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Collapsible and Spiky Wave for Dust Acoustic Waves in Dusty Plasmas

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ABSTRACT

In multicomponent dusty plasma, the Sagdeev Potential (SP) approach is employed to formulate the Energy Equation for arbitrary amplitude dust acoustic waves (DAWs), where an amount of electrons is trapped in potential well. The dependence of amplitude and width of the solitons of Sagdeev Potential on plasma parameters is widely discussed. The range of Mach number has determined for solitary waves (SWs) with the help of critical Mach number. The solution of the Energy Equation obtained, has been discussed by expanding the expression for SP in the higher terms of ϕ . The different solutions of Energy Equation give us SWs, breakable waves, collapsible waves and SWs with spiky and explosive nature. The role of temperature ratio on the transformation of SWs to collapsible waves is discussed. With the help of standard values of plasma parameters relevant to such plasma environment, the results so obtained, are discussed. These results may help us to explain the nature of SWs in different astrophysical situations.

1. Introduction

The research on SWs with small amplitude in the dynamics of dusty plasma is gradually becoming an emerging field due to the omnipresence of the different micron sized interplanetary dust particles (IDPs) (Messenger, 2000) in the astrophysical atmosphere. In initial days of studies about the different features of nonlinear waves with small amplitudes, (Washimi and Taniuti 1966) had formulated Korteweg-deVries (KdV) equation in ion acoustic waves in unmagnetized plasma, which afterward became a milestone in nonlinear dynamics. The KdV equation and its modified versions mKdV, KdV-Burgers and mKdV-Burger equations, etc. have been studies in a widespread way by many authors (Kato *et. al.* 1972), (Tappert 1972), (Tagare 1973), (Schamel 1973), (Das *et. al.* 1998), (Sarma and Dev 2014) to reduce inconsistency between theoretical anticipations and laboratory observations. The KdV family of equations have some limitations; especially it is lacking behind (Das *et. al.* 1998) in explain arbitrary amplitude of SWs. Contemporary researcher (Sagdeev 1966) looked into the characteristics of nonlinear waves having arbitrary amplitudes in unmagnetized isothermal plasma where $T_e \gg T_i$ with cold ions by time independent approach. In subsequent developments this approach is known as Sagdeev Potential approach. Many authors (Nejoh 1987, Banerjee and Maitra 2016) extensively used this approach to derive several nonlinear equations to show the possibilities of formation of SWs, double layer (DL), spiky and explosive mode etc. In the solar system, presence of the charged IDPs in the plasmas, the action and reaction between particles and outwardly applied magnetics and electric field is altered by the existence of the plasmas (Goertz 1989). The dust particles increase (decrease) the angular momentum of the Saturn's ring outside (inside) of the synchronous orbit, when the dust particles are reabsorbed by the ring (Goertz *et. al.* 1986). In another study (Rao *et. al.* 1990) had noticed that, if the dust is charged, a new dust acoustic mode for small amplitude SWs has to be generated. This theoretical observation had got boost by the experimental work of (Barkan *et. al.* 1995), which later encouraged the anticipation of several findings. (Ma and Liu 1997) had formulated dust-acoustic rarefactive solitons by injecting the simple charge equation $I_e + I_i = 0$. In a theoretical study (Mamun *et. al.* 1996) investigated the propagation of solitons in dusty plasma considering negatively charge dust particles through the SP approach. (Banerjee and Maitra 2016) investigated the role of dust particles and nonthermal effects on the SWs and DL in the plasma which contains electron, positron, ion and dust particles with the nonthermal electron and positron and reported that the nonthermal characteristic parameter, concentration of dust grain, temperatures of ion and positron have the capacity to modify the characteristics of wave propagation. The dynamical system in plasma produces density cavity in which some electrons to be abundant as trapped electron. So the influences of trapped electrons in

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shaping and spreading of the nonlinear wave in plasma have a great importance. The role of trapped particle in collisionless plasma was discussed by many authors (Schamel 1972, 1980, 1986, Alinejad 2011, Dev *et. al.* 2015) where they found that the trapped particles have significant influences on the propagation of waves like SWs and DL. (Sarma and Dev 2014) studies DAWs in warm dusty plasma with Boltzmann electron and ion distribution with constant dust charge with the help of the family of KdV equations and reported about the formation of compressive and rarefactive solitons and shock waves. (El-Taibany and Wadati 2007) tried to explain the dust-plasma interaction near the Mars for positively charged dust particles in nonthermal dusty plasma by means of the SP approach and also studied the role of nonthermal ions. They had reported that the nonthermal ions-charging currents desperately change the features of the dust acoustic SWs. In a report (Gosh *et. al.* 2008) has described that when the plasma system comes near to a single-ion population, extraordinarily narrow and spiky form of waves are obtained showing a different character of SWs. In a study about spiky SWs and explosive mode in presence of the trapped electron, (Nejoh 1990) reported that in a position where nonlinear ion-acoustic waves spreads in the interplanetary spaces, the spiky together with explosive (bursting) modes of SWs are found in the satellite observations (Gurnen *et. al.*1979, Temerin 1982, Kintner 1983). Generally spiky SWs are nearly associated with the energetic precursor effects in astrophysical situation (Nejoh 1988, 1992, 1994) and formation of these SWs breaks the chance of occurrence of the DL. The emission of solar- energetic- particles type of explosive events can be explained by the exclusive solution. In a recent report (Deka *et. al.* 2018) investigated the features of SWs in a relativistic degenerate ion beam driven magneto plasma and they have sought to explain the salient characteristics of the SWs of relativistic-degenerate plasma.

The primary aim of this study is to highlight the interactions of plasma parameters compatible for this plasma system by deriving the Energy Equation (Sagdeev Potential equation) for collapsible waves, breakable waves and waves in spiky and explosive mode. We have studied theoretically about the features of spiky and explosive waves along with collapsible waves in the different astrophysical situation. Studies of these phenomena on the SWs with arbitrary amplitudes still have necessity to explain the nonlinear waves of arbitrary amplitude in astrophysical situations.

