}{\mu}\left(1+\beta-\frac{1-\mu}{\sigma}\right)-\frac{\beta_d^2 k_z^2(1+\beta)^2}{2M^2\mu^2}+\frac{\beta_d^2 k_z^2(1-\mu)}{M^2\mu^2}\frac{1+\beta+2\sigma+\sigma^2}{\sigma(1+\sigma)^2} \tag{18}$$

This is of the form,

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2+S(\phi)=0 \tag{19}$$

Where

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$$S(\phi) = -\left[M^2\mu^2 \frac{\left(1+\beta+3\beta\phi+\beta\phi^2\right)\ \exp(\phi) - \sigma(1-\mu)\ \exp(\sigma\phi)}{\left\{\left(1+\beta\phi+\beta\phi^2\right)\ \exp(\phi) - (1-\mu)\ \exp(\sigma\phi)\right\}^3} + \beta_d \right]^{-2}$$

The equation (19) is called the Sagdeev potential equation. This is an energy integral which describes the behavior of an oscillatory particle having unit mass within a potential well $S(\phi)$. The velocity of the oscillatory particle at an instant ξ is $\frac{d\phi}{d\xi}$. $S(\phi)$ is a pseudopotential and is called Sagdeev potential.

4. Conditions for of Solitary Waves

By analyzing Sagdeev potential $S(\phi)$ we can study the characteristics of the solitons. The necessary conditions for the existence of solitary waves are,

(i) $S(0) = S(\phi_m) = 0$, where $|\phi_m|$ is the maximum amplitude of the wave, and $S(\phi) < 0$ between $\phi = 0$ and $\phi = \phi_m$

(ii) $S'(\phi) = 0$ at $\phi = 0$.

The above conditions can be verified by plotting the pseudopotential $S(\phi)$ against the plasma potential ϕ. Here ϕ and $S(\phi)$ represent the amplitude and width of the solitary wave respectively. Physically $S(\phi)$ represents the energy of the waves. The solitary waves are called compressive or rarefactive according as ϕ_m is positive or negative.

5. Results and Discussions

To investigate the solitary waves of arbitrary amplitude, we plot the pseudopotential $S(\phi)$ against the plasma potential ϕ, and numerically analyze the effect of various sets of parameters involved in the pseudopotential on the structures of the solitons. For the same values of the parameters the compressive as well as rarefactive solitary waves have been observed.

In figure 1, the amplitude and potential depth of compressive solitary wave are found to be slightly larger than those of rarefactive wave. The direction cosine $k_z = 0.05$ interprets that the direction of wave is along the perpendicular direction to the magnetic field. As both the waves, have large potential depth, it implies that they can accommodate large number of plasma particles, and hereby gain more energy.

In figure 2, for $k_z = 0.0005$, keeping other parameters unchanged, it has been observed that, the amplitudes of both compressive and rarefactive solitons are increased, while the potential depths become very smaller. This implies that, the solitary waves propagating in this direction carries lesser number of plasma particles in comparison to that propagating perpendicularly to the magnetic field.

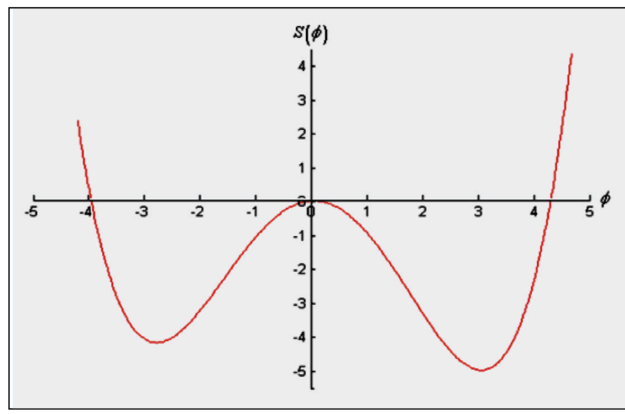

Figure 1: $S(\phi)$ vs ϕ for $\mu = 0.001, \sigma = 0.01, \beta_d = 0.05, \alpha = 0.5, M = 0.3, k_z = 0.05$

