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Stability Analysis of Integrated Pest Management with Impulsive Biological Control

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Abstract The aim of the present work is to study the dynamics of stage-structured pest control model including biological control, i.e. by releasing of natural enemies and infected pests periodically. It is assumed that only immature susceptible pests are attacked by natural enemies admitting Beddington DeAngelis functional response and mature susceptible pests are contacted by infected pests with bilinear incidence rate and become exposed. The sufficient condition for local stability of pest extinction periodic solution is derived by making use of Floquet's theory and small amplitude perturbation technique. The global attractivity of pest extinction periodic solution is also established by applying comparison principle of impulsive differential equations.

Keywords: Pest management, Impulsive differential equations, Stage-structuring, Beddington-DeAngelis functional response, Stability analysis.

1. INTRODUCTION

Farmers have a vast scope of pest control methods categorized into physical control, biological control, chemical control and wireless sensing. The most popular method to control pests is chemical control in which farmers spread the pesticides to eliminate or control the pest population at regular intervals. But the recent study shows that chemical pesticides leaves adverse effect to human being as well as other natural enemies of pests. Moreover Kotchen (1999) and Aktar et. al. (2009) showed that with regular use of chemical pesticides, a number of pests become resistant to the used pesticides which lead to farmer's loss and forces them to use strong pesticides. On the other hand, in biological control methods, the pest populace is suppressed by

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releasing the natural enemies of pests and /or by spreading infection among the pest population. For example Bollworm is a pest which attacks cotton and Bacillus Thruingirnsis is a natural enemy of Bollworm which saves the cotton. The use of one or more biological control is an alternative to chemical pesticides and combination of two or more biological control methods with or without use of chemical pesticides come under the category of Integrated Pest Management.

Further, the pest control models can be well constructed by making use of impulsive differential equations, as the pest controlling agents are released periodically. A number of authors have successfully developed pest controlling models by means of impulsive differential equations (Baek (2010), Negi and Gakkar (2007), Wang and Huang (2015), Yu et. al. (2011)). Recently, Xiang et. al. (2009) and Wang and Song (2010) worked on susceptible-exposed-infected (SEI) pest management models. Gupta et. al. (2017) studied the dynamics of plant-pest-virus-natural enemy food chain model. However susceptible-exposed-infected-natural enemy (SEIN) models are more important as they give more significant results from biological view point. Following is the SEIN pest management model studied by Mathur and Dhar (2016):

$$\begin{cases} \left.\begin{aligned} \frac{dS_1(t)}{dt} &= rS_1(t)\left(1-\frac{S_1(t)}{K}\right)-\frac{\beta_1 S_1(t)I_1(t)}{1+mS_1(t)}-\beta_2 S_1(t)N_1(t), \\ \frac{dE_1(t)}{dt} &= \frac{\beta_1 S_1(t)I_1(t)}{1+mS_1(t)}-(\alpha+\mu_1)E_1(t), \\ \frac{dI_1(t)}{dt} &= \alpha E_1(t)-\mu_1 I_1(t), \\ \frac{dN_1(t)}{dt} &= \frac{\gamma\beta_2 S_1(t)N_1(t)}{1+h\beta_2 S_1(t)}-\mu_2 N_1(t), \end{aligned}\right\} t \neq nT, \\ \left.\begin{aligned} \Delta S_1(t) &= 0, \\ \Delta E_1(t) &= 0, \\ \Delta I_1(t) &= p_1, \\ \Delta N_1(t) &= p_2, \end{aligned}\right\} t = nT, \quad n = 1,2,\ldots, \end{cases} \tag{1}$$

where S_1(t), E_1(t), I_1(t) and N_1(t) represents the densities of susceptible pests, exposed pests, infected pests and natural enemies respectively. In the absence of infected pests and natural enemies, the susceptible pests $S_1(t)$ grows logistically with r being intrinsic birth rate and K being carrying capacity;

$\dfrac{\beta_1 S_1(t) I_1(t)}{1+mS_1(t)}$ represents Holling Type-II interaction at which susceptible pests get exposed, β_2 is predation rate by natural enemy, γ is conversion rate of pests to natural enemy, μ_1 is natural death rates of exposed and infected pests, μ_2 is natural death rates of natural enemy.