2. Basic Equations and Formulation of Energy Equation

To investigate the properties of dust acoustic solitary wave with arbitrary amplitude in different form, we consider an unmagnetized plasma system consisting micron size dust particles having trapped electrons contaminated with dust-charge grain and Boltzmann ion distribution with negative dust charge fluctuation. To describe the basic equations, the ions are supposed to obey the Boltzmann relation (Das *et. al.* 1989). In fluid description, the nondimensional equations for dust charge dynamics can be written in the form given by:

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}\left(n_d u_d\right)=0 \tag{1}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}=z_d\frac{\partial \phi}{\partial \xi} \tag{2}$$

The equation (1) and (2) are accompanied by the Poissons' equation given by

$$\frac{\partial^2\phi}{\partial x^2}=z_d n_d+\mu_e n_e-\mu_i n_i \tag{3}$$

Although the electrons naturally move freely in the medium but a small fraction of them comes into the potential well and reduces energy continuously. As a result, this phenomenon, the electrons come back and kept inside the potential wells and finally are trapped therein. So the electron density can be considered as a sequence of free and trapped electrons can be described by the following distribution

$$n_e=1+\phi-\frac{4}{3}b\phi^{\frac{3}{2}}+\frac{1}{2}\phi^2-\frac{8}{15}b\phi^{\frac{5}{2}}+............. \tag{4}$$

where $n_\alpha\left(\alpha=d,e,i\right)$ are the density numbers of the dust, electron and ion particles. They are normalized by n_{d0}(number density of unperturbed dust particles), n_{e0} (number density of unperturbed electrons), n_{i0}(number density of unperturbed ions) respectively. The potential ϕ is normalized by $\frac{K_B T_{ef}}{e}$, is the Boltzmann constant, $\mu_e=\frac{n_{eo}}{z_{do}n_{do}}=\frac{1}{\delta-1}$, $\mu_i=\frac{n_{io}}{z_{do}n_{do}}=\frac{\delta}{\delta-1}$, $\delta=\frac{n_{i0}}{n_{e0}}, b=\frac{1-\beta}{\sqrt{\pi}}, c=\frac{1-\beta^2}{\sqrt{\pi}}, \beta=\frac{T_{ef}}{T_{et}}, =\frac{n_{io}}{n_{eo}};$ T_{ef}, and are T_{et} temperatures of free and trapped electrons. The term b quantifies the difference from the isothermality of plasma and b (>0) is the share of resonant electrons to the density of electrons. The conditions β = 1 and β = 0 represent the plasma system of the Maxwellian distribution and flat topped distribution of electrons respectively. In isothermal plasmas, it is evident to deduce the density of electrons by considering $b = 0$ and $c = 0$ but in the case of nonisothermal plasmas, b and c are taken as $0<b<1/\sqrt{\pi}$ and $0<c<1/\sqrt{\pi}$. The Boltzmann ion distribution (Sarma and Dev 2014) is given by

$$n_i=\exp\left(-\sigma_i\phi\right) \tag{5}$$

Here. $\sigma_i = \frac{T_e}{T_i}$ The amount of the dust charge is calculated by the currents gathered in the dust grain. The charging process of dust particles have been investigated by several authors (Northrop 1992, Barnes *et. al.* 1992, Sarma and Dev 2014). For present plasma system, we consider a simple model for dust grains charging as

$$I_e + I_i = 0 \tag{6}$$

Where I_e and I_i are the infinitesimal electron and ion currents charging the dust particles and are expressed (Wang *et. al.* 2016) as

$$I_e = A_1 n_e \exp(zQ_d) \tag{7}$$

$$I_i = A_2 n_i \exp(1 - \sigma_i z Q_d) \tag{8}$$

Where $A_1 = -\pi r_d^2 e n_{e0}\left(\frac{8k_B T_e}{\pi m_e}\right)^{1/2}, A_2 = \pi r_d^2 e n_{i0}\left(\frac{8k_B T_i}{\pi m_i}\right)^{1/2}$

and $z = \frac{e^2 z_{d0}}{r_d k_B T_e}$

The term r_d represents the radius of the dust particle and Q_d is the charge residing on the dust grains normalized to $e^2 z_{d0}$; T_i and T_e are the temperatures of ion and electron. With the help of Equations (7) & (8) and considering the real charge number (Wang *et. al.* 2016), $z_d = zQ_d$, the charge Equation (6) can be simplified to obtain the expression

$$z_d = \frac{-A\left(1 + \frac{\sigma_i^2 \phi^2}{2}\right) + \left(e^{\phi} - \frac{4}{3} b \phi^{\frac{3}{2}} - \frac{8}{15} c \phi^{\frac{5}{2}}\right)}{A\left(1 - \sigma_i - \frac{\sigma_i^3 \phi^2}{2}\right)} \tag{9}$$

Here $A = \frac{n_{i0}}{n_{e0}}\sqrt{\frac{T_i m_e}{T_e m_i}} = l\sqrt{\frac{1}{\kappa \sigma_i}} = \delta\sqrt{\frac{1}{\kappa \sigma_i}}$ and $\kappa = \frac{m_i}{m_i}$

The plasma model is simplified to discuss the role of varying parameters and other relevant small effect such as dissipation and dispersion has neglected in the dynamical system.

To derive the Energy Equation, the coordinate systems are transformed to time independent coordinate system through the stretching coordinate $\xi = x - Mt$, here M represents Mach number for the plasma wave. With the help of this coordinate, equations (1) and (2) can be written as

$$n_d = \frac{M}{u_d - M} \tag{10}$$

and $Mu_d + \frac{1}{2}u_d^2 = -\psi$ (11)

For simplicity, we have used a dimensionless function (Ma and Liu 1997) $\psi = -\int_0^{\phi} z_d \partial\phi$. The equations (10) and (11) which formulated the expression for dust number density as

