DOI: 10.15415/mjis.2012.11005

Probability Analysis of a Complex System Working in a Sugar Mill with Repair Equipment Failure and Correlated Life Time

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Abstract

The aim of this paper is to present a reliability analysis of a complex (SJP) system in a sugar mill with the assumption that repair equipment may also fail during the repair. This paper considers the analysis of a three-unit system with one big unit and two small identical units of a SJP System in a sugar mill. Failure and Repair times of each unit are assumed to be correlated. Using regenerative point technique various reliability characteristics are obtained which are useful to system designers and industrial managers. Graphical behaviors of MTSF and profit function have also been studied.

Keywords: Transition Probabilities, MTSF, Availability, Busy Period, Profit Function.

1. INTRODUCTION

Complex systems have widely studied in literature of reliability theory as a large number of researchers are making lot of contribution in the field by incorporating some new ideas/concepts. Two/Three-unit standby systems with working or failed stages have been discussed under various assumptions/situations by numerous researchers including [1-4], repair maintenance is one of the most important measures for increasing the reliability of the system, and they have assumed that repair equipment can never be failed, But in real situation repair equipment may also fail when repairman is busy in the repair of any one of the failed unit. They have also assumed that the failure and repair of units are independent of each other but in real life this is not so. To have an extensive study on such a practical situation including the above-mentioned important aspects, the sugar mills situated at Muzaffar Nagar in Uttar Pradesh were visited and the information was collected on the Sulphated Juice Pump(SJP) System Working therein, this system contains one big unit(B) and two small identical units(A)(one of them is in standby). Then, on the basis of the situation observed in this mill, we put a step towards this direction by analyzing the reliability and the profit of a 3-unit cold standby

Mathematical Journal of Interdisciplinary Sciences
Vol. 1, No. 1,
July 2012
pp. 57–66

SJP System, evaluating various measures of system effectiveness Taking these facts into consideration in this paper we investigate a complex system model assuming the possibility of repair equipment failure and correlated life time.

2. SYSTEM DESCRIPTION

Complex System consists of two small identical units of A and one big unit B both are connected in series, but A has two Sub unit connected in Parallel one of them is in standby mode, so both units A and B are initially operating but the operation of only one sub unit of A is sufficient for operating the system. There is single repair facility. When the repairman is called on to do the job, it takes a negligible time to reach at the system. When repair equipment fails during the repair of any failed unit, repairman starts the repair of repair-equipment first. The joint distribution of failure and repair times for each unit is taken to be bivariate exponential having density function. Each repaired unit works as good as new.

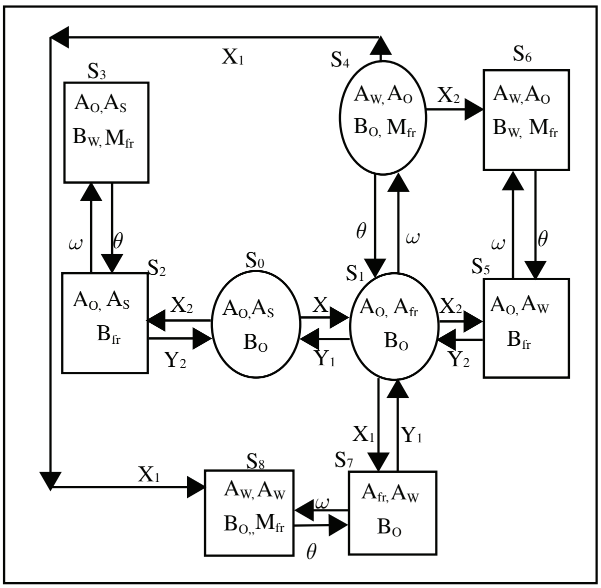

Figure1: Transition Diagram

3. NOTATIONS

For defining the states of the system we assume the following symbols:

$\mathbf{A_0}$	Unit A is in operative mode
$\mathbf{B_0}$	Unit B is in operative mode