In this paper, the model (1) is extended to a stage structured and Beddington-DeAngelis functional response model, which is more significant from biological view point. Stage structuring of pests is proposed due to the fact that most of the pests in their life history goes through two stages namely: immature larva and mature adult. DeAngelis et. al. (1975) and Beddington (1975) introduced a functional response given by $F=\dfrac{\alpha P_1}{a+bP_1+cP_2}$ independently but simultaneously for the reason that predator has to work hard to catch prey and so named as Beddington-DeAngelis functional response, where $P_1=P_1(t)$ represents the size of the prey species and $P_2=P_2(t)$ represents the size of predator species. Negi and Gakkhar (2007) studied the dynamics of Beddington-DeAngelis prey-predator mathematical model with impulsive harvesting. Cantrell and Cosner (2001) and Wang and Huang (2015) also discussed the prey-predator model using Beddington-DeAngelis type interactions.

The present work is organized in 4 sections. In section 2, a pest control model alongwith stage structuring and Beddington-DeAngelis interaction has been developed, where natural enemies and infected pests are released impulsively. Some important lemmas and boundedness of the system are established in section 3. Using comparison principles, small amplitude perturbation technique and Floquet's theory, sufficient conditions for local stability and global attractivity of pest extinction periodic solutions are obtained in section 4.

2. MATHEMATICAL MODEL

Before proposing the mathematical model describing the complex behavior of pest management, we make following assumptions:

(A1) Susceptible pest goes via two life stages namely, immature larva and mature adult.
(A2) Natural enemy attacks immature pest.
(A3) Infected pests contact with mature pest only.
(A4) Natural enemies and infected pests are released impulsively.

With these assumptions, following mathematical model is proposed:

$$\begin{cases}\left.\begin{aligned}&\frac{dS_1(t)}{dt}=rS_2(t)\left(1-\frac{S_1(t)+S_2(t)}{K}\right)-\frac{h_1S_1(t)N(t)}{1+\gamma_1S_1(t)+\gamma_2N(t)}-\alpha S_1(t),\\&\frac{dS_2(t)}{dt}=\alpha S_1(t)-\beta S_2(t)I(t),\\&\frac{dE(t)}{dt}=\beta S_2(t)I(t)-(\mu+d_1)E(t),\\&\frac{dI(t)}{dt}=\mu E(t)-d_2I(t),\\&\frac{dN(t)}{dt}=\frac{\eta h_1S_1(t)N(t)}{1+\gamma_1S_1(t)+\gamma_2N(t)}-d_3N(t),\end{aligned}\right\}t\neq nT,\\\left.\begin{aligned}&S_1(t^+)=S_1(t),\\&S_2(t^+)=S_2(t),\\&E(t^+)=I(t)+\theta_1,\\&N(t^+)=N(t)+\theta_{2,}\end{aligned}\right\}t=nT,n=1,2,\ldots,\end{cases}\tag{2}$$

where $S_1(t),S_2(t),E(t),I(t)$ and $N(t)$ are densities of immature susceptible pest, mature susceptible pest, exposed pest, infected pest and natural enemy respectively. Susceptible pest grows logistically with K being carrying capacity and r being growth rate, α is maturity rate of immature pests, h_1 is predation rate of immature susceptible pest by natural enemy, β is conversion rate of mature susceptible pest by infected pest to exposed pest, μ is the amount of exposed pests shifted to infected pests, η is the conversion rate of predation by natural enemy, d_1, d_2 are natural death rates of exposed and infected pests respectively and d_3 is natural death rate of natural enemies, θ_1 and θ_2 represents impulsively supplied amounts of infected pests and natural enemies when $t = nT, n = 1,2, \ldots$, where T represents the impulsive period.

3. PRELIMINARIES

The solution of system (2) is expressed by $Y(t)=(S_1(t),S_2(t),E(t),I(t),N(t))'$ and is a piecewise continuous function $Y:\mathrm{R}_+\times\mathrm{R}_+^5\to\mathrm{R}_+$, that is, $Y(t)$ is continuous in the interval $(\mathrm{nT},(\mathrm{n}+1)\mathrm{T}],\mathrm{n}\in\mathrm{Z}_+$ and $Y(nT^+)=\lim_{t\to nT^+}Y(t)$ exists. The smoothness properties of variables of the system (2) guarantee the existence as well as uniqueness of solution (See Lakshmikantham et. al. (1989) for further details).

Before proving the main results, we firstly state and establish some lemmas which are useful in next section.