$$n_d = \left(1 - \frac{2\psi}{M^2}\right)^{\frac{1}{2}} \tag{12}$$

After using Equation (9), the dimensionless function can be expressed as

$$\begin{aligned}
\psi = &-\frac{15}{\sigma_i^{3/2}\sqrt{2-2\sigma_i}}\exp\left[-\frac{\sqrt{2-2\sigma_i}}{\sigma_i^{3/2}}\right] \\
&\times\left[\exp\left[\frac{2\sqrt{2-2\sigma_i}}{\sigma_i^{3/2}}\right]\mathrm{Ei}\left[\phi-\frac{\sqrt{2-2\sigma_i}}{\sigma_i^{3/2}}\right]-\mathrm{Ei}\left[\phi+\frac{\sqrt{2-2\sigma_i}}{\sigma_i^{3/2}}\right]-\exp\left[\frac{\sqrt{2-2\sigma_i}}{\sigma_i^{3/2}}\right]\mathrm{Ei}\left[-\frac{\sqrt{2-2\sigma_i}}{\sigma_i^{3/2}}\right]+\mathrm{Ei}\left[\frac{\sqrt{2-2\sigma_i}}{\sigma_i^{3/2}}\right]\right] \\
&+\frac{82^{1/4}}{\sigma_i^3\left(-\frac{\sigma_i^3}{-1+\sigma_i}\right)^{3/4}}\left[\tan^{-1}\left[\sqrt{\phi}\left(\frac{\sigma_i^3}{2-2\sigma_i}\right)^{1/4}\right]\left(2\sqrt{2}c-5b\sqrt{-\frac{\sigma_i^3}{-1+\sigma_i}}\right)-\tanh^{-1}\left[\sqrt{\phi}\left(\frac{\sigma_i^3}{2-2\sigma_i}\right)^{1/4}\right]\left(2\sqrt{2}c+5b\sqrt{-\frac{\sigma_i^3}{-1+\sigma_i}}\right)\right] \\
&-\frac{15\sqrt{2}A\tan^{-1}\left[\frac{\phi\sigma_i^{3/2}}{\sqrt{2}\sqrt{-1+\sigma_i}}\right]\sqrt{-1+\sigma_i}\sigma_i^2}{\sigma_i^{9/2}} \\
&+\frac{1}{\sigma_i^3}\left[\frac{16}{3}\sqrt{\phi}(15b+2c\phi)+15\sqrt{2}A\tan^{-1}\left[\phi\sqrt{\frac{\sigma_i^3}{-2+2\sigma_i}}\right]\sqrt{\frac{\sigma_i^3}{-1+\sigma_i}}+15A\phi\sigma_i^2\right]
\end{aligned} \tag{13}$$

Where Ei represents the exponential integral function[1] Ei(z). To formulate the Sagdeev Potential for our solution along with the Energy Equation, we have normalized the dust charge by $z_d = \frac{\omega}{15\omega_0}$, where the dust surface floating-

[1] $\mathrm{Ei}(z) = -\int_{-z}^{\infty}(e^{-t}/t)dt$, where the principal value of the integral is taken

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potential (El-Labany and El-Taibany 2003) in an infinite place compared to the unperturbed plasma potential is given by ω_0. The quantity ω_0 can be obtained from the equation (9) by using $\phi=0$ and is evaluated as

$$\omega_0=\frac{1-\sigma_i}{1-A} \tag{14}$$

Further, imposing the boundary condition $\frac{d\phi}{d\xi}\to 0$ as, $\xi\to\pm\infty$ the basic sets of Equations (1)-(3) are transformed to the form

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2+V(\phi)=0 \tag{15}$$

The Equation (15) is known as Energy Equation and the Sagdeev Potential $V(\phi)$ is written by the following expression

$$\begin{aligned}V(\phi)&=\frac{1}{\delta-1}\left[1-\left(exp(\phi)-\frac{8}{15}b\phi^{\frac{5}{2}}-\frac{16}{105}c\phi^{\frac{7}{2}}\right)\right]+\frac{\delta}{\sigma_i(\delta-1)}\left[1-exp(-\sigma_i\phi)\right]\\&+M^2\left[1-\left(1-\frac{2\psi}{M^2}\right)^{\frac{1}{2}}\right]=\frac{1}{2}B_1\phi^2-\frac{2}{5}B_2\phi^{\frac{5}{2}}-\frac{1}{3}B_3\phi^3-\frac{2}{7}B_4\phi^{\frac{7}{2}}\end{aligned} \tag{16}$$

Where $B_1=\dfrac{15-30A+15A^2+AM^2-15\delta+30A\delta-15A^2\delta-M^2\delta-AM^2\sigma_i+AM^2\delta\sigma_i+M^2\delta\sigma_i^2-AM^2\delta\sigma_i^2}{2M^2(A-1)(\delta-1)(\sigma_i-1)}$,

$$B_2=\frac{8(\delta-A)b}{15(A-1)(\delta-1)}$$

$$\begin{aligned}B_3=&\frac{1}{6M^4(A-1)(\delta-1)(\sigma^i-1)^2}\{675-2025A+202A^2-675A^3+45M^2-45AM^2+AM^4-675\delta\\&+2025A\delta-2025A^2\delta+675^3\delta-45M^2\delta+45AM^2\delta-M^4\delta+M^4\sigma_i^3-AM^4\sigma_i^3-M^4\delta\sigma_i^3+AM^4\delta\sigma_i^3\\&-45M^2\sigma_i+45AM^2\sigma_i-2AM^2\sigma_i+45M^2\delta\sigma_i-45AM^2\delta\sigma_i+2M^4\delta\sigma_i-M^4\sigma_i^3\sigma_i+AM^4\sigma_i^3\sigma_i\\&+M^4\delta\sigma_i^3\sigma_i-AM^4\delta\sigma_i^3\sigma_i+2AM^4\sigma_i^3-2M^4\delta\sigma_i^3-AM^4\sigma_i^4+M^4\delta\sigma_i^4\}\end{aligned}$$

$$\begin{aligned}B_4=&\frac{8}{105M^2(A-1)(\delta-1)(\sigma_i-1)}\{105b-105Ab-5cM^2+7AcM^2-105b\delta\\&+105Ab\delta-2cM^2\delta+5cM^2\sigma_i-7AcM^2\sigma_i+2cM^2\delta\sigma_i\}\end{aligned}$$

The analysis on the SP involves (Sah and Goswami 1994, Shukla and Mamun 2002) some conditions to form solitions otherwise there is a possibility to form double layers. The conditions are namely.

(i) $V(\phi)=0$ in the vicinity of $\phi=0$ for undisturbed conditions, where $\xi\to\pm\infty$,

(ii) $\left(\frac{dV(\phi)}{d\phi}\right)_{\phi=0}=0$ acts as comprehensive charge neutrality $(n_{e0}+z_{do}n_{d0}=n_{i0})$ of the equilibrium..

