

Exponentially Weighted Moving Average (EWMA) Chart Based on Six Delta Initiatives

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Abstract A control chart is a graphical device for representation of the data for knowing the extent of variations from the expected standard. The technique of control chart was suggested by W.A. Shewhart of Bell Telephone Company based on three sigma limits. M. Harry, the engineer of Motorola has introduced the concept of six sigma in 1980. In six sigma initiatives, it is expected to produce 3.4 or less number of defects per million of opportunities. Moderate distribution proposed by Naik and Desai is a sound alternative of normal distribution, which has mean and mean deviation as pivotal parameters and which has properties similar to normal distribution. Naik and Tailor have developed various control charts based on this distribution. In this paper an attempt is made to construct a control chart based on six delta initiatives for exponentially weighted moving average chart. Suitable Table for mean deviation is also constructed and presented for the engineers for making quick decisions.

Keywords Moderate distribution, EWMA, Six Delta.

1. INTRODUCTION

The technique of quality control was developed by W. A. Shewhart (1931). It was based on 3sigma control limits. Mikel Harry (1980), the engineer of Motorola has introduced the concept of six sigma. He developed methods for problem solving that combined formal techniques, particularly relating to measurement, to achieve measurable savings in millions of dollars. The companies, which are practicing Six Sigma, are expected to produce 3.4 or less number of defects per million opportunities

R.Radhakrishnan and P.Balamurugan (2010, 2010, and 2016) have developed six sigma based control charts for the number of defectives, exponentially weighted moving average and standard deviation.

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Naik and Desai have proposed an alternative of normal distribution called moderate distribution, which has mean (μ) and mean deviation (δ) as pivotal parameters and which has properties similar to normal distribution. Naik and Taylor (2015, 2016) have suggested 3δ (3 mean deviation) control limits based on moderate distribution. On the basis of 3 control limits, they have developed \bar{X} -chart, R-chart, s-chart and d-chart. Taylor (2016) has also developed moving average and moving range chart and exponentially moving average chart under moderateness assumption.

Similar to six sigma concept, the concepts of six delta can be developed. So here an attempt is made to develop six delta concepts similar to six sigma concept. The six sigma control limits are based on normality assumption and the control limits are determined by using standard deviation (σ -sigma) of the statistic, whereas the six delta control limits are based on moderateness assumption and the control limits are determined by using mean deviation (δ -delta) of the statistic. In six delta initiatives, it is expected to produce 1.7 or less number of defects per million of opportunities. If the companies practicing Six Delta initiatives use the control limits, then no point fall outside the control limits because of the improvement in the quality of the process. Taylor has proposed sample standard deviation(s) chart and sample mean deviation (d) chart based on six delta initiatives. Here an attempt is made to construct a control chart based on six delta initiatives for exponentially weighted moving average. Suitable Table for mean deviation is also constructed and presented for the engineers for making quick decisions.

2. CONCEPTS AND TERMINOLOGIES

A. Upper specification limit (USL)

It is the greatest amount specified by the producer for a process or product to have the acceptable performance.

B. Lower specification limit (LSL)

It is the smallest amount specified by the producer for a process or product to have the acceptable performance.

C. Tolerance level (TL)

It is the difference between USL and LSL, $TL = USL - LSL$

D. Process capability (Cp)

This is the ratio of tolerance level to six times mean deviation of the process.

$$Cp = (TL / 6\sqrt{\frac{\pi}{2}}\delta) = (TL/10.6369\delta) = (USL-LSL)/10.6369\delta$$

E. Mean deviation ($\delta_{6\delta}$): For many purposes mean deviation is the most useful measure of dispersion of a set of numbers. It is the mean of absolute deviation.

F. Quality Control Constant (M_{md})

The constant M_{md} is introduced in this paper to determine the control limits based on six delta initiatives for sample mean deviation.

G. Constant Factor (α)

It is a constant, $0 < \alpha \leq 1$, which is used to calculate exponentially weighted moving average.

