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Influence of Mathematical and Astronomical Developments in Medieval Kerala on Vāstuśāstra

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ABSTRACT

The contribution in the field of mathematics is unparalleled. The concept of zero and the place value system is alone sufficed to place India in a high pedestal. Historians were generally under the impression that Indian supremacy in Mathematics came to an end with Bhaskaracharya (1114–1185) who is also known as Bhaskara II.Recent researches and publications of books like 'Crest of the peacock' written by George Gheverghese Joseph, has brought out the marvelous achievements of Southern India, especially Kerala region after the period of Bhaskaracharya which produced many results surpassing the Europeans in its indigenous style till the advent of Western Education system in early 19th Cent. This medieval contribution includes mathematical analysis and first steps in Calculus and many outstanding discoveries in astronomy. These contributions in Mathematics as well as in Astronomy are now grouped and studied under the title "Kerala School". Accordingly, Sangamagrama Madhava (14th Cent.), doyen of Kerala School, is recognized as the 'Father of infinitesimal Analysis'. In this paper the attempt is made to analyse the influence of Kerala School in the development of traditional building science and architecture. This branch of knowledge is generally categorized under the term 'Vāstuśāstra'.`

1. Introduction

The South India, witnessed an unprecedented advancement in Mathematics during 1300-1700 and further sustained its glory up to 1850. This part of peninsular India tucked between the mountain ranges and Arabian Sea, in the South-West tip, was politically undisturbed during this period unlike Northern India, which was facing continuous threats and attacks from outside. With fertile land blessed with forty rivers and two monsoon seasons and tropical climate made Kerala a well-known location for pepper and other spices from time immemorial. Thus, the rulers of this part of the country were immensely rich through spice trade and were magnanimous and keen in offering boundless patronage for intellectual pursuits. During this medieval period, Arts, Music, Drama,Literature and Sciences like Mathematics, Astronomy, Vāstuvidyā (Architecture & Building Technology) and Āyurveda (indigenous medicine) flourished with no comparison to other parts of the country.

With regard to Mathematics and Astronomy, remarkable contributions were made by Mādhava (c. 1340 – c. 1425), Vaṭeśśeri Parameśvara (c. 1380–1460), Dāmodara (c.1410-c.1500), Kelallūr Nīlakaṇṭha (14 June 1444–1544) Jyeṣṭhadeva (c. 1500–c. 1575) Citrabhānu (16th century) Śaṅkara Vāriyar (c. 1500– c. 1560), Acyuta Piṣāroṭi (c. 1550 –7 July 1621). Putumana Somayāji (c.1660-1740. In the field of Mathematics, Infinitesimal Analysis and the concept of Calculus were the important contributions from historical point of view. In the history of mathematics and astronomy, these contributions from Kerala are currently studied under the title 'Kerala School'. This study has gained worldwide acceptance and acclaim. Mathematical and astronomical inventions served the main purpose of refinement of astronomical theories, its computational techniques and revalidation, revision, reconstruction of various astronomical constants required for the calculations of true longitudes of Sun, Moon, planets and nodes to predict exactly the day-to-day movements of these heavenly objects with respect to a given place and time and predict celestial phenomena like eclipses, conjunction of planets etc. accurately. In the West also, the development of mathematics in the seventeenth century was initially related to Astronomy and it got shifted to other areas of science and technology later. In particular, the Calculus played an important role in deriving many results in Physics and Engineering during

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nineteenth century once Calculus got prime acceptance after Cauchy wrote Cours d'Analyse (1821). In that work, Cauchy endeavored to make Calculus rigorous and to do away with algebraic approaches.It is to be investigated in a similar manner whether the 'Kerala School', where Calculus originated, had made any impact on Engineering and Technology and whether it triggered any research in other applied sciences. By and large, it is generally understood that no remarkable technological advancement was made and the 'invented mathematical ideas' remained within the mathematical and astronomical sphere at theoretical level. The class structure which prevailed in the society during the period might have prevented the exchange of knowledge and ideas between different sections of society. By and large, those who work with hand and those who work with head were not connected in the sharing of knowledge. The only notable exception to the above compartmentalization was the field of 'Architecture and Building Science'. This field broke all the barriers of so called 'caste' or 'class' structure and even the lower tier worker involved in construction were made literate in Sanskrit, the sacred language specifically to get acquainted with the Science behind it. They were taught Sanskrit to get them trained in the 'Building Science'. This intentional move might be an outcome to attain perfection in construction and to ensure the desired construction quality and to harvest a flawless end-product. This has also had far reaching social impact.