In the standard potential approach, the ultimate solitary wave requisite is

(iii) $\left(\frac{d^2V(\phi)}{d\phi^2}\right)_{\phi=0}<0$

Apart from above assigned conditions, $V(\phi)$ yields the requirements $V(\phi)=0=V(\phi_m);V'(\phi)=0=V'(\phi_m)$; and $V(\phi)$ should be negative between the region $\phi=0$ and $\phi=\phi_m$, here ϕ_m is the upper limit of ϕ for refractive solitons. Thus for maximum measure of ϕ, we can write the following relation

$$V(\phi_m)=B_1\phi_m{}^2-B_2\phi_m{}^{\frac{5}{2}}-B_3\phi_m{}^3-B_4\phi_m{}^{\frac{7}{2}} \tag{17}$$

As the second order derivative of $V(\phi)$ alters its sign so the critical Mach number M_c can be obtain from the third condition as

$$M_c=\frac{\sqrt{15(1-\delta)(1-A)^2}}{\sqrt{(1-\sigma_1)(\delta-A+\delta(1-A))}} \tag{18}$$

For survival of SWs locally, the Mach number must confined (Sarma and Dev 2014) by the relation $M_c\le M\le M_{max}$. The value of the critical Mach number M_c is considered as the least possible value of M and for its bigger values the SWs may occur. The upper limit M_{max} of M provides the biggest positive root of Equation (17) which is also essential for creation of SWs.

3. Studies of Arbitrary Amplitude DAWs

To solve the Energy Equation we will go forward step by step with different approximation of ϕ in the expansion of $V(\phi)$ with usual boundary conditions to display the nonlinear behaviour of solitons.

3.1 Expansion of the SP upto $\left(\frac{5}{2}\right)^{th}$ degree term

Retaining the expansion up to $\left(\frac{5}{2}\right)^{th}$ order of $V(\phi)$ and using this result in (15) we have

$$\left(\frac{d\varnothing}{d\xi}\right)^2 = B_1\phi^2 - \frac{4}{5}B_2\phi^{\frac{5}{2}} \tag{19}$$

The corresponding solution of Equation (19) admits the soliton profile as

$$\phi = \phi_{1m} sech^4\left[-\frac{B_1^{\frac{1}{2}}}{4}\xi\right] \tag{20}$$

where $\phi_{1m} = \left(\frac{5B_1}{4B_2}\right)^2$ represents the amplitude for the wave. This type of result has been already obtained by many authors for ion-acoustic solitary waves of compressive type in connection to the solution of mKdV (Shukla and Mamun 2002) and Sagdeev potential (Tappert 1972, Sah and Goswami 1994) equations. At the same time the constants involved are dependent on the plasma parameters as well as dust charge fluctuations and dust temperatures (Sah and Goswami 1994).

3.2 Expansion of the SP upto 3rd degree term

To obtain better approximation in solitonic solution, the expansion of $V(\phi)$ may consider upto order in ϕ and then Equation (15) reduces to the form

$$\left(\frac{d\varnothing}{d\xi}\right)^2 = B_1\phi^2 - \frac{4}{5}B_2\phi^{\frac{5}{2}} - \frac{2}{3}B_3\phi^3 \tag{21}$$

In order to evaluate the nonlinear wave solution from the Equation (21), a transformation (Das *et. al.* 1998), $\phi(\xi) = \psi^2(\xi)$, is brought in with proper boundary conditions. Then the Equation (21) acknowledges the solitonic solution as

$$\phi = \left[\frac{B_1}{B_2} + \left(\frac{B_2^2 + 4B_1B_3}{B_1^2}\right)^{\frac{1}{2}} cosh\left(\frac{(B_1)^{\frac{1}{2}}}{2}\right)^{\frac{1}{2}}\right]^{-2} \tag{22}$$

The equivalent solution of the type of (20) has been obtained by several authors such as Tagare and Chakrabarty (1974) for mixed nonlinearity of KdV and mKdV equations and Das *et. al.* (1998) for Sagdeev potential equations.

3.3 *Expansion of the SP upto $\left(\frac{7}{4}\right)^{th}$ degree term*

To get yet better estimate of solitonic solution, we may retain terms upto $\left(\frac{7}{4}\right)^{th}$ order of ϕ in the expansion of $V(\phi)$ and then Equation (15) reduces to

$$\left(\frac{d\varnothing}{d\xi}\right)^2 = B_1\phi^2 - \frac{4}{5}B_2\phi^{\frac{5}{2}} - \frac{2}{3}B_3\phi^3 - \frac{4}{7}B_4\phi^{\frac{7}{2}} \tag{23}$$

Firstly, the Equation (23) is modified by the extra term $\phi^{\frac{7}{2}}$ and this additional term, which will be seen, acts a fascinating character on the formation of the dust acoustic solitary wave in different nature. To obtain the different natures of the solitonic solution from the modified Energy Equation (23), we employ the same transformation as used to solve the Equation (21).

In situation where $B_2 = B_3 = 0$, we will have a solitonic solution like as

$$\phi = \phi_{2m}\, \mathrm{sech}^{4/3}\left[\xi / w_1\right] \tag{24}$$

where $\phi_{2m} = \left(\frac{B_1}{B_4}\right)^{\frac{2}{3}}$ and $w_1 = \frac{4}{3\sqrt{B_1}}$ represent the amplitude and width of the soliton. This form of solution matches precisely with the solution provided by (El-Labany and El-Taibany 2003). In an area where $\phi(\xi)$ fulfils the condition $0 < \phi(\xi) < \phi_0$, the solution of the Equation (23) gives rise to a solitonic profile as

$$\phi_s = \mathrm{sech}^4\left[\pm\frac{1}{2}\sqrt{k_1\phi_0}\left(\phi_0 - \sqrt{\phi}\right)\xi\right] \tag{25}$$

where $\phi_0 = -\frac{B_3}{B_4}$ and $k_1 = \frac{B_4}{4}$

The Equation (25) gives a different type of spiky solitary waves which is often found (Nejoh 1994) in interplanetary space. The equivalent solution of the solution (23) has been obtained by (El-Labany and El-Taibany 2003) for dust acoustic solitary wave and (Das *et. al.* 1998) for ion acoustic solitary wave for $0 < \phi_s < \phi_0$.
In the region $\phi(\xi) < 0$, i.e. $\sqrt{\phi} < 0$, the solution for the Equation (23) is obtained as

$$\phi_E = \phi_0^2 \text{cosech}^4 \left[\pm \frac{1}{2} \sqrt{k_1 \phi_0} \left(\phi_0 - \sqrt{\phi} \right) \xi \right] \quad (26)$$

The solution profile (26) represents an explosive profile of the SWs. This type of solution profile is similar to the results as obtained by (Nejoh 1994), (Das *et. al.* 1998) and Naranmandula and Wang (2005). But along with the SWs (spiky and explosive nature), there is a possibility of formation of DL in the whole region $0 < \sqrt{\phi} < \phi_0$ and the profile of solution of this DL is obtained as

$$\phi_D = \phi_0^2 \tan\text{h}^4 \left[\pm \frac{\sqrt{k_1 \phi_0}}{2} \phi^{\frac{1}{4}} \left(\phi_0 - \chi \right)^{\frac{1}{2}} \right] \xi \quad (27)$$

The results similar to (El-Taibany and Wadati 2007) have been obtained by (Das *et. al.* 1998) as well as (El-Labany and El-Taibany 2003).

