Math. J. Interdiscip. Sci.Vol. 7, No. 2, March 2019, pp.77–80

Mathematical Journal of Interdisciplinary Sciences

Journal homepage: https://mjis.chitkara.edu.in/

k-Contra Harmonic Mean Labeling of Snake Related Graphs

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ARTICLE INFORMATION

Received: 17 September, 2018
Revised: 11 October, 2018
Accepted: 06 February, 2019

Published online: March 6, 2019

Keywords:
k-Contra harmonic labeling, k-Contra harmonic graph, Triangular snake, Double Triangular snake.

DOI: https://doi.org/1015415/mjis.2019.72010

ABSTRACT

A graph $G(p, q)$ is said to have a *k*-Contra Harmonic mean labeling if there exists an injection $f: V \to \{k-1, k, k+1, \ldots, k+q-1\}$ such that the induced map f is defined by $f^*(uv) = \left\lceil \frac{f(u)^2 + f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2 + f(v)^2}{f(u)+f(v)} \right\rfloor$ is a bijection from E to $\{k, k+1, \ldots, k+q-1\}$. A graph that admits a *k*-Contra Harmonic Mean Labeling(k-CHML) is called *k*-Contra Harmonic Mean Graph(k-CHMG).

1. Introduction

In (Narasimhan *et. al.* 2013) introduced the Contra Harmonic mean labeling. In this paper we prove that some snake related graphs. Throughout this paper *k* denote any positive integer >0. For all other terminology and notations we follow (Harary 1988), (Gallian, 2018).

2. *Main Results*

Theorem 2.1

The graph $TS_m\,(m \geq 3)$ is a *K*–Contra Harmonic mean graph for any *k*.

Proof

Let $\{u_l, 1 \leq l \leq m,\ v_l, 1 \leq l \leq m-1\}$ be the vertices and $\{a_l, b_l, c_l,\ 1 \leq l \leq m-1\}$ be the edges which are denoted as in Figure 2.1.

First we label the vertices:

For $1 \leq l \leq m-1$ $f(v_l) = k+3l-3$

For $1 \leq l \leq m$ $f(u_l) = k+3l-4$

Then the induced edge labels are:

$$f^*(a_1) = k+2$$

For $2 \leq l \leq m-1$ $f^*(a_l) = k+3l-2$ $f^*(c_1) = k+1$

$$f^*(c_l) = k+3l-1$$

For $1 \leq l \leq m-1$ $f^*(b_l) = k+3l-3$

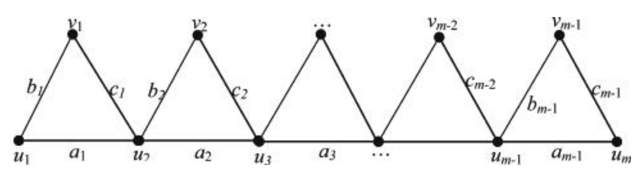

Figure 2.1. Ordinary labeling of TS_m

Hence, the edge labels are all distinct.

4-CHML of TS_5 is shown in Figure 2.2.

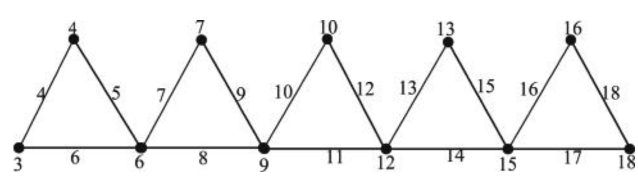

Figure 2.2: 4-CHML of TS_5

Theorem 2.2

The graph $A(TS_m)\,(m \geq 4)$ is a *k*-Contra Harmonic mean graph for any *k*.

ISSN No.: 2278-9561 (Print) ISSN No.: 2278-957X(Online) Registration No. : CHAENG/2013/49583 Periodicity: Bi-Annually

Proof

Let $\left\{u_l, 1 \le l \le m, v_l, 1 \le l \le \frac{m}{2}\right\}$ be the vertices and $\left\{a_l,\ 1 \le l \le m-1,\ b_l, c_l,\ 1 \le l \le \frac{m}{2}\right\}$ be the edges which are denoted as in Figure 2.3.