Incidentally prominent writers of Vāstuśāstra during this period were closely associated with 'Kerala School' as evident from the fact that Cennās Nārāyaṇan Nampūtiripāṭ (b.1428), the author of the most renowned text, '*Tantrasamuccaya* (TS)' the code of rituals for Kerala temples, has included a chapter on Vāstuvidyā depicting various aspects of temple construction. The author's knowledge in Mathematics and Astronomy is evident from this chapter. He was a contemporary of Parameśvara (1360 -1460 AD) who introduced the revolutionary system of 'Dṛggaṇitam' in Indian astronomy. The author of *Manuṣyālacandrikā* (MC), Tirumaṅgalattu Nīlakaṇṭhan (b. 1510) was a student of the famous mathematician-astronomer Nīlakaṇṭha Somayāji (1444-1544 CE) who wrote *Tantrasaṅgraha* (1500CE). He was also a colleague of Jyeṣṭhadeva (b1510 CE) who wrote *Yuktibhāṣa*. Another popular treatise named *Śilpiratna*-SR (1635 CE) was written by Śrīkumāran. He also authored a text called '*Bhāṣāśilpiratna*' in Malayalam verse, which is roughly a concise translation of the former text. Śrīkumāran had also association with Trikanthiyur, the seat of learning established by Nīlakaṇṭha. Thus, it could be inferred that there was close association with the Mathematicians and they themselves had acquired ample knowledge in Mathematics and Astronomy. This should have had definite impact on the epistemological and philosophical approach to the building science as well.

Writing new books on the subject itself is an indication of some innovation that had evolved in the field. Traditionally, new books were written only when new ideas/approaches were to be presented, otherwise scholars ventured to write only interpretations on well-known texts, called 'vyākhyāna' to exhibit their command over the subject. Both MC and SR refer to TS as the prime text, which indicates that TS had introduced new concepts that were not seen in earlier texts.

2. Traditional Architecture of Kerala

Traditional Kerala Architecture is unique in many aspects when it is compared to the architecture styles in the rest of Indian sub-continent; so also is the Vāstu literature. The development of Kerala style of temple architecture has taken place in around 200 CE subsequent to the stabilisation of migration of people from Northern India. This happened as a part of establishing sixty four grāmas (settlements) from Southern Karnataka area in the North to Chenganoor in the South extending to about 600 kilometres in Kerala. Three stages can be traced in the evolution of Kerala architecture. The first stage began from 4th Century to 10th Century, the second stage from 10th Century to 13th Century and the last stage subsequent to 13th Century. During these stages, when a new style suited to the geography, climate, need and raw material availability was developed, there was also a need for new reference books. As occurred in other spheres of knowledge i.e. Mathematics, Astronomy, Ayurveda etc., there has been revitalization of *Vāstuśāstra* also. Most evidently, the use of rock and stone masonry was reduced and style became more timber intensive and airy. Some of the Indian Vāstu literature is as old as Vedic texts. The texts available on the subject were numerous as evident from the mentioning of more than twenty five reference texts in the preamble of MC. However, as mentioned above there were requirements for new reference texts as new ideas and concepts were being evolved. This may be a reason for the inclusion of a chapter in the Codes for Rituals in Kerala Temples called *Tantrasamuccaya* (TS) written by Cennās Nārāyaṇan Nampūtiripāṭ (b. 1426 CE). Otherwise the possibility is that Cennās introduced the changes and his ideas were presented in TS. Subsequently Tirumaṅgalatt Nīlakaṇṭhan (b. 1500 CE) and Śrīkumāran (c.1590) wrote MC and SR incorporating developments in the school of thought put forward by *Cennās*, in TS. In this we can notice parallels in the Astronomical developments in Kerala which commenced with Dṛggaṇita in the same period. After TS, the second and third stages of Kerala Architectural pursuits were mainly according to TS and MC and SR produced based on same philosophical approach. This could also be considered under the umbrella of 'Kerala School'