4. Result and Discussion

To study the nature of the maximum Mach number, SP and solitary wave for different parameters, we use some standard values of plasma parameters applicable to our model (Shukla and Mamun, 2002, Dev 2017), where $n_{eo} = 3\times10^{14} m^{-3}, n_{io} = 5\times10^{14} m^{-3}, T_i = 0.16eV$, $\beta = 0.1-0.7, T_e = 0.1eV, T_e = 0.1eV, \delta = 1.062$. The value of critical Mech number is obtained as $M_c = 0.93$

To determine the possibility of existence of arbitrary amplitude of the SWs, the expression (17) is numerically analysed and is displayed it in Figure 1. We have seen that for particular values of parameters related to this plasma model, the upper limit (M_{max}) of M, which is the M-cordinate point; the graph cross the M-axis which is similar to the report as mentioned by (H. Alinejad 2011) and it helps to find the range of Mach number. It is observed that The value of M_{max} depend on temperature ratio σ_i.

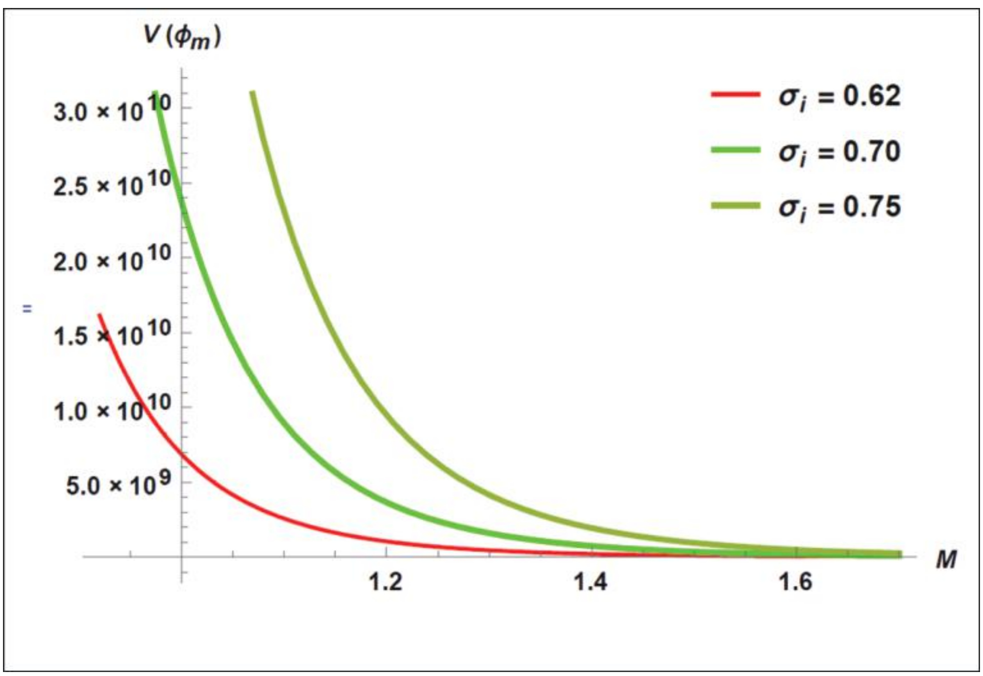

Figure 1. Nature of $V(\phi_m)$ for the equation (19) against the Mach number *M* for different values of σ_i

The behaviour of the SP given by Equation (16) has been analysed numerically and dependence of the rarefactive solitons on plasma parameters are shown in Figure 2. It has been observed that in Figs 2(a–c) the depth and width of soliton increases with increasing values of Mach number *M*, temperature ratio σ_i and δ. These phenomena happened perhaps increase of Mach number, ion density ratio and temperature; increase the area of rarefactive SP and increase the restoring force (Alinejad 2011) which step-up the frequency of oscillation. As a result it enhances the speed of phase velocity of the solitons for unvarying wave length and contributing to propagate the solitons with higher velocity. These observations agree with the result as shown by (El-Labany and El-Taibany 2003). In Figure 3 (d), it is observed that variation of trapping parameter β does not have effect on variation of solitons of the SP. Figure 3 (e) displays the behaviour of SD with ϕ and and σ_i find that depth of rarefactive soliton decreases σ_i as increases.

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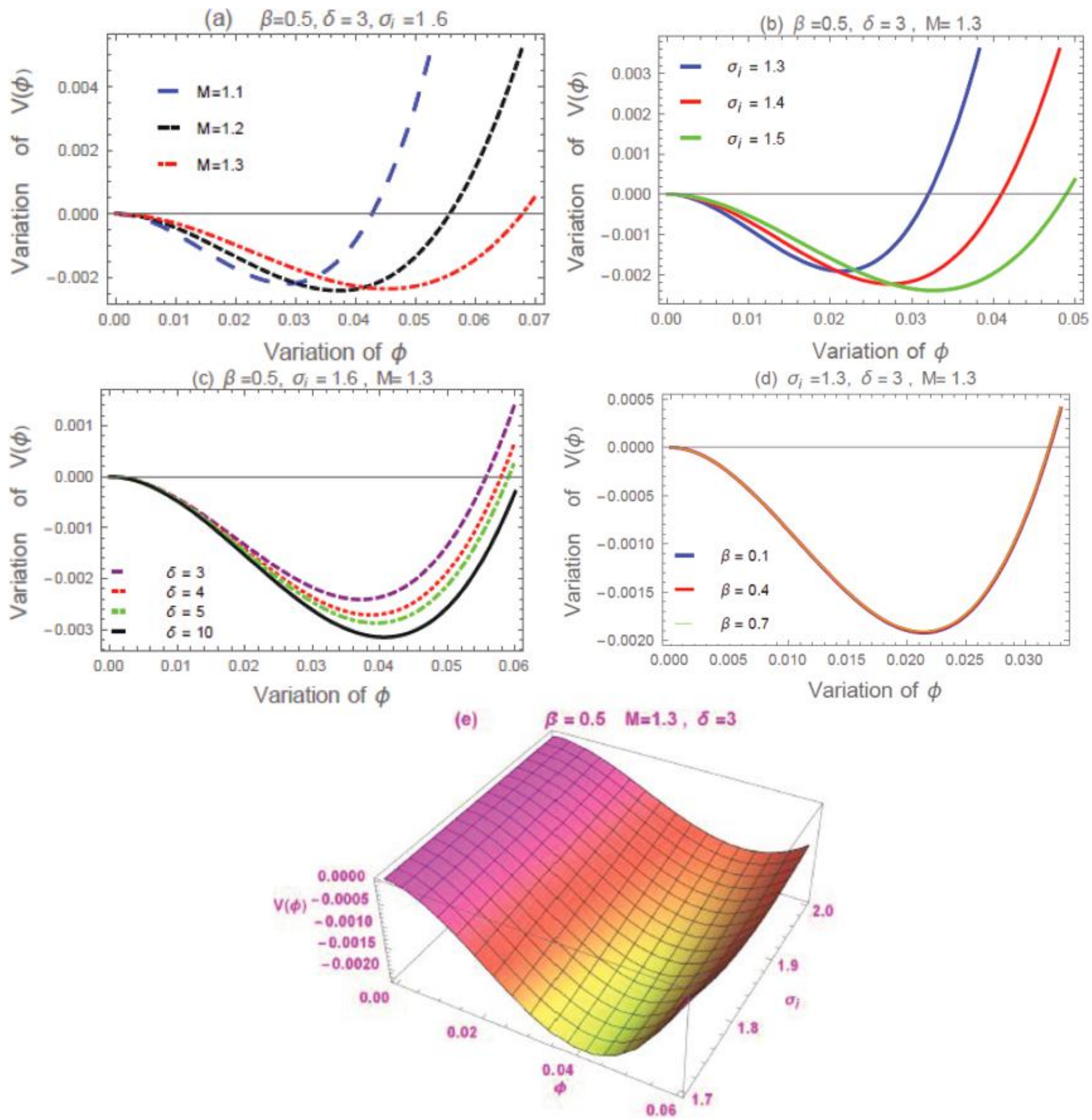

