Math. J. Interdiscip. Sci.Vol. 7, No. 2, March 2019, pp.157–160

CHITKARA

Mathematical Journal of Interdisciplinary Sciences

Journal homepage: https://mjis.chitkara.edu.in/

Performance of Moving Average (MA) Chart Under Three Delta Control Limits and Six Delta Initiatives

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ARTICLE INFORMATION

Received: 13 December, 2018
Revised: 06 January, 2019
Accepted: 25 February, 2019

Published online: March 6, 2019

Keywords:
Moderate distribution, Moving average, Mean deviation, Six Delta.

DOI: https://doi.org/1015415/mjis.2019.72020

ABSTRACT

An SQC chart is a graphical tool for representation of the data for knowing the extent of variations from the expected standard. This technique was first suggested by W.A. Shewhart of Bell Telephone Company based on 3σ limits. M. Harry, the engineer of Motorola has introduced the concept of six sigma in 1980. In 6σ limits, it is presumed to attain 3.4 or less number of defects per million of opportunities. Naik V.D and Desai J.M proposed an alternative of normal distribution, which is named as moderate distribution. The parameters of this distribution are mean and mean deviation. Naik V.D and Tailor K.S. have suggested the concept of 3-delta control limits and developed various control charts based on this distribution. Using these concepts, control limits based on 6-delta is suggested in this paper. Also the moving average chart is studied by using 6-delta methodology. A ready available table for mean deviation is prepared for the quality control experts for taking fast actions.

1. Introduction

The conventional quality control charts developed by (Shewhart 1931) were based on normality assumptions and control limits are calculated using 3 times standard deviation distance from the expected level of quality. The six sigma approach was first suggested Mikel Harry in 1980. At that time he was working as an engineer in Motorola. (Radhakrishnan and Balamurugan 2010, 2011 and 2016) have developed various types of control charts based on six sigma approach. (Naik and Desai 2015) have proposed an alternative of normal distribution called moderate distribution, which has location parameter as mean (μ_0) and scale parameter as mean deviation(δ). (Naik and Tailor 2015) have suggested 3δ (3 mean deviation) control limits based on moderate distribution. On the basis of 3δ control limits, they have developed $\bar{X}$-chart, R-chart, s-chart and d-chart (Tailor 2016) has also developed moving average and moving range chart and exponentially moving average chart under moderateness assumption.

Similar to six sigma concept, six delta concepts is suggested by Tailor K.S. The six sigma control limits are based normality assumption and the control limits are determined by using standard deviation (σ-sigma) of the statistic, whereas the six delta control limits are based on moderateness assumption and the control limits are determined by using mean deviation (δ-delta) of the statistic. In six sigma approach, it is presumed to attain 3.4 or less number of defects per million of opportunities whereas in six delta approach, it is presumed to attain 1.7 or less number of defects per million of opportunities. (Tailor 2017) has proposed sample standard deviation(s) chart, sample mean deviation (d) chart and exponentially weighted moving .average (EWMA) chart based on six delta initiatives. In this paper, control limits based on 6-delta is suggested. Also the moving average chart under moderateness assumption is studied by using 6-delta methodology. A ready available table for mean deviation is also prepared for the quality control experts for taking fast actions.

2. Some Useful Terms

A. upper specification limit (U.S.L)
It is the acceptable maximum value of an item suggested by the quality control expert.

B. lower specification limit (L.S.L)
It is the acceptable minimum value of an item suggested by the quality control expert.

C. level of tolerance (T.L)
It is computed as T.L = U.S.L − L.S.L

D. process capability (*Cp*)
It is defined as,

ISSN No.: 2278-9561 (Print) ISSN No.: 2278-957X(Online) Registration No. : CHAENG/2013/49583 Periodicity: Bi-Annually

Cp = (T.L / 6 $\sqrt{\frac{\pi}{2}}\delta$) = (**T.L** /10.6369 δ) = (U.S.L – L.S.L)/10.6369 δ

E. Mean deviation ($\delta_{6\delta}$):
It is the most intuitively and rationally defined measure of dispersion.

F. Quality Control Constant (S_{md})
The constant S_{md} is introduced to computzze six delta based control limits for the said chart.