Lemma 1 [Lakshmikantham et. al (1989)] Let the function $w \in PC'\left[R^+, R\right]$ and w(t) be left-continues at t_k, k = 1, 2, ... the following inequalities

$$\begin{cases} w'(t) \le g(t)w(t) + h(t), t \ge t_0, t \ne t_k, \\ w(t_k^+) \le d_k w(t_k) + b_k, t = t_k, k = 1,2,\ldots\ldots \end{cases} \tag{3}$$

where $g, h \in PC\left[R^+, R\right]$, b_k are constants and d_k are positive constants, then

$$\begin{aligned} w(t) \le\ & w(t_0)\prod_{t_0<t_k<t} d_k e^{\int_{t_0}^{t} g(s)ds} + \sum_{t_0<t_k<t}\left(\prod_{t_k<t_j<t} d_j e^{\int_{t_0}^{t} g(s)ds}\right)b_k \\ & + \int_{t_0}^{t} \prod_{s<t_k<t} d_k e^{\int_s^t g(\sigma)d\sigma} h(s)ds, t \ge t_0. \end{aligned} \tag{4}$$

If the inequalities in (3) are reversed, then inequality (4) also holds exactly but for inequality with reversed sign.

Lemma 2 There exists a positive constant *H*, such that $S_1(t) \le H, S_2(t) \le H$, $E(t) \le H, I(t) \le H$ and $N(t) \le H$ for all solutions $Y(t) = (S_1(t), S_2(t), E(t), I(t), N(t))$ of system (2) with *t* being big enough.

Proof: Let us define

$$W(t) = \eta S_1(t) + \eta S_2(t) + \eta E(t) + \eta I(t) + N(t) \tag{5}$$

and let $0 < d < min\{d_1, d_2, d_3\}$.

Then for $t \ne nT$, we obtain that $D^+W(t) + dW(t) \le \frac{(r+d)^2 \eta K}{4r} = H_0$.

When $t = nT, W(t^+) \le W(t) + \theta_1 + \theta_2$.

Using Lemma 1, for any $t \in (nT, (n+1)T]$, we obtain

$$\begin{aligned} W(t) &\le W(0)e^{-dt} + \int_0^t H_0 e^{-d(t-s)}ds + \sum_{0<nT<t}(\theta_1 + \theta_2)e^{-d(t-nT)} \\ &\to \frac{H_0}{d} + \frac{(\theta_1+\theta_2)e^{-dT}}{e^{dT}-1}, \text{as } t \to \infty, \end{aligned}$$

which implies that *W*(*t*) is uniformly bounded. So the definition of *W*(*t*) implies that ∃ a constant

$H := \frac{H_0}{d} + \frac{(\theta_1+\theta_2)e^{-dT}}{e^{dT}-1}$ such that $S_1(t) \le H, S_2(t) \le H, E(t) \le H, I(t) \le H$ and $N(t) \le H$ for all *t* big enough.

Now we proceed to find pest extinction periodic solutions for the model (2). For the case of pest-extinction periodic solution, consider the following pest extinction sub-system

$$\begin{cases} \left.\begin{aligned} \frac{dI(t)}{dt} &= -d_2 I(t), \\ \frac{dN(t)}{dt} &= -d_3 N(t), \end{aligned}\right\} t \neq nT, \\ \left.\begin{aligned} I(t^+) &= I(t) + \theta_1, \\ N(t^+) &= N(t) + \theta_{2,} \end{aligned}\right\} t = nT. \end{cases} \tag{6}$$

Using Lemma 3.3 of Jatav and Dhar (2014), we get that

$$\breve{I}(t) = \frac{\theta_1 \exp(-d_2(t-nT))}{1-\exp(-d_2 T)}, \breve{I}(0^+) = \frac{\theta_1}{1-\exp(-d_2 T)}$$

and

$$\breve{N}(t) = \frac{\theta_2 \exp(-d_3(t-nT))}{1-\exp(-d_3 T)}, \breve{N}(0^+) = \frac{\theta_2}{1-\exp(-d_3 T)}$$

are positive solutions of the subsystems, which are globally asymptotically stable.

4. STABILITY ANALYSIS

In this section, sufficient conditions are obtained for local stability and global attractivity of pest- extinction periodic solution.