Figure 2. The variation of the SP against ϕ for different values of (a) Mach number M = 1.1(blue dashing upper curve), M=1.2(black dotted middle curve), M = 1.3(red dot dashed lower curve); (b) temperature ratio σ_i = 1.3 (upper blue line), σ_i =1.4 (middle red line), σ_i = 1.5 (lower green curve) and (c) δ =3(dashing purple upper curve), δ =4(dotted red curve), δ =5(lower green dotdashed curve)), δ =10(lowest solid curve), (d) tapping parameter β = 0.1, 0.4 and 0.7 (e) behaviour of Sagdeev potential against ϕ and σ_i

With help Figure 3 we have investigated the primary features (amplitude and width) of dust acoustic wave's solitons of the expression (20) for parameters variation. It has been observed that the amplitude and width of the DAWs decrease when the values of β, σ_i, δ increase. But opposite phenomenon is seen when M increase. The increments in β, σ_i, δ and reduction in M led to decrease the wave potentials. So we may conclude that β, σ_i, δ and M are vying parameters to settle the characteristics of solitons. Comparing the four figures of Figure 3, it has observed that increments in trapping parameter β and temperature ratio σ_i create solitons with high amplitudes. This phenomenon may happen perhaps large number of resonant electrons increases the nonlinear effect of the DAWs. In Figure 3(e), it is observed that for higher values of temperature ratio σ_i (> 1)these SWs transform to collapsible SWs.

Figure 4 shows the solitonic solitons of the expression (22) which is obtained by solving the Energy Equation (15) considering higher order terms of ϕ upto 3^{rd} order. The graph shows collapsible wave. This type of wave was predicted by Labany and Taibany for higher expansion in ϕ and agrees with our graph.

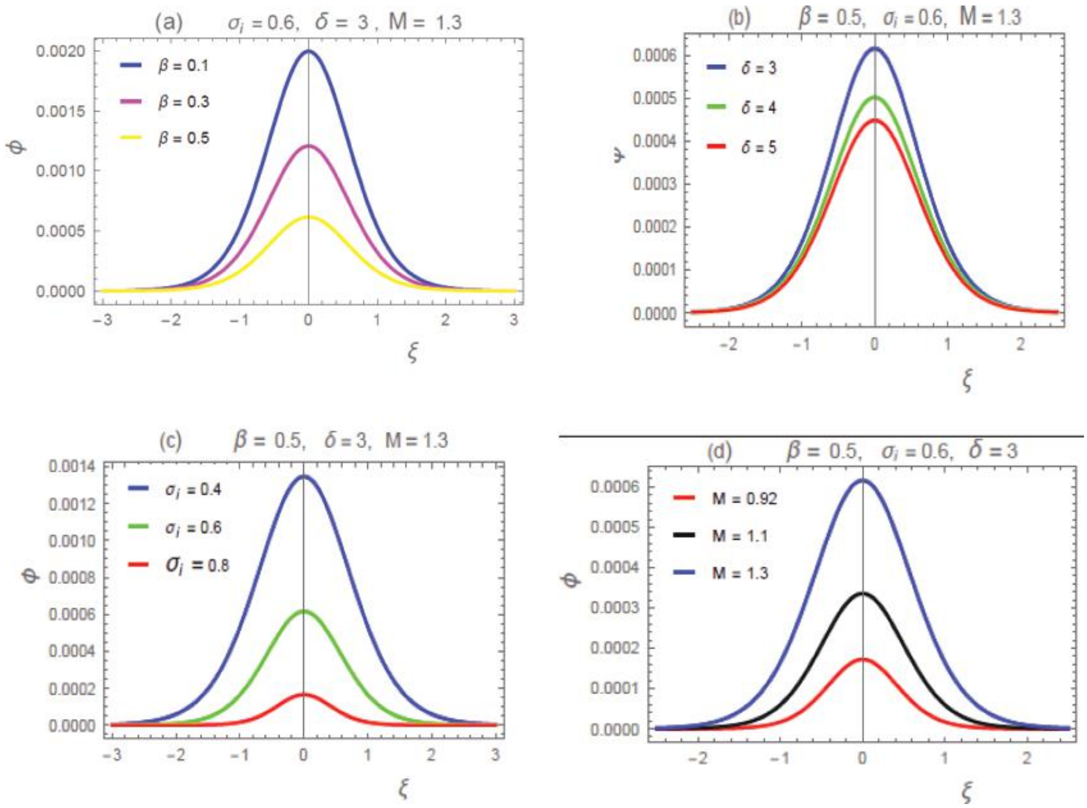

Figure 3. ϕ is plotted against ξ of the solution (20) for different values of plasma parameters (a) for $\sigma_i = 0.6$, δ =.3, M=1.3 with $\beta =$ 0.1 (upper blue curve), $\beta = 0.3$, (middle magenta curve), $\beta = 0.5$, (lower yellow curve); (b) for $\sigma_i = 0.6$, $\beta = 0.5$, M = 1.3 with $\delta = 3$ (upper blue curve), δ =4(middle green curve), $\delta = 5$ (lower red curve); (c) for $\beta = 0.5$, $\delta = 3$, M=1.3, with $\sigma_i = 0.4$(upper blue curve), σ_i = 0.6 (middle green curve), $\sigma_i = 0.8$ (lower red curve); (d) for $\beta = 0.5, \sigma_i = 0.3, \delta$ =.3 with M= 0.92 (upper blue curve), M=1.1(middle black curve), M=1.3 (lower red curve).