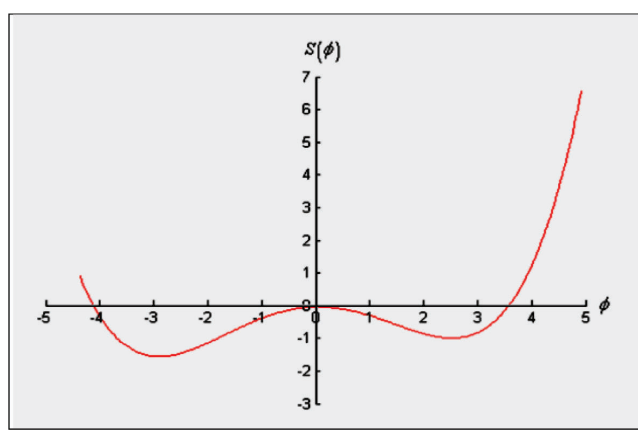

Figure 2: $S(\phi)$ vs ϕ for $\mu = 0.001, \sigma = 0.01, \beta_d = 0.05,$ $\alpha = 0.5, M = 0.3, k_z = 0.0005$

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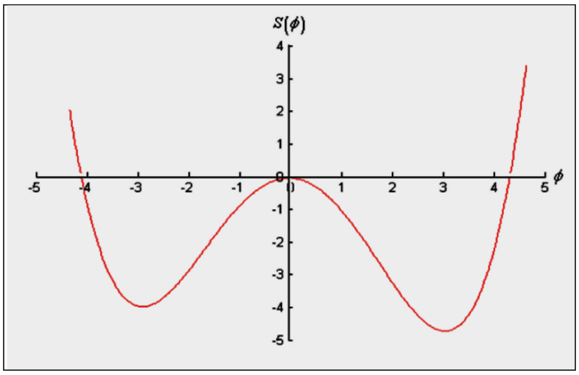

Figure 3: $S(\phi)$ vs ϕ for $\mu = 0.001, \sigma = 0.01, \beta_d = 0.05,$
$\alpha = 0.5, M = 1.1, k_z = 0.05$

Considering $k_z = 0.05$ and 0.0005, if the Mach number is increased to 1.1 from 0.3 (keeping other parameters unchanged) we have observed that in the first case (figure 3), amplitudes of both the waves become larger, but potential depths become smaller. But, in the second case (figure 4), amplitude and width both are found to be decreased in compressive waves than those of rarefactive waves. Thus, the greater Mach number can increase the amplitude, but decrease the potential depth of the solitary waves with direction cosine $k_z = 0.05$. Further, the same Mach number is the cause of existence of very weak compressive solitons in compare to rarefactive one.

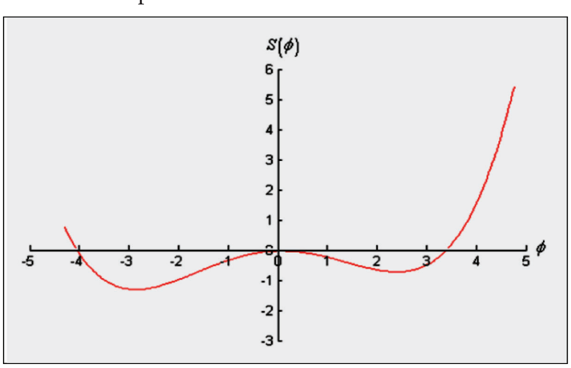

Figure 4: $S(\phi)$ vs. ϕ for μ=0.001,σ=0.01,$\beta_d = 0.05,$
$\alpha = 0.5, M = 1.1, k_z = 0.0005$

In figure 5 and figure 6, considering $M = 0.3$ and increasing α from 0.5 to 0.95, we have observed the solitary waves for $k_z = 0.05$ and 0.0005 respectively. In figure 5, (for $k_z = 0.05$), both compressive and rarefactive solitons have the same amplitude, but greater potential depth is found in compressive waves than that of rarefactive wave. But in figure 6, for compressive soliton both width and amplitude are found to be decreased than rarefactive soliton ($k_z = 0.0005$).