$\mathbf{A_{fr}}$	Unit A is in failure mode
$\mathbf{B_{fr}}$	Unit B is in failure mode
w	Constant rate of failure of repair- equipment
θ	Constant rate of repair-equipment's repair
$\mathbf{A_w}$	unit A in failure mode but in waiting for repairman
$\mathbf{B_w}$	unit B in failure mode but in waiting for repairman
$\mathbf{M_{fr}}$	unit A in failure mode but in waiting for repairman
$\mathbf{B_w}$	unit B in failure mode but in waiting for repairman
$\mathbf{X_{i(i=1,2)}}$	random variables representing the failure times of A and B unit respectively for i=1,2
$\mathbf{Y_{i(i=1,2)}}$	random variables representing the repair times of A and B unit respectively for i=1,2
$f_{i(x,y)}$	joint pdf of (x_i,y_i);i=1,2 $=\alpha_i\beta_i(1-r_i)e^{-\alpha_i x-\beta_i y}I_0(2\sqrt{(\alpha_i\beta_i r_i xy)});X,Y,\alpha_i,\beta_i>0;$ $0\le r_i<1,$ where $I_0(2\sqrt{\alpha_i\beta_i r_i xy})=\sum_{j=0}^{\infty}\frac{(\alpha_i\beta_i r_i xy)^j}{(j!)^2}$
$k_i\mathbf{(Y/X)}$	conditional pdf of Y_i given X_i=x is given by $=\beta_i e^{-\alpha_i r_i x-\beta_i y}I_0(2\sqrt{(\alpha_i\beta_i r_i xy)})$
$g_i\mathbf{(.)}$	marginal pdf of X_i= $\alpha_i(1-r_i)e^{-\alpha_i(1-r_i)x}$
$h_i\mathbf{(.)}$**:**	marginal pdf of Y_i= $\beta_i(1-r_i)e^{-\beta_i(1-r_i)y}$
$q_{ij}(.),Q_{ij}(.)$	pdf &cdf of transition time from regenerative states pdf &cdf of transition time from regenerative state S_i to S_j.
μ_i	Mean sojourn time in state S_i.
$\oplus$	Symbol of ordinary Convolution $A(t)\oplus B(t)=\int_0^t A(t-u)B(u)du$
$\odot$	symbol of stieltjes convolution $A(t)\odot B(t)=\int_0^t A(t-u)dB(u)$

3.1 Transition Probability and Sojourn Times

The steady state transition probability can be as follows

$P_{01}=\frac{\alpha_1(1-r_1)}{\phi}$

$P_{32}=1$

$P_{02}=\frac{\alpha_2(1-r_2)}{\phi}$

$P_{41}=\frac{\theta}{\phi}$

$P_{10}=\frac{\beta\ (1-r_1)}{\beta\ (1-r\ )+\omega+\phi}$

$P_{46}=P_{45.6}=\frac{\alpha_1(1-r_1)}{\theta+\phi}$

$P_{14}=\frac{\omega}{\beta\ (1-r\ )+\omega+\phi}$

$P_{48}=P_{47.8}=\frac{\alpha_2(1-r_2)}{\theta+\phi}$

$P_{15}=\frac{\alpha_2(1-r_2)}{\beta\ (1-r\ )+\omega+\phi}$

$P_{51}=\frac{\beta_2(1-r_2)}{\omega+\beta_2(1-r_2)}$

$P_{17}=\frac{\alpha_1(1-r_1)}{\beta\ (1-r\ )+\omega+\phi}$

$P_{56}=\frac{\omega}{\omega+\beta_2(1-r_2)}$

$P_{20}=\frac{\beta_2(1-r_2)}{\beta_2(1-r_2)+\omega}$

$P_{71}=\frac{\beta_1(1-r_1)}{\omega+\beta_1(1-r_1)}$

$P_{23}=\frac{\omega}{\beta_2(1-r_2)+\omega}$

$P_{78}=\frac{\omega}{\omega+\beta_1(1-r_1)}$

$P_{01}+P_{02}=1$

$P_{41}+P_{47.8}+P_{45.6}=1$

$P_{10}+P_{14}+P_{15}+P_{17}=1$

$P_{20}+P_{23}=1$

$P_{41}+P_{46}+P_{48}=1$

$P_{71}+P_{78}=1$

$P_{32}=1,\ P_{87}=1\ ,\ P_{65}=1$

$P_{56}+P_{51}=1$ (1-26)