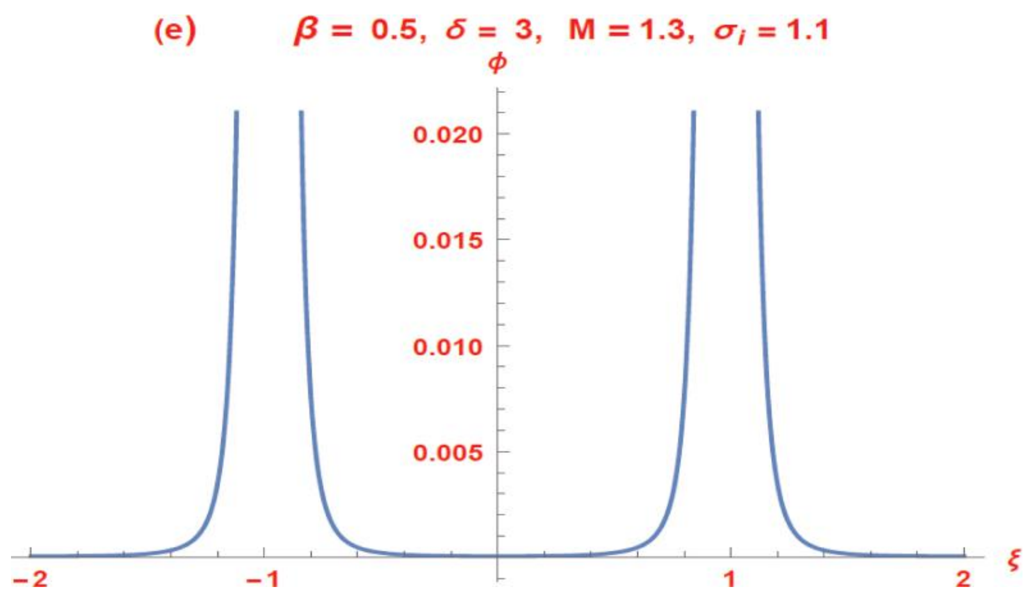

Figure 3(e). ϕ is plotted against ξ of the solution (20) representing the collapsible wave for different values of plasma parameters for $\beta = 0.5$, M= 1.3 , $\delta = 3$ with $\sigma_i = 1.1$

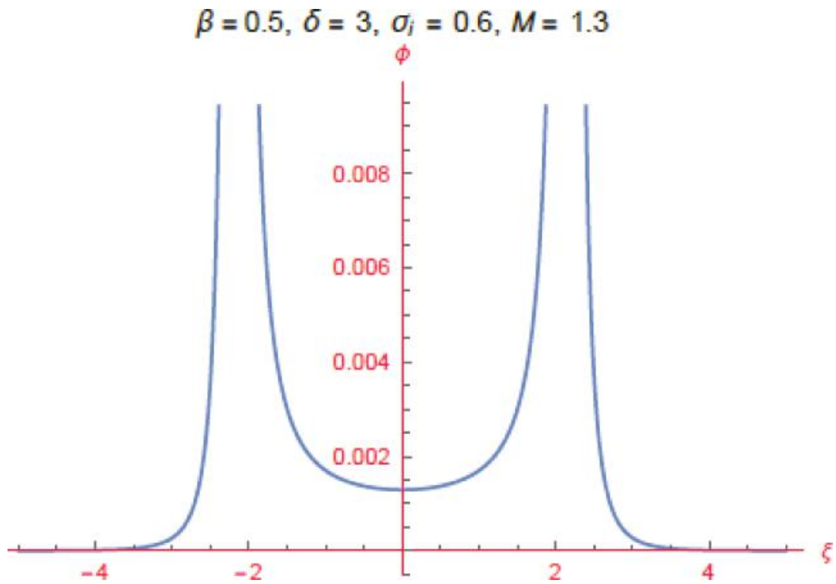

Figure 4. ϕ is plotted against ξ of the solution (22) representing the collapsible wave for different values of plasma parameters for σ_i =0.6, δ = 3, β = 0.5, M = 1.3

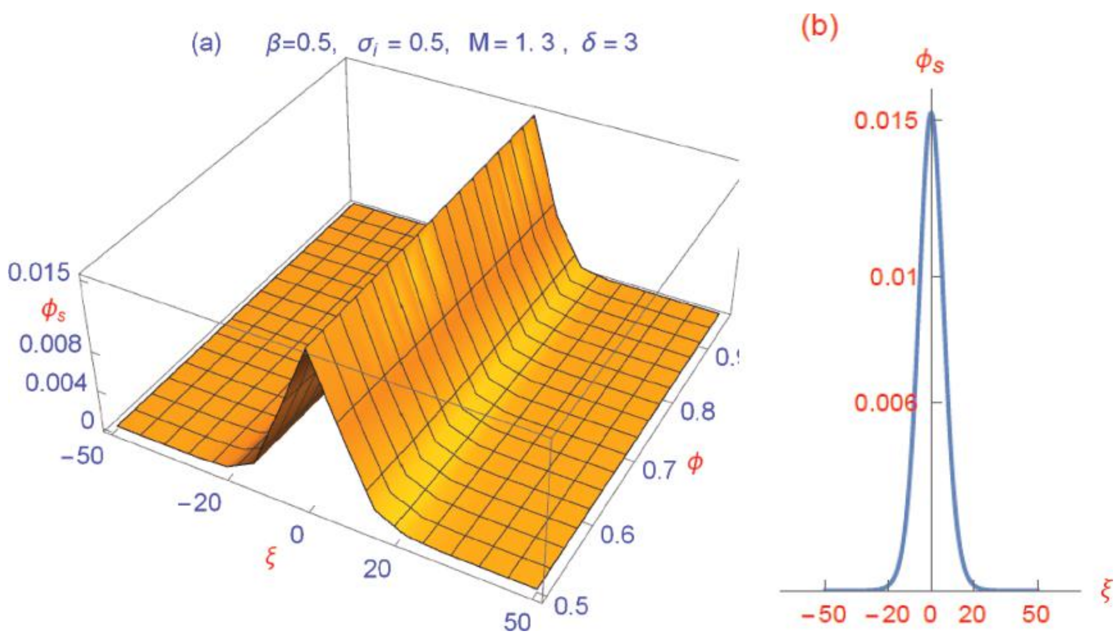

Figure 5. Displays the spiky solitary wave solution of the Equation (25) for β = 0.5, σ_i = 0.3 M = 1.3, δ =3 representing the spiky wave potential variation (a) against ξ and ϕ ; (b) against ξ