Theorem 4.1 Let $(S_1(t), S_2(t), E(t), I(t), N(t))$ be arbitrary solution of the system (2), then pest-extinction periodic solution $\left(0,0,0,\breve{I}(t),\breve{N}(t)\right)$ is locally stable if and only if

$$\sigma = \frac{h_1\theta_2}{d_3}\log\left(\frac{1-\exp(-d_3T)(1-\gamma_2\theta_2)}{1-\exp(-d_3T)+\gamma_2\theta_2}\right) \\ -\alpha T + \frac{\alpha r}{\beta\theta_1 d_2}\exp(d_2T)\left(1-\exp(-d_2T)\right)^2 < 0.$$

Proof: In order to establish local stability of pest-extinction periodic solution $\left(0,0,0,\breve{I}(t),\breve{N}(t)\right)$, we define

$$S_1(t)=\phi_1(t),S_2(t)=\phi_2(t),E(t)=\phi_3(t),I(t)$$
$$=\check{I}(t)+\phi_4(t),N(t)=\breve{N}(t)+\phi_5(t),$$

where $\phi_i(t), i=1,2,\ldots,5$ are small amplitude perturbations, then the impulsive system (2) can be expanded in the following linearized form:

$$\begin{cases}\left.\begin{aligned}\frac{d\phi_1(t)}{dt}&=r\phi_2(t)-\frac{h_1\phi_1(t)\breve{N}(t)}{1+\gamma_2\breve{N}(t)}-\alpha\phi_1(t),\\ \frac{d\phi_2(t)}{dt}&=\alpha\phi_1(t)-\beta\check{I}(t)\phi_2(t),\\ \frac{d\phi_3(t)}{dt}&=\beta\check{I}(t)\phi_2(t)-(\mu+d_1)\phi_3(t),\\ \frac{d\phi_4(t)}{dt}&=\mu\phi_3(t)-d_2\phi_4(t),\\ \frac{d\phi_5(t)}{dt}&=\frac{\eta h_1\breve{N}(t)\phi_1(t)}{1+\gamma_2\breve{N}(t)}-d_3\phi_5(t),\end{aligned}\right\}t\neq nT,\\ \left.\begin{aligned}\phi_1(t^+)&=\phi_1(t),\\ \phi_2(t^+)&=\phi_2(t),\\ \phi_3(t^+)&=\phi_3(t),\\ \phi_4(t^+)&=\phi_4(t),\\ \phi_5(t^+)&=\phi_5(t),\end{aligned}\right\}t=nT,\end{cases} \tag{7}$$

Let $\Phi(t)$ be the fundamental matrix of (7), it must satisfy

$$\frac{d\Phi(t)}{dt}=\begin{pmatrix}-\frac{h_1\breve{N}(t)}{1+\gamma_2\breve{N}(t)}-\alpha & r & 0 & 0 & 0\\ \alpha & -\beta\check{I}(t) & 0 & 0 & 0\\ 0 & \beta\check{I}(t) & -(\mu+d_2) & 0 & 0\\ 0 & 0 & \mu & -d_2 & 0\\ \frac{\eta h_1\breve{N}(t)}{1+\gamma_2\breve{N}(t)} & 0 & 0 & 0 & -d_3\end{pmatrix}\Phi(t)=A\Phi(t), \tag{8}$$

where A is equivalent to

$$\begin{pmatrix} -\frac{h_1\breve{N}(t)}{1+\gamma_2\breve{N}(t)}-\alpha+\frac{\alpha r}{\beta\check{I}(t)} & 0 & 0 & 0 & 0 \\ \alpha & -\beta\check{I}(t) & 0 & 0 & 0 \\ 0 & \beta\check{I}(t) & -(\mu+d_1) & 0 & 0 \\ 0 & 0 & \mu & -d_2 & 0 \\ \frac{nh_1\breve{N}(t)}{1+\gamma_2\breve{N}(t)} & 0 & 0 & 0 & -d_3 \end{pmatrix}$$

The linearization of impulsive conditions of (2), i.e. equations sixth to tenth of (2) becomes

$$\begin{pmatrix} \phi_1(t^+) \\ \phi_2(t^+) \\ \phi_3(t^+) \\ \phi_4(t^+) \\ \phi_5(t^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_1(t) \\ \phi_2(t) \\ \phi_3(t) \\ \phi_4(t) \\ \phi_5(t) \end{pmatrix}$$