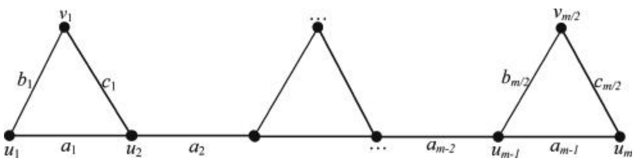

Figure 2.3. Ordinary labeling of $A(TS_m)$

First we label the vertices:

For $1 \le l \le \frac{m}{2}$ $f(v_l) = k + 4l - 5$

For $1 \le l \le m$ $f(u_l) = k + 2l - 2$

Then the induced edge labels are:

$$f^*(a_1) = k + 1$$

For $2 \le l \le m-1$

$$f^*(a_l) = \begin{cases} k + 4l - 6 & \text{if } l \text{ is odd} \\ k + 2l - 1 & \text{if } l \text{ is even} \end{cases}$$

For $1 \le l \le \frac{m}{2}$ $f^*(b_l) = k + 4l - 4$

$$f^*(c_1) = k + 2$$

For $2 \le l \le \frac{m}{2}$ $f^*(c_l) = k + 4l - 3$

Hence, the edge labels are all distinct.

7-CHML of $A(TS_4)$ is shown in Figure 2.4.

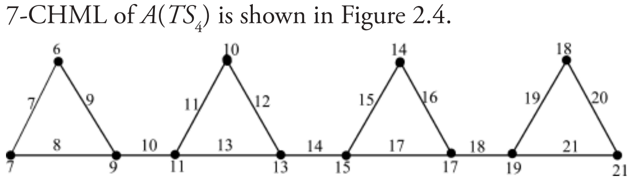

Figure 2.4. 4-CHML of $A(TS_4)$

Theorem 2.3

The graph $DA(TS_n)$ $(n \ge 4)$ is a *k*-Contra Harmonic mean graph for any *k*.

Proof

Let $\left\{u_l, v_l, 1 \le l \le \frac{n}{2},\ w_l, 1 \le l \le n\right\}$ be the vertices and $\left\{a_l,\ 1 \le l \le n-1,\ b_l, c_l, d_l, e_l,\ 1 \le l \le \frac{n}{2}\right\}$ be the edges which are denoted as in Figure 2.5.

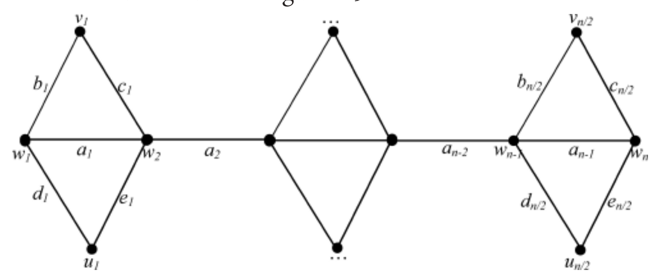

Figure 2.5: Ordinary labeling of $DA(TS_n)$

First we label the vertices:

For $1 \le l \le \frac{n}{2}$ $f(v_l) = k + 6l - 7$ $f(u_l) = k + 6l - 2$

$$f(w_l) = \begin{cases} k + 3l - 3 & \text{if } l \text{ is odd} \\ k + 3l - 4 & \text{if } l \text{ is even} \end{cases}$$

Then the induced edge labels are:

$$f^*(a_1) = k + 1$$

For $2 \le l \le n-1$ $f^*(a_l) = k + 3l - 1$

For $1 \le l \le \frac{n}{2}$ $f^*(b_l) = k + 6l - 6$

$$f^*(c_1) = k + 2$$

For $2 \le l \le \frac{n}{2}$ $f^*(c_l) = k + 6l - 5$

$$f^*(d_1) = k + 4$$

For $2 \le l \le \frac{n}{2}$ $f^*(d_l) = k + 6l - 3$

$$f^*(e_1) = k + 3$$

For $2 \le l \le \frac{n}{2}$ $f^*(e_l) = k + 6l - 2$

Hence, the edge labels are all distinct.

6-CHML of $DA(TS_4)$ shown in Figure 2.6.

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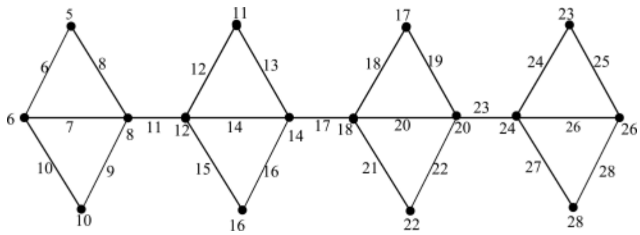

Figure 2.6. 4-CHML of $DA(TS_4)$

Theorem 2.4

The graph QS_m is a k-Contra Harmonic mean graph for any k.