Mean sojourn times:

$$\mu_0=\frac{1}{\phi} \qquad \mu_1=\frac{1}{\beta(1-r)+\omega+\phi}$$

$$\mu_4=\frac{1}{\theta+\phi} \tag{27-29}$$

4. ANALYSIS OF CHARACTERISTICS

4.1 MTSF (Mean Time to System Failure)

To determine the MTSF of the system, we regard the failed state of the system as absorbing state, by probabilistic arguments, we get

$$\phi_0(t) = Q_{01} \otimes \phi_1(t) + Q_{02}$$

$$\phi_1(t) = Q_{10} \otimes \phi_0(t) + Q_{14} \otimes \phi_4(t) + Q_{15} + Q_{17}$$

$$\phi_4(t) = Q_{41} \otimes \phi_1(t) + Q_{48} + Q_{46} \tag{30-32}$$

Taking Laplace Stieltjes transforms of these relations and solving for $\phi_0^{**}(s)$,

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)} \tag{33}$$

Where

$$N = \mu_0(1 - P_{14}P_{41}) + \mu_1(P_{01} + P_{02}P_{41}) + \mu_2(P_{01}P_{14} + P_{02})$$

$$D = (1 - P_{14}P_{41}) - P_{01}P_{10} + P_{02}P_{41}P_{10} \tag{34-35}$$

4.2 Availability Analysis

Let $A_i(t)$ be the probability that the system is in up-state at instant t given that the system entered regenerative state i at t=0.using the arguments of the theory of a regenerative process the point wise availability $A_i(t)$ is seen to satisfy the following recursive relations

$$A_0(t) = M_0(t) + q_{01} \oplus A_1(t) + q_{02} \oplus A_2(t)$$

$$A_1(t) = M_1 + q_{10} \oplus A_0(t) + q_{14} \oplus A_4(t) + q_{15} \oplus A_5(t) + q_{17.7} \oplus A_1(t) + q_{18.7} \oplus A_8(t)$$

$$A_2(t) = q_{20} \oplus A_0(t) + q_{23} \oplus A_3(t)$$

$$A_3(t) = q_{32} \oplus A_2(t)$$

$$A_4(t) = M_4 + q_{41} \oplus A_1(t) + q_{45.6} \oplus A_5(t) + q_{47.8} \oplus A_7(t)$$

$$A_5(t) = q_{51} \oplus A_1(t) + q_{56} \oplus A_6(t)$$

$$A_6(t) = q_{65} \oplus A_5(t)$$

$$A_7(t) = q_{71} \oplus A_1(t) + q_{78} \oplus A_8(t)$$

$$A_8(t) = q_{87} \oplus A_7(t) \tag{36-44}$$

Now taking Laplace transform of these equations and solving them for $A_0^*(s)$, we get

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)} \tag{45}$$

The steady state availability is

$$A_0 = \lim_{s\to 0}(sA_0^*(s)) = \frac{N_1}{D_1} \tag{46}$$

Where

$$\begin{aligned} N_1 = {} & \mu_0(1-P_{23})[P_{71}P_{18.7}(1-P_{56}P_{65}) + P_{78}P_{51}(P_{45.6}P_{14} + P_{15}) \\ & -P_{78}(1-P_{11.7} - P_{14}P_{41}) + P_{78}P_{65}P_{56}(1-P_{11.7} - P_{14}P_{41})] \\ & +(1-P_{23})P_{78}P_{01}[(\mu_1 + \mu_4 P_{14}) + P_{56}P_{65}(\mu_1 + \mu_4 P_{14})] \end{aligned}$$

$$\begin{aligned} D_1 = {} & \mu_0 P_{78}P_{10}P_{20}P_{51} - \mu_1 P_{78}P_{01}P_{20}P_{51} + \mu_2(P_{78}P_{51}(1-P_{14}P_{47.8} - P_{02}P_{10}) \\ & -P_{18.7}P_{51} - P_{78}P_{11.7}) + \mu_3 P_{51}(1-P_{11.7} - P_{14}P_{41} - P_{01}P_{10} \\ & -P_{71}P_{14}P_{47.8})\mu_4(P_{01}P_{51}P_{14}P_{20}P_{78}P_{51}) + (\mu_6 + \mu_5)[(P_{20}P_{01}P_{78}(P_{15}P_{45.6} + P_{51})] \\ & +\mu_7 P_{01}[P_{20}(1-P_{11.7}) - P_{51}P_{20}(P_{14} + P_{18.7}) - P_{51}P_{14}(1-P_{47.8})] \\ & +\mu_8 P_{02}P_{20}P_{51}[P_{78}(P_{10} - P_{14}P_{47.8}) + P_{18.7}] \end{aligned}$$