G. Span (w)
This is a value, computed from the two subsequent moving averages.

3. Three Delta Control Limits for Moving Average Chart

Suppose that the main variable of the process x follows **moderate distribution**. The mean of x is E(x) = μ_0 and mean deviation of x is $\delta_x = \delta'$. Let $x_1, x_2, \ldots\ldots x_n$ are the sample observations taken from the production process. The moving average of span w at time i is defined as

$$M_i = \frac{x_i + x_{i-1} + \ldots + x_{i-w+1}}{w} \tag{1}$$

The variance of the moving average M_i is derived as,

$$V(M_i) = \frac{1}{w^2}\sum_{j=i-w+1}^{i} V(x_j) = \frac{1}{w^2}\sum_{j=iw+1}^{i}\sigma^2 = \frac{\sigma^2}{w}$$

Hence, its standard deviation $= \frac{\sigma}{\sqrt{w}}$ (2)

So, its mean deviation $= \frac{\sqrt{\frac{\pi}{2}}\delta}{\sqrt{w}}$ (3)

On the basis of 3δ criteria suggested by Naik and Tailor the control limits for proposed chart can be represented as follows.

$$CL_{3\delta} = \mu_0 \tag{4}$$

$$LCL_{3.} = \mu_0 - 3\frac{\sqrt{\frac{\pi}{2}}\delta}{\sqrt{w}} = \mu_0 - 5.3184\frac{\delta}{\sqrt{w}} \tag{5}$$

$$LCL_{3\delta} = \mu_0 + 3\frac{\sqrt{\frac{\pi}{2}}\delta}{\sqrt{w}} = \mu_0 + 5.3184\frac{\delta}{\sqrt{w}} \tag{6}$$

Where μ_0 is specified value of the mean and δ is the process mean error.

4. Six Delta Based Control Limits for Moving Average Chart

First of all, we have to fix the level of tolerance (T.L) and process capability (C_p) to determine the process mean deviation δ (termed as $\delta_{6\delta}$) , which is calculated from Cp = (T.L /10.6369 δ). For a suggested values of T.L and C_p, the values of $\delta_{6\delta}$ are calculated, and presented in table 2. The value of S_{md} is obtained by using $P(Z \le S_{md}) = 1 - \alpha_1$, where $\alpha_1 = 1.7 \times 10^{-6}$ and is a standard moderate variate. Thus, the six delta control limits for the proposed chart are determined as,

$$CL_{6\delta} = \mu_0 \tag{7}$$

$$LCL_{6\delta} = \mu_0 - \frac{S_{md}\delta_{6\delta}}{\sqrt{w}} \tag{8}$$

$$UCL_{6\delta} = \mu_0 + \frac{S_{md}\delta_{6\delta}}{\sqrt{w}} \tag{9}$$

5. An Empirical Study for Moving Average Chart and Comparison of Three Delta Limits Against Six Delta Initiatives

To illustrate the use of moving average chart with three delta and six delta limits, a data set is taken from (Montgomery 2007). The data, together with the corresponding moving averages of span five (w = 5) are shown in Table 1. The target mean is taken to be 0 and process mean deviation is taken to be 1. Three delta and six delta control limits are computed from this data set, and control charts are plotted under these two limits.

Table 1. Data set

Observation		
1	9.45	9.45
2	7.99	8.72
3	9.29	8.91
4	11.66	9.5975
5	12.16	10.11
6	10.18	10.256
7	8.04	10.266
8	11.46	10.7
9	9.2	10.208

10	10.34	9.844
11	9.03	9.614
12	11.47	10.3
13	10.51	10.11
14	9.4	10.15
15	10.08	10.098
16	9.37	10.166
17	10.62	9.996
18	10.31	9.956
19	8.52	9.78
20	10.84	9.932
21	10.9	10.238
22	9.33	9.98
23	12.29	10.376
24	11.5	10.972
25	10.6	10.924
26	11.08	10.96
27	10.38	11.17
28	11.62	11.036
29	11.31	10.998
30	10.52	10.982

(a) Three delta control limits for M. A chart:
Here the target mean (μ_0) is taken as 10, process mean deviation (δ) is taken to be 1 and $w = 5$. The three delta control limits are computed using equations (7), (8), (9) and are found as,