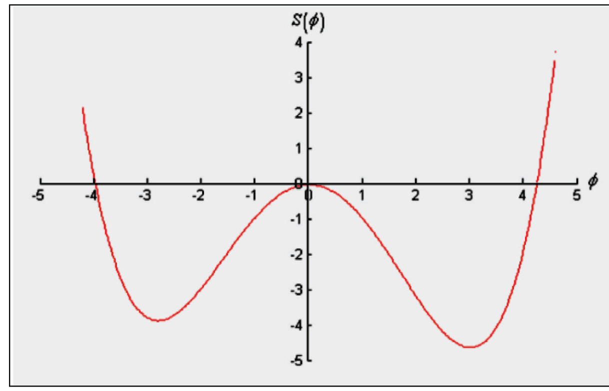

Figure 5: $S(\phi)$ vs. ϕ for $\mu = 0.001$,σ=0.01,$\beta_d = 0.05,$
$\alpha = 0.95, M = 0.3, k_z = 0.05$

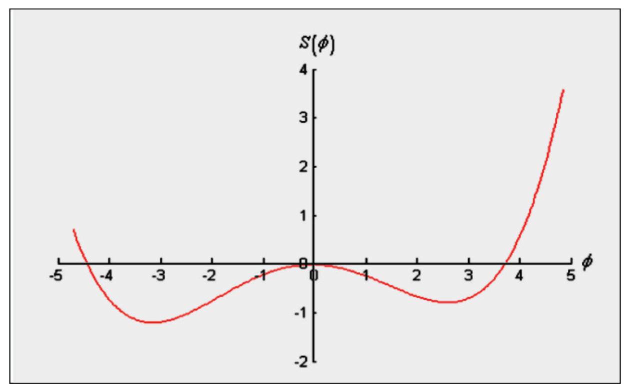

Figure 6: $S(\phi)$ vs. ϕ for $\mu = 0.001, \sigma = 0.01, \beta_d = 0.05,$
$\alpha = 0.95, M = 0.3, k_z = 0.0005$

Further for the same Mach number $M = 0.3$ and same direction cosine $k_z = 0.05$ if α is decreased to 0.1, significant in both width and amplitude of compressive wave is observed than those of rarefactive wave.

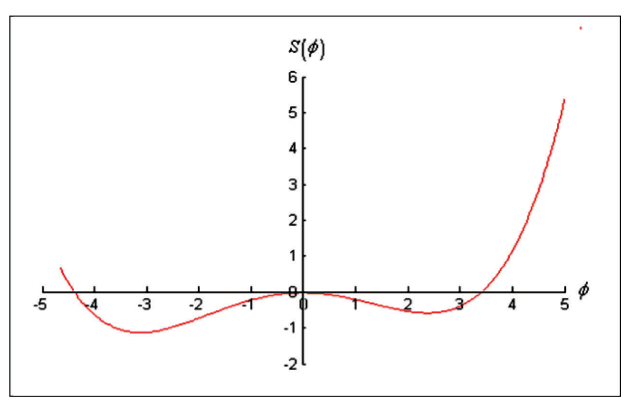

Figure 7: $S(\phi)$vs.ϕ for $\mu = 0.001$,$\sigma = 0.01$,$\beta_d = 0.05,$
$\alpha = 0.1$,$M = 0.3, k_z$=0.05

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Conclusions

In this work on magnetized plasma, the basic equations consisting of dust with negative charge, nonthermal ions and Boltzmann distributed electrons are combined to form an energy integral, called the Sagdeev potential equation by applying nonperturbative approach. The study of arbitrary amplitude solitary waves shows that, the conditions for existence of both subsonic rarefactive and compressive solitary waves are being supported by our fluid model of plasma. The characteristics properties of both rarefactive and compressive solitons are investigated under different situations. The solitary wave propagation is studied in different direction to the magnetic field. Findings could be of interest in understanding some new facts of nonlinear behaviour of plasma acoustic waves and may helps in space and astrophysical research.

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