(47-48)

4.3 Busy Period Analysis Of The Repairman

Let $B_i(t)$ be the probability that the repairman is busy at instant t, given that the system entered regenerative state I at t=0.By probabilistic arguments we have the following recursive relations for $B_i(t)$

$$B_0(t) = q_{01} \oplus B_1(t) + q_{02} \oplus B_2(t)$$

$$B_1(t) = W_1 + q_{10} \oplus B_0(t) + q_{14} \oplus B_4(t) + q_{15} \oplus B_5(t) + q_{17.7} \oplus B_1(t) + q_{18.7} \oplus B_8(t)$$

$$B_2(t) = W_2 + q_{20} \oplus B_0(t) + q_{23} \oplus B_3(t)$$

$$B_3(t) = W_3 + q_{32} \oplus B_2(t)$$

$$B_4(t) = W_4 + q_{41} \oplus B_1(t) + q_{45.6} \oplus B_5(t) + q_{47.8} \oplus B_7(t)$$

$$B_5(t) = W_5 + q_{51} \oplus B_1(t) + q_{56} \oplus B_6(t)$$

$$B_6(t) = W_6 + q_{65} \oplus B_5(t)$$

$$B_7(t) = q_{71} \oplus B_1(t) + q_{78} \oplus B_8(t)$$

$$B_8(t) = W_8 + q_{87} \oplus B_7(t) \tag{49-57}$$

Taking Laplace transform of the equations of busy period analysis and solving them for $B_0^*(s)$,we get

$$B_0^*(s) = \frac{N_2(s)}{D_1(s)} \tag{58}$$

In the steady state

$$B_0 = \lim_{s\to 0}(sB_0^*(s)) = \frac{N_2}{D_1} \tag{59}$$

Where

$$\begin{aligned} N_2 = \mu_0(1-P_{23})[P_{71}P_{18.7}(1-P_{56}P_{65}) + P_{78}P_{51}(P_{45.6}P_{14} + P_{15}) \\ -P_{78}(1-P_{11.7} - P_{14}P_{41}) + P_{78}P_{65}P_{56}(1-P_{11.7} - P_{14}P_{41})] \end{aligned} \tag{60}$$

D_1 is already specified.

4.4 Expected Number of Visits by the Repairman

We defined as the expected number of visits by the repairman in (0,t],given that the system initially starts from regenerative state S_i

By probabilistic arguments we have the following recursive relations for $V_i(t)$

$$V_0(t) = q_{01} \oplus (1+V_1(t)) + q_{02} \oplus (1+V_2(t))$$

$$V_1(t) = q_{10} \oplus V_0(t) + q_{14} \oplus V_4(t) + q_{15} \oplus V_5(t) + q_{11.7} \oplus V_1(t) + q_{18.7} \oplus V_8(t)$$

$$V_2(t) = q_{20} \oplus V_0(t) + q_{23} \oplus V_3(t)$$

$$V_3(t) = q_{32} \oplus V_2(t)$$

$$V_4(t) = q_{41} \oplus V_1(t) + q_{45.6} \oplus V_5(t) + q_{47.8} \oplus V_7(t)$$

$$V_5(t) = q_{51} \oplus V_1(t) + q_{56} \oplus V_6(t)$$

$$V_6(t) = q_{65} \oplus V_5(t)$$

$$V_7(t) = q_{71} \oplus V_1(t) + q_{78} \oplus V_8(t)$$

$$V_8(t) = q_{87} \oplus V_7(t) \tag{61-69}$$

Taking Laplace stieltjes transform of the equations of expected number of visits

And solving them for, $V_0^{**}(s)$ we get

$$V_0^{**}(s) = \frac{N_3(s)}{D_1(s)} \tag{70}$$

In steady state

$$V_0 = \lim_{s\to 0}(sV_0^*(s)) = \frac{N_3}{D_1} \tag{71}$$

where
$$N_3 = \mu_0(1-P_{23})P_{47.8} + \mu_2(P_{02} + P_{51}P_{18.7} + P_{02}P_{41}P_{48}) + (\mu_4)P_{02}P_{48}P_{78} + \mu_8(1 + P_{47.8}P_{02}) \tag{72}$$