Therefore the monodromy matrix of (7) can be written as

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \Phi(T).$$

From (8), we get that $\Phi(T)=\Phi(0)\exp\left(\int_0^T Adt\right)$, where $\Phi(0)$ is the identity matrix. So the eigen values of the monodromy matrix M are

$$\lambda_1 = \exp\left(-\int_0^T \left(\frac{h_1(t)\breve{N}(t)}{1+\gamma_2\breve{N}(t)}+\alpha-\frac{\alpha r}{\beta\check{I}(t)}\right)dt\right),$$
$$\lambda_2 = \exp\left(-\int_0^T \left(\beta\check{I}(t)\right)\right) = \exp\left(-\frac{\beta\theta_1}{d_2}\right) < 1,$$
$$\lambda_3 = \exp\left(-(\mu+\mathrm{d}_1)\mathrm{T}\right) < 1,$$

$$\lambda_4 = \exp(-d_2 T) < 1,$$
$$\lambda_5 = \exp(-d_3 T) < 1.$$

Thus the Floquet's theory of differential equations with impulse effect implies that the pest- extinction periodic solution of the system (2) is locally stable if $|\lambda_1| < 1$, i.e. $\sigma < 0$. Hence the result.

Theorem 4.2 Let $(S_1(t), S_2(t), E(t), I(t), N(t))$ be any solution of (2), then the pest-extinction periodic solution $\left(0,0,0,\check{I}(t),\breve{N}(t)\right)$ of (2) is globally attractive provided σ < 0, σ as defined in theorem above.

Proof: If $(S_1(t), S_2(t), E(t), N(t), N(t))$ is arbitrary solution of model (2), from fourth and ninth equation of the model, we have

$$\begin{cases} \dfrac{dI(t)}{dt} \geq -d_2 I(t), t \neq nT, \\ I(t^+) = I(t) + \theta_{1,} t = nT. \end{cases} \tag{9}$$

Consider the following comparison system

$$\begin{cases} \dfrac{dw_1(t)}{dt} = -d_2 w_1(t), t \neq nT, \\ w_1(t^+) = w_1(t) + \theta_{1,} t = nT. \end{cases} \tag{10}$$

Using the Lemma 3.3 of Jatav and Dhar (2014), we get that the above system possess global asymptotic periodic solution

$$w_1^*(t) = \frac{\theta_1 \exp(-d_2(t-nT))}{1-\exp(-d_2 T)}, nT < t \leq (n+1)T, n \in Z_+.$$

In view of Lemma 3.3 of Jatav and Dhar (2014) and the comparison theorem, we obtain $I(t) \geq w_1(t)$ and $w_1(t) \to w_1^*(t) as t \to \infty$.

Then $\exists$ an integer k_1 such that

$$I(t) \geq w_1(t) > \check{I}(t) - \in_0, nT < t \leq (n+1)T, n > k_1. \tag{11}$$

Now from fifth and tenth equations of (2), we obtain the following sub-system

$$\begin{cases} \dfrac{dN(t)}{dt} \geq -d_3 z(t), t \neq nT, \\ N(t^+) = N(t) + \theta_{2,} t = nT. \end{cases} \tag{12}$$

As in the previous manner, we acquire that sub-system (12) assumes a periodic solution and there exist integer $k_2 (k_2 > k_1)$ such that

$$N(t) \geq \breve{N}(t) - \in_0, nT < t \leq (n+1)T, n > k_2. \tag{13}$$

Now the first equation of model (2) can be expressed as

$$\frac{dS_1(t)}{dt} \leq -\left(\frac{h_1\left(\breve{N}(t) - \in_1\right)}{1 + \gamma_1 S_1(t) + \gamma_2\left(\breve{N}(t) - \in_1\right)} + \alpha - \frac{\alpha r}{\beta \check{I}(t)} \right) S_1(t).$$

Integrating the above equation between the pulses, we get

$$S_1((n+1)T) \leq S_1(nT) \exp\left(\int_{nT}^{(n+1)T} -\left[\frac{h_1\left(\breve{N}(t) - \in_1\right)}{1 + \gamma_1 S_1(t) + \gamma_2\left(\breve{N}(t) - \in_1\right)} + \alpha - \frac{\alpha r}{\beta \check{I}(t)} \right] dt \right).$$

After the subsequent pulses, we get below mentioned stroboscopic map

$$S_1((n+1)T^+) \leq S_1(nT^+) \exp\left(\int_{nT}^{(n+1)T} -\left[\frac{h_1\left(\breve{N}(t) - \in_1\right)}{1 + \gamma_1 S_1(t) + \gamma_2\left(\breve{N}(t) - \in_1\right)} + \alpha - \frac{\alpha r}{\beta \check{I}(t)} \right] dt \right), = S_1(nT^+) q,$$

where $q = \exp\left(\int_{nT}^{(n+1)T} -\left[\frac{h_1\left(\breve{N}(t) - \in_1\right)}{1 + \gamma_1 S_1(t) + \gamma_2\left(\breve{N}(t) - \in_1\right)} + \alpha - \frac{\alpha r}{\beta \check{I}(t)} \right] dt \right) < 1$, as $\sigma < 0$.

