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A Note on IIR Filters with Random Parameters

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Abstract

The infinite impulse response (IIR) filter of an AR(1) process is studied under random parameter assumption. The statistical properties of random transfer function are derived. Stochastic process modeling is also considered. Finally, a conclusion section is given.

Keywords: AR(1) process; Beta distribution; IIR filter; Random parameter; Transfer function

1 INTRODUCTION AND MAIN RESULTS.

A filter is a useful tool which have many applications such as smoothing and prediction. There are many types of filters such as low and high pass filters. The filters are designed for time and frequency domains. Some of them are moving averages, Bessel, Butterworth, Chebyshev and Wiener filters.

One of important member of filters class is the IIR. When the input process for this filter is a white noise, then the output process would be an autoregressive process. The transfer function of an IIR is given by

$$H^*(z) = \frac{b_0}{a_0 + \sum_{j=1}^{p} a_j z^{-j}}.$$

For more details on IIR filters, see Lyons (2011). For example, for an AR(1) process,

$$H(z) = H_z = \frac{1}{1-\phi z},$$

over $|\phi| < 1$.

The H_z also provides a well-defined framework for studying the stochastic process with random parameters. To this end, suppose that ϕ is a random variable distributed uniformly on (-1,1). Then

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$$P\left(H_z \le h_z\right)=\begin{cases}\dfrac{1-u_z}{2} & z\le 0,\\ \dfrac{1+u_z}{2} & z\ge 0,\end{cases}$$

where $u_z = \dfrac{1-h_z^{-1}}{z}$. That is, $P\ (H_z \le h_z) = 0.5\ (1+\text{sign}\ (z)u_z)$. For example, when $z > 0$, and by assuming $P\ (H_z \le h_z) = 1 - \alpha$, for some given significant level α, we have

$$h_z = \frac{1}{1-\left(1-2\alpha\right)z},$$

which is the transfer function of an AR(1) process with coefficient $1 - 2\alpha$. Also, one can see that

$$E\left(H_z\right)=z^{-2}\left\{\log\left(\sqrt{\frac{1+z}{1-z}}\right)-z\right\},$$

and

$$E\left(H_z^2\right)=|\,z\,|^{-3}\left\{\log\left(\sqrt{\frac{1+z}{1-z}}\right)-z\right\}.$$

The beta distribution is a very flexible distribution and a widely used approach in many fields of statistics is to approximate a complicated distribution by a beta distribution. In most cases, it is enough the support of complicated distribution is bounded for example it remains between 0 and 1. For example, Zhang and Wu (2002) showed that the distribution of Kolmogorov-Smirnov statistic can be globally approximated by a general beta distribution. Therefore, for another choice, suppose that $\dfrac{\phi+1}{2}$ has beta distribution $F_{\beta eta}$. Then,

$$P\left(H_z \le h_z\right)=\begin{cases}1-F_{\beta eta}\left(\dfrac{1+u_z}{2}\right) & z\le 0,\\ F_{\beta eta}\left(\dfrac{1+u_z}{2}\right) & z\ge 0.\end{cases}$$

Example 1: Stochastic process modeling. Here, we are going to suppose that ϕ depends on time t (time varying parameter). Indeed, it is a stochastic process. Without loss of generality let $\phi_t > 0$. For example, let

$$\phi_t = \gamma\phi_{t-1},$$

where γ has a beta distribution. Then, we find that

$$H_t(z) = H_{t-1}(\gamma z).$$

This identity helps us to obtain all statistical properties of $H_t(z)$, using a Monte Carlo simulation method.

2 CONCLUSIONS

The statistical properties of random transfer function of an IIR filter of an AR(1) process with random parameter are derived. Stochastic modeling is also considered.

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