Proof

Let $\{u_l, 1 \le l \le m, v_l, 1 \le l \le 2m-2\}$ be the vertices and $\{a_l, b_l, c_l, d_l, 1 \le l \le m-1\}$ be the edges which are denoted as in Figure 2.7.

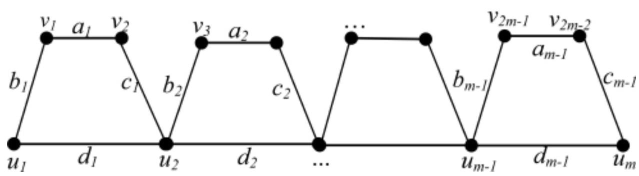

Figure 2.7. Ordinary labeling of QS_m

First we label the vertices:

For $1 \le l \le 2m-2$ $f(v_l) = k+2l-2$

For $1 \le l \le m$ $f(u_l) = k+4l-5$

Then the induced edge labels are:

$$f^*(a_1) = k+3$$

For $2 \le l \le m-1$ $f^*(a_l) = k+4l-2$

For $1 \le l \le m-1$ $f^*(b_l) = k+4l-4$

$$f^*(c_1) = k+2$$

For $2 \le l \le m-1$ $f^*(c_l) = k+4l-1$

For $1 \le l \le m-1$ $f^*(d_l) = k+4l-3$

Hence, the edge labels are all distinct.

2-CHML of QS_4 shown in Figure 2.8.

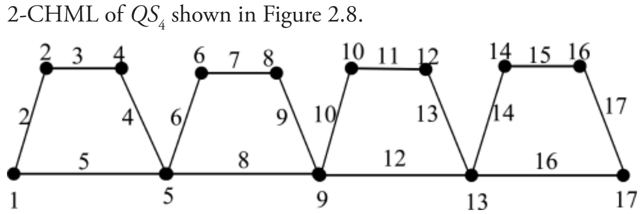

Figure 2.8. 4-CHML of QS_4

Theorem 2.5

The graph $A(QS_m)$ $(m \ge 4)$ is a k-Contra Harmonic mean graph for any k.

Proof

Let $\{u_l, v_l, 1 \le l \le m\}$ be the vertices and $\left\{a_l, b_l, c_l, 1 \le l \le \frac{m}{2}, d_l, 1 \le l \le m-1\right\}$ be the edges which are denoted as in Figure 2.9.

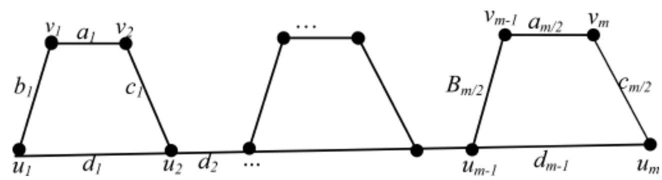

Figure 2.9. Ordinary labeling of $A(QS_m)$

First we label the vertices:

For $1 \le l \le m$ $f(v_l) = k+4l-5$

For $1 \le l \le m$ $f(u_l) = k+2(l-1)$

Then the induced edge labels are:

For $1 \le l \le \frac{m}{2}$ $f^*(a_l) = k+5l-4$ $f^*(b_l) = k+5(l-1)$

$$f^*(c_1) = k+2$$

For $2 \le l \le \frac{m}{2}$ $f^*(c_i) = k+5i-2$

$$f^*(d_1) = k+3$$

For $2 \le l \le m-1$

$$f^*(d_l) = \begin{cases} \dfrac{2k+5l-1}{2} & \text{if } l \text{ is odd} \\ \dfrac{2k+5l-2}{2} & \text{if } l \text{ is even} \end{cases}$$

Hence, the edge labels are all distinct.

8-CHML of $A(QS_3)$ is shown in Figure 2.10.

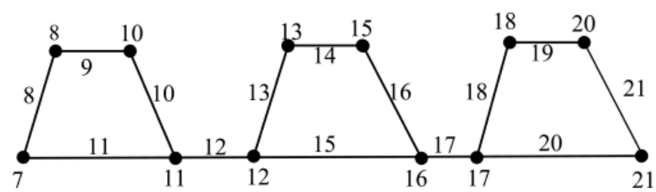

Figure 2.10. 4-CHML of $A(QS_3)$

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