D_1 is already specified

5. PROFIT ANALYSIS

The expected total profit incurred to the system in steady state is given by

$$P = C_0A_0 - C_1B_0 - C_2V_0 \tag{73}$$

Where

C_0 =Revenue/unit up time of the system

C_1 =Cost/unit time for which repairman is busy

C_2 =Cost/visit for the repairman

6. CONCLUSION

For a more clear view of the system characteristics w.r.t. the various parameters involved, we plot curves for MTSF and profit function in figure-2 and figure-3 w.r.t the failure parameter (α) of unit A for three different values of correlation coefficient, between X and Y , while the other parameters are kept fixed as

$$\alpha_2 = .005,\ \beta_1 = .02,\ \beta_2 = 0.01,\ \theta = 0.001,\ C_0 = 400,\ C_1 = 200,\ C_2 = 40,\ \omega = .004$$

From the fig.-2 it is observed that MTSF decreases as failure rate increases irrespective of other parameters. the curves also indicates that for the same value of failure rate,MTSF is higher for higher values of correlation coefficient(r),so here we conclude that the high value of r tends to increase the expected life time of the system. From the fig.-3 it is clear that profit decreases linearly as

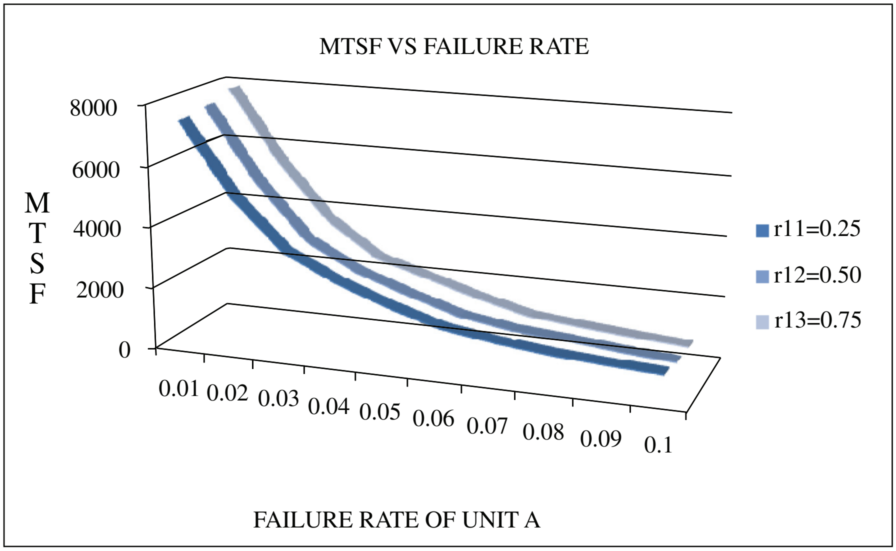

Figure 2: MTSF vs Failure Rate

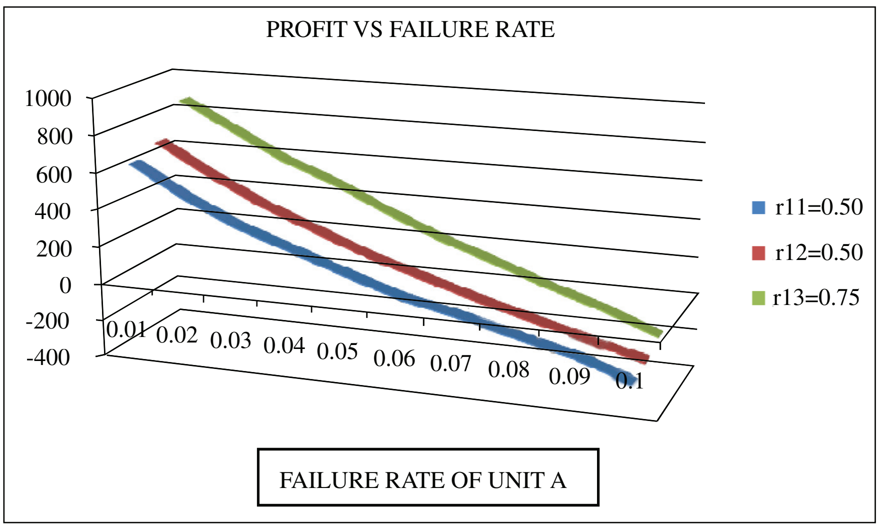

Figure 3: Profit vs Failure Rate

failure rate increases. Also for the fixed value of failure rate, the profit is higher for high correlation (r).

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