Figure 5 illustrate the wave profile of the Equation (25), which is the solution of the Equation (23) retaining the power of ϕ upto $\left(\frac{7}{2}\right)^{th}$ order. This is a new type of spiky solitary wave of amplitude ϕ_S.This is similar graphical representation with (Nejoh 1994) for electrostatic drift waves of inhomogeneous magnetized plasma. The spiky solitary wave is restricted to a region of positive potential. The existence of the spiky solitary wave in the astrophysical situation may be one reason to form spokes (Messenger 2000) in the Saturn's rings.

The Equation (26) is numerically analysed for the values of parameters $\beta = 0.5, \sigma_i = 0.6 \text{M} = 1.3, \delta = 3, \phi = -65$ and we have found the amplitude becomes infinite and renamed the solution as explosive solitary wave as predicted by El-

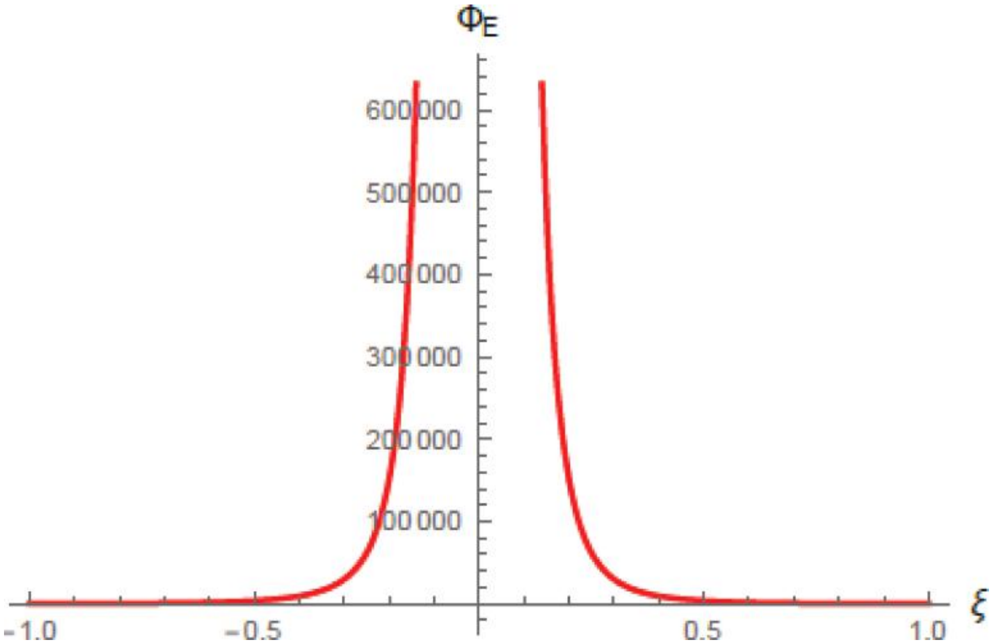

Figure 6. Represents the explosive solitary wave solution of the equation (26) for $\beta = 0.5$, $\sigma_i = 0.6$ M=1.3, δ =3, $\varnothing = -65$

(Labany and El-Taibany 2003) and (Das *et. al.* 1998) which is shown in Figure6. The results we have obtained here is good agreement with the reports of many authors (Naranmandula and Wang 2005, Deka *et. al.* 2018) for higher order expansion. Basically, these types of phenomenon depend on the coefficient of ϕ, which are functions of trapped electron coefficient, dust temperature, number of ions etc. The exclusive mode solutions are limited to an area where potential of the waves cross the upper limit of potential. It is observed that the spiky mode of the SWs exists within the area $0 < \phi(\xi) < \phi_0$ while the explosive mode of the SWs grows in the region $\phi(\xi) < 0$. So both mode of the solitary wave cannot be materialized simultaneously i.e. if explosive mode appears then the spiky solitary wave disappears.

Conclusion

In this work we have derived an Energy Equation, which contains Sagdeev Potential with the help of the nonperturbative approach in presence of trapped electrons, Boltzmann ions distribution and dust charge variation in a dusty plasma system. The SP has analysed imposing different condition which formed different types of SWs with rarefactive solitons. The dispersion relation is analytically derived in form of critical Mach number, and domain for the SWs is obtained in form of Mach number limit. The studies about the developments in nonlinearity of the Energy Equation for arbitrary amplitudes have been made for three different cases by expanding the SP in series in terms of ϕ. The solution of the Energy Equation gives solitary wave when the power of is considered upto $\left(\frac{5}{2}\right)^{th}$ order. These results are similar to the solution of KdV type equation which is already obtained by many authors. In this study we have found that the parameters β, σ_i, δ have opposite effects on the solitons of the DAWs as the Mach number M act on it, also the DAW's solitons transform to collapsible wave for greater value of $\sigma_i (> 1)$. When the power of ϕ is retained upto 3rd order, the solution of the Energy Equation gives us collapsible or breakable wave. The spiky types solitary wave solution is obtained when the power of ϕ is took upto $\left(\frac{7}{2}\right)^{th}$ order with the range $0 < \phi_s < \phi$. By further investigation we watch that the solution provides explosive solitary wave when $\sqrt{\phi} < 0$ and there is a possibility of double layer when the range is considered as $0 < \sqrt{\phi} < \phi_0$. In this report different kind of nonlinear mode of waves has been discussed independently. Many examples of each mode have been reported in space observations (Alfvén 1982, Temerin *et. al.*1982, Smith and Brecht 1985), so the results so obtained may helpful to better understanding the different astro-physical phenomena.

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