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A Note on Monitoring Fuzzy Financial Returns

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Abstract This paper presents change point analysis for stock market time series where it is assumed the rate of return on securities are approximated as LR-fuzzy numbers. We consider the change point detection in the mean and variance of returns. The methods are proposed and their theoretical aspects are studied. A real data set is also considered. Finally, a conclusion section is given.

Keywords: Change point; Cusum; LR-Fuzzy number; Membership function; Monitoring; Rate of return; Rolling analysis; Securities

1 INTRODUCTION

Monitoring plays an important role in time series analysis. In practice, some characteristics (mean, variance or both) of a specified time series may change over periods of times. That is, they are fixed through a period and differ from one period to the next. This phenomena is referred as change point, monitoring, regime shift and surveillance.

Change point analysis has been received considerable attentions for financial time series. For example, Hillebrand and Schnabl (2003) studied change point detection in volatility of Japanese foreign exchange intervention under GARCH modeling. Halunga *et al*. (2009) detected changes in the order of integration of US and UK inflation. An excellent reference in change point analysis is Csorgo and Horvath (1997). However, in all of these examples, financial time series are regarded as sequence of random variables. In this paper, we consider rate of returns of securities as fuzzy numbers.

Two main properties of every financial market are uncertainty and vague. Monitoring under these conditions is difficult task. Although, stochastic models may consider the uncertainty, however, they do not respect to vague. Therefore the fuzzy logic produces a good inferential setting for monitoring stock market data. Fuzzy approach is applied for making inference in financial problems by Leon et al. (2002), Bermudez et al. (2005).

Hence, following Zulkifli *et al*. (2009), we suppose that the rate of returns are LR-fuzzy numbers $A = (a_l, a_u, c, d)_{LR}$ defined by membership function

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$$\mu_A(r) = \begin{cases} L(\frac{a_l - r}{c}) & r \in [a_l - c, a_l] \\ 1 & r \in [a_l, a_u] \\ R(\frac{r - a_u}{d}) & r \in [a_u, a_u + d] \\ 0 & \text{o.w} \end{cases}$$

Here, $[a_l, a_u]$ is the peak of A and L, R are two even reference functions defined on $[0, 1] \to [0, \infty]$, strictly increasing and upper semi-continuous on

$$supp(A) = \{r; \mu_A(r) > 0\}.$$

This paper is organized as follows. Section 2 considers the cusum, rolling and cusum of square statistics for change point detection in mean and variance of returns. Theoretical justifications are given in section 3. A real data set is given in section 4. Conclusions are given in section 5.

2 CHANGE IN MEAN AND VARIANCE

In this section, we propose the cusum and rolling methods for change point detection in means of returns. Also, we study the shift in variance of return series using the cusum of square test statis-tic.

2.1 Cusum method

Let $R_j = (a_{lj}, a_{uj}, c_j, d_j)_{LR}$, $j = 1, 2, ..., n$ denote the rate of returns. Following Hawkins (1977) and Bermudez et al. (2005), the k-th cusum statistic is given by

$$s_k = \sum_{i=1}^{k} (e_i - \overline{e}),\ k = 1, 2, ..., n-1,$$

where

$$e_j = a_{uj} - a_{lj} + \frac{1}{2}(d_j - c_j), j = 1, 2, ...n.$$

So, we conclude that by plotting s_k over number of observations $k = 1, 2, ..., n - 1$, if there exists some maximum or minimum in these four plots, we conclude that there are some change points.

2.2 Rolling method

The rolling analysis is useful technique to monitor a time series. It is done by estimating parameters over a rolling window with fixed length during the given sample. Therefore, for monitoring an asset returns in mean, we compute $\overline{e}_k^m$

, where $\bar{e}_k^m$ is the mean *ej* for $j = k, ..., k + m - 1, k = 1, 2, ..., n - m + 1$. That is, we compute means over a rolling window with fixed length *m*. If there is a change in means of returns then these plots capture these instabilities.