LCL = 7.6216, CL = 10 and UCL = 12.3784

(b) Six-delta limits for M. A chart:
For a given data set, U.S.L = 11.17, L.S.L = 8.72, T.L = 11.17 − 8.72 = 2.45 and C_p = 1.5. The value of $\delta_{6\delta}$ = 0.1536, which is found from the Table 2, $S_{md} = 5.815$ which is calculated from $P\left(Z \le S_{md}\right) = 1-\alpha_1$, where = 1.7 × 10^{-6}. The other values are chosen to be as μ_0 = 10 and w = 5. Hence, six delta limits for M.A chart for a suggested T.L and are computed using equations (7), (8), (9) and are found as,

$$LCL_{6\delta} - 9.6006, CL_{6\delta} = 10, UCL_{6\delta} = 10.3994$$

Table 2. Values of $\delta_{6\delta}$ for a suggested C_p and TL

T.L C_p	2.41	2.42	2.43	2.44	2.45
1.0	0.2266	0.2275	0.2285	0.2294	0.2303
1.1	0.2060	0.2068	0.2077	0.2085	0.2094
1.2	0.1888	0.1896	0.1904	0.1912	0.1919
1.3	0.1743	0.1750	0.1757	0.1765	0.1772
1.4	0.1618	0.1625	0.1632	0.1638	0.1645
1.5	0.1510	0.1517	0.1523	0.1529	0.1536
1.6	0.1416	0.1422	0.1428	0.1434	0.1440
1.7	0.1333	0.1338	0.1344	0.1349	0.1355
1.8	0.1259	0.1264	0.1269	0.1274	0.1280
1.9	0.1192	0.1197	0.1202	0.1207	0.1212
2.0	0.1133	0.1138	0.1142	0.1147	0.1152
2.1	0.1079	0.1083	0.1088	0.1092	0.1096
2.2	0.1030	0.1034	0.1038	0.1043	0.1047
2.3	0.0985	0.0989	0.0993	0.0997	0.1001
2.4	0.0944	0.0948	0.0952	0.0956	0.0960
2.5	0.0906	0.0910	0.0914	0.0918	0.0921

(c) M. A-charts for data set given in Table 1 based on three delta and six delta limits

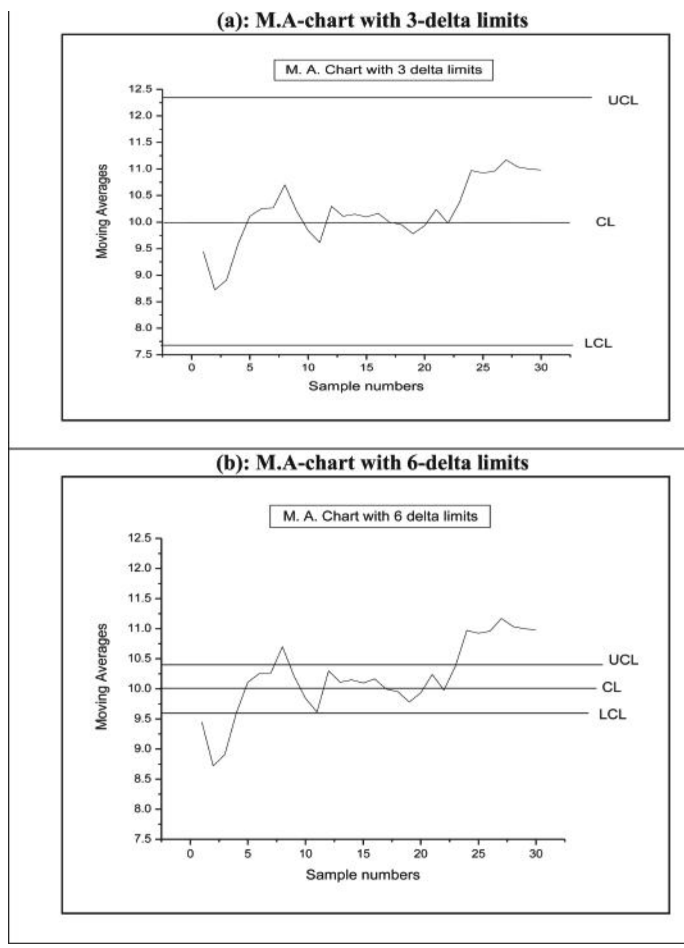

Figure 1

Summary and Conclusion

In this paper, moving average chart is discussed under three delta and six delta control limits with an illustration. From figure 1, it can be seen that the production process is in statistical control when we are applying 3-delta control limits but the process is out of control in six-delta control limits. If we compare the UCL and LCL of both the types of charts, six delta control limits are always smaller than the three delta control limits. So it can be concluded that the chart based on six delta control limits are more effective towards detecting the shift in the value of moving averages than the charts under three delta control limits. This is a next generation control chart technique and it will replace existing six sigma technique. So it is recommended that the control charts under six delta control limits should be used for the best results.

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ISSN No.: 2278-9561 (Print) ISSN No.: 2278-957X (Online) Registration No. : CHAENG/2013/49583