3. THREE DELTA CONTROL LIMITS FOR EXPONENTIAL WEIGHTED MOVING AVERAGE (EWMA) CHART

Taylor has proposed exponentially weighted moving average (EWMA) chart under the moderateness with 3δ control limits. Suppose that the main variable of the process x follows **moderate distribution**. The mean of x is $E(x) = \mu$ and mean deviation of x is $\delta_x = \delta'$. The EWMA function is defined as,

$$Z_t = \alpha x_t + (1 - \alpha)Z_{t-1}, \text{ where } 0 < \alpha \leq 1.$$

If the individual \bar{X} are independent random variables with variance $\frac{\sigma^2}{n}$, then the variance of Z_t is defined as

$$\sigma_t^2 = \frac{\sigma^2}{n} \left(\frac{\alpha}{2 - \alpha} \right) [1 - (1 - \alpha)^{2t}]$$

Therefore

$$\sigma_t = \left[\frac{\sigma^2}{n} \left(\frac{\alpha}{2 - \alpha} \right) [1 - (1 - \alpha)^{2t}] \right]^{\frac{1}{2}} = \frac{\sigma}{\sqrt{n}} \left[\left(\frac{\alpha}{2 - \alpha} \right) [1 - (1 - \alpha)^{2t}] \right]^{\frac{1}{2}} \quad (1)$$

Since we are assuming moderateness, the mean error of Z_t is defined as

$$\delta_t = \sqrt{\frac{\pi}{2}} \frac{\delta}{\sqrt{n}} \left[\left(\frac{\alpha}{2 - \alpha} \right) [1 - (1 - \alpha)^{2t}] \right]^{\frac{1}{2}} \quad (2)$$

Thus, the 3δ - control limits of EWMA chart can be determined as follows

$$\text{Central line (C.L.)} = E(\bar{X}_t)$$

$$= \bar{\bar{X}}_t \quad (3)$$

$$\text{Lower control limit (L.C.L.)} = E(\bar{X}_t) - 3\delta_t$$

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$$\begin{aligned} &= \bar{\bar{X}}_t - 3\sqrt{\frac{\pi}{2}} \frac{\delta}{\sqrt{n}} \left[\left(\frac{\alpha}{2-\alpha} \right) [1 - (1-\alpha)^{2t}] \right]^{\frac{1}{2}} \\ &= \bar{\bar{X}}_t - 5.3184 \frac{\delta}{\sqrt{n}} \left[\left(\frac{\alpha}{2-\alpha} \right) [1 - (1-\alpha)^{2t}] \right]^{\frac{1}{2}} \end{aligned} \quad (4)$$

Upper control limit (U.C.L) = $E(\bar{X}_t) + 3\delta_t$

$$\begin{aligned} &= \bar{\bar{X}}_t + 3\sqrt{\frac{\pi}{2}} \frac{\delta}{\sqrt{n}} \left[\left(\frac{\alpha}{2-\alpha} \right) [1 - (1-\alpha)^{2t}] \right]^{\frac{1}{2}} \\ &= \bar{\bar{X}}_t + 5.3184 \frac{\delta}{\sqrt{n}} \left[\left(\frac{\alpha}{2-\alpha} \right) [1 - (1-\alpha)^{2t}] \right]^{\frac{1}{2}} \end{aligned} \quad (5)$$

Where $\bar{\bar{X}}_t$ and δ_t are typically estimated from preliminary data as sample mean and sample mean deviation.

As α is small and if t increases, the effect of starting value soon dissipates and the mean error converges to its asymptotic value.

$$\text{i.e } \delta_t = \sqrt{\frac{\pi}{2}} \frac{\delta}{\sqrt{n}} \left[\left(\frac{\alpha}{2-\alpha} \right) \right]^{\frac{1}{2}}$$

The control limits for EWMA chart are usually based on the asymptotic mean deviation of the statistic. Hence asymptotic 3-control limits for this chart can be derived as following way,