When we compare the Vāstu stipulations depicted in *Bṛhatsaṃhitā* (6th Cent.) remarkable difference is noticeable. With TS, 'Digyoni' concept based on perimeter was firmly established and meticulously followed from that period. The length measurement unit, *'Aṅgula'***,** was standardised during that period. Kerala style buildings including temples are essentially with hipped or pyramidal roof made of timber thatched with clay tiles or copper plates. Their supporting structure consisted of columns supported on firm base and wall plates or beams supported by the columns to carry hipped roof .The column base supported on firm bed formed on well compacted earth/firm rock which forms the foundation. Thus the structural system mainly consists of columns, its base and column capital beams/ wall plates, rafters, ridge piece and reapers. Of these the column base and columns are generally made of wood, solid rock or well-built masonry. The roofing support is essentially wooden frame work with rafters, ridge piece, battens (reaper) etc. made of hard wood. The roof elements are mostly timber construction and rarely solid rock-cut panels. Regarding building materials, *Śilpiratna* (Chapter: 14:Verse: 1) states that rock, brick, lime, timber, clay tiles, mud blocks and metals like iron, copper and alloys are the main building materials.

The vertical load carrying member is called pillar or a column in general. The classification of columns according to modern structural engineering is as follows depending upon their slenderness ratio, i.e. the ratio between the length(L) and its least lateral dimension(D).

1. Pedestals
2. Short columns
3. Intermediate columns and
4. Long columns.

Previously columns were designed based on fixed ratios called 'classical order' of columns as we can see in the ancient Roman and Greek design. When we refer to ancients Vastu texts also we can only find constant L/D ratios mentioned. To cite examples *Matsyapurāṇa* specifies (Chapter 250) that the dimension of the pillar shall be 7L /80 where L, is the height of the pillar. This works out to L/D = 80/7=11.2. In the *Bṛhatsaṃhitā*, it is said that minimum lateral dimension of column at top shall be 81/800 of height. (Chapter: Verse : 27). This will be approximately 1/10 when lateral dimension of column at base. As stated earlier for classifying a column in to pedestal, short column etc. and to specify the limits, clear understanding of structural behaviour and mathematical analysis are imperative. Astonishingly, MC Verse 24 Chapter: 5 states as follows:

> *stambhoccābdhīṣu shaḍbhūdharavasunavadigrudrabhāga ikataḥ syāt*
> *stambhādhovistṛtistadvasunavadaśarudrāṃśahīnogratā raḥ*

Lateral dimension (D) shall be 1/4, 1/5, 1/6, 1/7, 1/8, 1/9, 1/10 or 1/11 of its height(L). While referring to the clause 25.1.1 of IS 456-2000, Indian Standard: Plain and reinforced Concrete -Code of Practice, a column or a strut is a compression member, the effective length of which exceeds (3) three times the least lateral dimension and hence when L/D is less than 3 it will be under the category of Pedestals and not columns. MC has precisely set the lower limit as 1/4. Again according to the same code the clause 25.1.2 states that the compression member may be considered as short when both the slenderness with respect to the least is less than 12. Here also MC has set the upper limit as 11 ensuring it is always below twelve to ensure column is 'short' to avoid any chance of buckling. When other international as well as Indian codes are examined we can find that this limit of 12 or conservatively 11 holds well for all most all building materials. The British code and American code are more conservative. The upper limit set by the British Code: BS 8110-1; 1997 is 10. Referring to clause 4.6.2,masonry code ((IS 1905-1987) it could be seen that the same range is set.

In this context, *Manuṣyālacandrikā* (MC) reveals that the author has clearly understood the behaviour of the columns under the application of loads, as they have set lower limit as well as the upper limit for the short column scientifically. As stated above, the rule in the MC stating both the lower limit as 4 and upper limit as 11 is amazing. Is this just a coincidence? Any chance that Tirumaṅgalatt Nīlakaṇṭhan carrying out a mathematical analysis to find the above limits. The possibility cannot be ruled out though derivation for the slenderness limits are not stated in any available medieval Vāstu texts. There are instances where the scholars of Kerala School using Calculus for deriving a curve for profiling a vessel for making a globe rotating synchronous with time (equivalent to a water clock). While interpreting the Verse 22 in *Āryabhaṭīyam* (Part III: *Golapāda*), Nilakaṇṭha Somayāji (NS), the teacher of TN, states as follows:

> *" kāṣṭhamayaṃ samāvṛttaṃ samantataḥ samaguruṃ laghuṃ golam*
> *pāratatailajalaistaṃ bhramayet svadhiyā ca kālasamam" iti..*
> *kāṣṭhamayaṃ na lohādimayam….. svayamapyabhyūhya jñeyaḥ" .*

Here the requirement is to profile a vessel with a bottom opening so as the water level in the vessel falls synchronous with time. (Note that in a cylindrical vessel water will initially fall quickly and then it will slow down due to reduction of head) For this NS suggests to profile (*pariṇahabhedaḥ kartavyaḥ*) the vessel. He adds that "*asya yuktiḥ svayamapi jñātuṃ śakyā, gaṇitayuktivat durvijñeyatābhāvāt*", which means that the logic (required in equation) in this context is also derivable (by ourself) as in the case of any other mathematical problem. Thus it becomes a problem in Hydraulics and applied mathematics. Let us derive the equation for profiling.

Assume a circular vessel with radius R at any height h

Then:

Velocity of draining water $v = \sqrt{2gh}$ where g acceleration due to gravity and h head (height) of water in the vessel.

Let 'a' area of small opening at the bottom though which water drains. And Cd be the coefficient of discharge.

Then water drained out in time

$$dt = \Delta Q = C_d.a,v.dt = C_d.a.\sqrt{2gh}.dt \qquad \ldots\ldots(1)$$

Let drop of height in water level be *dh* and radius at that height be *R*

Then $\Delta Q = \pi R^2.dh$ (2)

Equating (1) & (2)

$$\Delta Q = C_d.a,v.dt = C_d.a.\sqrt{2gh}.dt = \pi R^2.dh$$

Then $\sqrt{h} = \left[\dfrac{\pi}{C_d.a.\sqrt{2g}}\right].R^2.\dfrac{dh}{dt}$

But in order to achieve synchronisation, $\dfrac{dh}{dt}$ shall be constant.

Then $\sqrt{h} = \left[\dfrac{\pi}{C_d.a.\sqrt{2g}}\right].R^2.C$

Or

$$h = \left[\frac{C\ .\pi}{C_d.a.\sqrt{2g}}\right]^2.CR^{4.}$$

$$R^4 = \left[\frac{2g.a^2.C_d{}^2}{C^2\pi^2}\right]h = K.h$$

Where K is a constant

$$R = K.\sqrt[4]{h}$$

That the radius (R) shall reduce with height (h) satisfying the equation $R = K.\sqrt[4]{h}$ where K is constant. Then the water level will drop proportional to time (linearly). This is the profiling (Pariṇāhabhedaḥ) implied by NS. This is a solid proof for the fact that 'Applied Mathematics and Science' was also under the reach of Kerala school.

Another instance is available in MC itself. Regarding the determination of true cardinal directions (Geographical East and North), several methods have been discussed in several texts, of which MC states only one method. This method suggests a correction with respect to a popular method to make the E-W and N-S line more precise. The theory behind the minute correction also cannot be derived without proper understanding of the Astronomy behind it. *Śilpiratna* (1635) had stated the following formula:

'Bhabhuja' is $\pm h \tan\phi + \dfrac{R\sin\lambda.R\omega\sqrt{h^2+s^2}}{R..R\cos\phi}$

$\lambda, \phi, \omega, h, s$ & R are respectively the sun's instant declination, latitude of the place, maximum declination, height of śaṅku (gnomon), length of shadow, R (Radian). A later renowned Astronomy book belonging to Kerala school, *Sadratnamālā* (1819 CE) also states same formula for finding directions.

As mentioned earlier, astronomers, mathematicians and Vāstu experts had close associations and most of the scholars had studied all the three subjects. Tirumaṅgalatt Nīlakaṇṭhan's colleague has written very popular Mathematics text *Yuktibhāṣa* (1530 CE). In this text while deriving the infinite series expansion for PI , example for demonstrating the properties of similar triangle has cited the relation between the length of the collar pin slot and the rafter in a square pavilion. The literal translation of the portion is as follows:

"Then there is a second antecedent figure (triangle). For that E-W line itself is the Base. From the end of the Base, two segments together, along the side of the square, is Height or opposite side. The second radial line from the centre of the circle becomes Hypotenuse. Thus is the second antecedent figure (triangle). Then consider, the consequent figure corresponding to this. The line from the end of point of the first radial line, perpendicular to the second radial line, is the Base. From the meeting point of this Base to the end of the radial line is the Height. The second segment in the side of the square is the hypotenuse. This is the second consequent figure (triangle). When the second rafter becomes the antecedent hypotenuse, the two spans of the rafters together becomes the antecedent height (opposite side of the right triangle). Hence the length of the second rafter will have more length than the first rafter. Proportionately the length of the slot for the wooden pin also will be more. That (the length of the slot) will be the hypotenuse of consequent figure (triangle) and will be parallel to the eave line, which is the opposite side of the antecedent figure (triangle). How the inclination of the rafter and the inclination of the slot are varying proportionally, in the same way the antecedent figures (triangle) and consequent figures (triangle) considered here are also correlated. Here at the end of the E-W line, the second segment in the side of the square is multiplied by the radius, which is the base of the antecedent figure (triangle) and divided by the second radial line (which is the fourth proportion). The result is the base of the consequent figure (Right triangle). Assuming this base as opposite side (height) and then portion of the second radial line between the meeting point of this height and the centre of the circle as adjacent side (base) and the first radial line as hypotenuse there exists a third triangle here also."

In the above, for understanding the similarity of triangles, an analogy with rafters and slits (holes) provided in the rafters to receive wooden pin (ties between rafters) is given. Here as stated earlier, a square pavilion with pyramidal shaped roof with central ridge piece and rafters originating from centre ridge piece in radial direction to seat on to the wall plate. These rafters will be usually tied by providing another square wooden piece, which is locally called Vala. These wooden pins pass through the rafters, and hence there should be holes (slots) to receive these pins. The outer most line of rafters is called eave -reaper is called vamata. The interactions between Vāstu experts and Mathematicians of those days are evident from this quote.

Another noticeable feature in Kerala Vāstu texts is that they have updated the old texts in the light of developments in Mathematics and Astronomy. In the *Tantrasamuccaya* TS, the value of π is given as $355/113$, which is a very good approximation, Further value of π given in SR (1635CE) is 3.141592653 (33215/104348) which is correct up to 9 decimal places. Mentioning of a high accuracy value for π is definitely under the influence of Kerala School of Mathematics where Mādhava (c. 1340-c.1425) gave a rapidly converging series of π, obtaining the infinite series, using the first 21 terms to compute an approximation of π, he obtains a value correct to 11 decimal places (3.14159265359). After Mādhava in 15th cent later books have adopted more accurate values other than 22/7, by finding the convergents according to convenience and need. The continued fraction and convergents were known and widely used in Indian Mathematics as a part of kuttākāra, the solution of indeterminate equations much before its emergence in Europe with the writings of the Dutch mathematician, Christian Huygens (1629-95)

3. Conclusion

Under the influence of Kerala School of Mathematics and Astronomy, Vāstuśāstra in Kerala, which is locally known as 'Taccuśāstra' also, underwent many notable revisions and updating. Consequently a new philosophical and epistemological approach evolved in this field of Vāstuśāstra also. This unbroken tradition has made Kerala Vāstu something unique when compared to those existing in other parts of India. The new features which were embedded could be summarized as follows:

1. TS (15th Cent.) codified the newly evolved temple rituals as well as its architecture. The new code of conduct for rituals and code of practice for design and construction of structures, streamlined the temple tradition. This also influenced the residential architecture as evident from the preamble of MC where it is explicitly stated that it is written in accordance with TS.
2. As the style evolved, Kerala style was more timber intensive. More emphasis was given to 'modular' type of construction which in turn helped for optimization of the use of timber as wood cutting was made easy as there was interrelation between various structural elements such as wall plates, rafters, ridge piece and reapers.
3. Units of measurements were standardised in terms of absolute measurements and the same was popularised.
4. A systematic approach was introduced and meticulously followed for room dimensioning, which ensured systematic and quality construction. Also position of rooms were prescribed according to the local (Kerala) climate and geography deviating from the old texts which might have probably originated in North India.
5. In the design of column beam etc., a mathematical approach was brought in and range for slenderness ratio, span-depth ratio etc. was introduced anticipating certain structural engineering theories introduced three centuries later in the West.
6. Advances in Astronomy were made use in formulating the methods for finding true cardinal direction etc.

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