Thus $S_1(nT^+) \leq S_1(0^+) q^n$ and so $S_1(nT^+) \to 0$ as $n \to \infty$. Therefore there exists an $\in_2 > 0$, small enough and an integer k_3 ($k_3 > k_2$), such that $0 < S_1(t) < \in_2, nT < t \leq (n+1)T, n > k_3$.

In system (2), from second equation, we see

$$\frac{dS_2(t)}{dt} \le \beta S_2(t)\left(\check{I}(t) - \in_1\right).$$

In a similar manner, there exists $k_4(k_4 > k_3)$, such that $S_2(t) \le \in_2, nT < t \le (n+1)T, n > k_4$.
In system (2), from third equation, we have

$$\frac{dE(t)}{dt} \le -(\mu + d_1)E(t),$$

Integrating the above equation between the pulses, we attain

$$E((n+1)T) \le E(nT)\exp\left(\int_{nT}^{(n+1)T} -(\mu + d_1)dt\right).$$

After the subsequent pulses, we get below mentioned stroboscopic map

$$E((n+1)T^+) \le E(nT^+)\exp(-(\mu + d_1)T).$$

Therefore $E(nT^+) \le E(0^+)\exp(-n(\mu + d_1)T)$ and so $E(nT^+) \to 0$ as $n \to \infty$. Thus $E(t) \to 0$ as $n \to \infty$.
So there exists an $\in_3 > 0$, small enough, there exist an integer $k_5(k_5 > k_4)$, such that $0 < E(t) < \in_3, nT < t \le (n+1)T, n > k_5$.
Again from fourth and ninth equation of system (2), we have

$$\begin{cases} \dfrac{dI(t)}{dt} \le \mu \in_2 - d_2 I(t), t \ne nT, \\ I(t^+) = I(t) + \theta_{1,} t = nT. \end{cases} \tag{14}$$

Again considering comparison system

$$\begin{cases} \dfrac{dw_1(t)}{dt} \le \mu \in_2 - d_2 w_1(t), t \ne nT, \\ w_1(t^+) = w_1(t) + \theta_{1,} t = nT, \end{cases} \tag{15}$$

and making use of Lemma 3.3 of Jatav and Dhar (2014), we see that the system (15) has following periodic solution

$$w_1^*(t) = \frac{\mu \in_2}{d_2} + \frac{\theta_1 \exp(-d_2(t-nT))}{1-\exp(-d_2 T)}, nT < t \le (n+1)T, n \in Z_+,$$

which is globally asymptotically stable. In view of Lemma 3.3 of Jatav and Dhar (2014) and the comparison theorem of the impulsive differential equations, we have $I(t) \le w_1(t)$ and $w_1(t) \to w_1^*(t) as t \to \infty$.

Then $\exists$ an integer $k_6 (k_6 > k_5)$ such that

$$I(t) \le w_1(t) < w_1^*(t) + \in_0, nT < t \le (n+1)T, n > k_6.$$

From fifth and tenth equation of system (2), we obtain that

$$\begin{cases} \frac{dN(t)}{dt} \le (\eta h_1 \varepsilon_2 - d_3) N(t), t \ne nT \\ N(t^+) = N(t) + \theta_2, t = nT. \end{cases}$$

In a similar manner, there exists an integer $k_7 (k_7 > k_6)$ such that

$$N(t) \le w_2^*(t) + \in_0, nT \le t \le (n+1)T, n \ge k_7,$$

where $w_2^*(t)$ is the solution of corresponding comparison system.

Since $\varepsilon_1, \varepsilon_2$ and ε_3 are arbitrary small positive constants, so $w_1^*(t) \to \check{I}(t)$ and $w_2^*(t) \to \breve{N}(t)$ as $\varepsilon_2 \to 0.$ Therefore we have $S_1(t) \to 0, S_2(t) \to 0,$ $, E(t) \to 0, I(t) \to \check{I}(t)$ and $N(t) \to \breve{N}(t)$. Hence the pest-extinction periodic solution $\left(0,0,0,\check{I}(t),\breve{N}(t)\right)$ of (2) is globally attractive.

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