Central line (C.L) = $E(\bar{X}_t)$

$$= \bar{\bar{X}}_t \quad (6)$$

Lower control limit (L.C.L) = $E(\bar{X}_t) - 3\delta_t$

$$\begin{aligned} &= \bar{\bar{X}}_t - 3\sqrt{\frac{\pi}{2}} \frac{\delta}{\sqrt{n}} \left[\left(\frac{\alpha}{2-\alpha} \right) \right]^{\frac{1}{2}} \\ &= \bar{\bar{X}}_t - 5.3184 \frac{\delta}{\sqrt{n}} \left[\left(\frac{\alpha}{2-\alpha} \right) \right]^{\frac{1}{2}} \end{aligned} \quad (7)$$

Upper control limit (U.C.L) = $E(\bar{X}_t) + 3\delta_t$

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$$\begin{aligned}
 &= \bar{\bar{X}}_t + 3\sqrt{\frac{\pi}{2}} \frac{\delta}{\sqrt{n}} \left[\left(\frac{\alpha}{2-\alpha} \right) \right]^{\frac{1}{2}} \\
 &= \bar{\bar{X}}_t + 5.3184 \frac{\delta}{\sqrt{n}} \left[\left(\frac{\alpha}{2-\alpha} \right) \right]^{\frac{1}{2}} \quad (8)
 \end{aligned}$$

4. SIX DELTA BASED CONTROL LIMITS FOR EXPONENTIAL WEIGHTED MOVING AVERAGE (EWMA) CHART

Fix the tolerance level (TL) and process capability (C_p) to determine the process mean deviation δ (termed as $\delta_{6\delta}$), which is calculated from $C_p = (TL/10.6369\delta)$. For a specified TL and C_p of the process, the values of $\delta_{6\delta}$ is calculated, and presented in table 2. The value of M_{md} is obtained by using $P(Z \leq M_{md}) = 1 - \alpha_1$, where $\alpha_1 = 1.7 \times 10^{-6}$ and Z is a standard moderate variate. Thus, the control limits for six delta based control chart for EWMA are determined as,

$$CL_{\delta_{6\delta}} = \bar{\bar{X}}_t \quad (9)$$

$$LCL_{\delta_{6\delta}} = \bar{\bar{X}}_t - M_{md} \frac{\delta_{6\delta}}{\sqrt{n}} \left[\frac{1}{n} \left(\frac{\alpha}{2-\alpha} \right) \left[1 - (1-\alpha)^{2t} \right] \right]^{\frac{1}{2}} \quad (10)$$

$$UCL_{\delta_{6\delta}} = \bar{\bar{X}}_t + M_{md} \frac{\delta_{6\delta}}{\sqrt{n}} \left[\frac{1}{n} \left(\frac{\alpha}{2-\alpha} \right) \left[1 - (1-\alpha)^{2t} \right] \right]^{\frac{1}{2}} \quad (11)$$

Similarly, asymptotic 6δ -control limits for this chart can also be derived.

5. AN EMPIRICAL STUDY FOR EWMA CONTROL CHART AND COMPARISON OF THREE DELTA LIMITS AGAINST SIX DELTA INITIATIVES

To illustrate the use of EWMA control chart with three delta and six delta limits, a data set is taken from Lucas J. M and Crosier R.B. The data, together with the corresponding EWMA values, are shown in Table 1. The target value is taken to be 0. Three delta and six delta control limits are computed from this data set, and control charts are plotted under these two limits.

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Table 1: Data set.

t	Observed value	EWMA = $\alpha + (1-\alpha)$
0	–	0.0
1	1.0	0.25
2	–0.5	0.063
3	0.0	0.047
4	–0.8	–0.165
5	–0.8	–0.324
6	–1.2	–0.543
7	1.5	–0.032
8	–0.6	–0.174
9	1.0	0.119
10	–0.9	–0.135
11	1.2	0.198
12	0.5	0.274
13	2.6	0.855
14	0.7	0.817
15	1.1	0.887
16	2.0	1.166
17	1.4	1.224
18	1.9	1.393
19	0.8	1.245

(a) Three delta control limits (asymptotic) for EWMA chart:

Here the parameters of the EWMA are chosen to be $\alpha = 0.25$, $\delta = 1$, $n = 1$ and the target mean is taken as zero. Hence, the three delta control limits are found as,

$$LCL = -2.010, CL = 0 \text{ and } UCL = 2.010$$

(b) Control limits (asymptotic) based on six delta initiatives for EWMA chart:

For a given data set $USL = 1.393, LSL = -0.032, TL = 1.393+0.032 = 1.425 \approx 1.43$ and $C_p = 1.5$. The value of $\delta_{6\delta} = 0.0896$, which is found from the Table 2, $M_{md} = 5.815$ which is calculated from $P(Z \leq M_{md}) = 1 - \alpha_1$, where $\alpha_1 = 1.7 \times 10^{-6}$. The other parameters are chosen to be $\alpha = 0.25, n = 1$ and the target mean is taken as zero.. Hence, the control limits based on six delta initiatives for EWMA chart for a specified TL and M_{md} are determined as, $CL_{6\delta} \equiv 0, LCL_{6\delta} = -0.1969, UCL_{6\delta} = 0.1969$

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Table 2: Values of $\delta_{6\delta}$ for a specified C_p and TL

TL Cp	1.41	1.42	1.43	1.44	1.45
1.0	0.1326	0.1335	0.1344	0.1354	0.1363
1.1	0.1205	0.1214	0.1222	0.1231	0.1239
1.2	0.1105	0.1112	0.1120	0.1128	0.1136
1.3	0.1020	0.1027	0.1034	0.1041	0.1049
1.4	0.0947	0.0954	0.0960	0.0967	0.0974
1.5	0.0884	0.0890	0.0896	0.0903	0.0909
1.6	0.0828	0.0834	0.0840	0.0846	0.0852
1.7	0.0780	0.0785	0.0791	0.0796	0.0802
1.8	0.0736	0.0742	0.0747	0.0752	0.0757
1.9	0.0698	0.0703	0.0708	0.0713	0.0717
2.0	0.0663	0.0667	0.0672	0.0677	0.0682
2.1	0.0631	0.0636	0.0640	0.0645	0.0649
2.2	0.0603	0.0607	0.0611	0.0615	0.0620
2.3	0.0576	0.0580	0.0585	0.0589	0.0593
2.4	0.0552	0.0556	0.0560	0.0564	0.0568
2.5	0.0530	0.0534	0.0538	0.0542	0.0545

(c) EWMA-charts for data set given in Table 1 based on three delta and six delta limits:

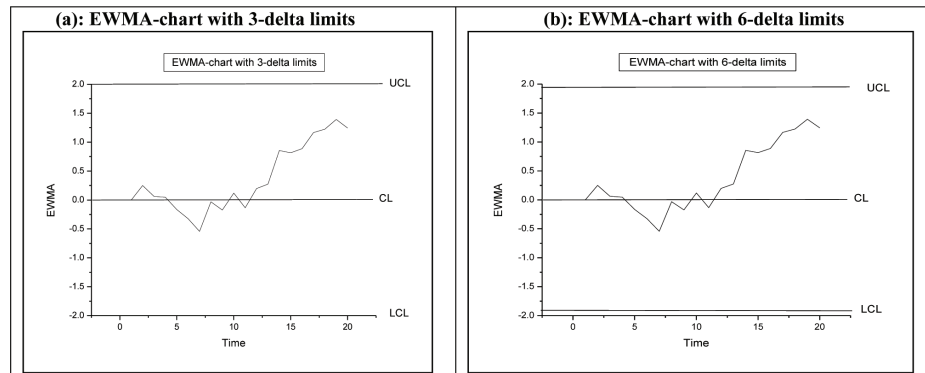


Figure 1

SUMMARY AND CONCLUSION

In this paper, EWMA chart is discussed under three delta and six delta control limits with an illustration. From figure 1, it can be seen that the production process is in statistical control in both the cases. If we compare the UCL and LCL of both the types of charts, six delta control limits are always smaller than the three delta control limits. So it can be concluded that the chart under six delta control lifts are more effective towards detecting the shift in the value of EWMA than the charts under three delta control limits. This is a next generation control chart technique and it will replace existing six sigma technique. So it is recommended that the control charts under six delta control limits should be used for the best results.

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