2.3 Cusum of square method

Inclan and Tiao (1994) used cumulative sum of square for retrospective detection of changes of variance. Following their approach, we propose the

$$\mathrm{IT} = \max_k |D_k|$$

as test statistic for shift detection in variance of returns, where

$$D_k = \frac{\sum_{i=1}^{k} v_i}{\sum_{i=1}^{n} v_i} - \frac{k}{n},$$

with $v_j = \frac{1}{2}(a_{uj} - a_{lj}) + (d_j - c_j), j = 1,2,....,n$

3 THEORETICAL JUSTIFICATION

In this section, we propose some justification for selecting e_i and v_j to use in cusum, rolling and cusum of square test statistic. First, note that R_j is a non-fuzzy variable and it is a random variable. Then,

$$R_j = \mu_j + \varepsilon_j, j = 1,2,....,n.$$

To check the constancy of μj , following Hawkins (1977), the k-th cusum statistic is given by $U_k = \sum_{j=1}^{k}(R_j - \bar{R})$. The overall cusum test statistic for testing the change point among observations *Rj* is given by

$$U = \max_{1 \le k \le n-1} |U_k|.$$

Theorem 1. Assuming R_j being random (non-fuzzy) variables, with com-mon finite variance $\sigma^2 < \infty$, then U_k is close uniformly on k to its expectation given by $\theta_k \sum_{j=1}^{k}(\mu_j - \bar{\mu})$.

Proof. One can see that $E_k = |U_k - \theta_k| = \left|\sum_{j=1}^{k} (\varepsilon_j - \bar{\varepsilon})\right|$. According to the Donsker and continuous mapping theorem, it is seen that

$$\sqrt{n} \sup_{0<t<1} E_{[nt]} \xrightarrow{d} \sup_{0<t<1} |B(t)|,$$

where $B(\cdot)$ is standard Brownian bridge on [0, 1]. This shows that $\max_{1 \le k \le n-1} E_k = O_p(n^{-1/2})$, and this completes the proof.

Remark 1. Suppose that there is a change point in mean process μ_j. That is, they are (say) μ_1^* before change point k_0 and shifts to μ_2^* after time point k_0. Let $\delta = \mu_1^* - \mu_2^*$. It is seen that

$$\theta_k = \begin{cases} k(1-k_0/n)\delta & k \le k_0, \\ k_0(1-k/n)\delta & k \ge k_0 + 1. \end{cases}$$

Therefore, $|\theta_k|$ takes its maximum on k_0 and for constructing the cusum statistic, it is enough to compute $\max_{1 \le k \le n-1} |E(U_k)|$.

Theorem 2. In the fuzzy setting again U_j is close e_j .

Proof. See Bermudez *et al*. (2005).

Remark 2. Following Bermudez et al. (2005) and Zulkifli et al. (2009), one can show that, in the fuzzy setting, $\mu_j = e_j$. The argument for rolling is the same. This justifies proposing the cusum and rolling statistics in above mentioned form in a fuzzy setting. Arguments for cusum of square test statistic, in a non-fuzzy approach, are similar cusum statistic, see Inclan and Tiao (1994). Again, following Bermudez et al. (2005), it is seen that the semi-var(R_j) = v_j . The above Theorem (Theorem 2) and Remarks justifies estimation of θ_k by U_k.

Remark 3. As we learned from a referee, this method may be studied from robustness perspective. Because, considering stock returns as fuzzy numbers may be let us in safety margins since we may not consider small changes as a sharp change points and in practice real change points will be detected, an event which is not happened in non-fuzzy setting.

4 REAL DATA SET

Here, we analyze the TEPIX (Tehran Exchange Price Index). The data set is log-daily return of mentioned stock for period 1 Feb 2005 to 20 May 2009. We have plotted the series and the usual (when the returns are not considered as fuzzy numbers) cusum of the series in Figures 1 and 2, respectively. Both plots indicate that there is no change point among observations. Following Zulkifli et al. (2009), a_{uj} and a_{lj} are asset *j*-th return at 60-th and 40-th percentiles, respectively. The c_j is asset *j*-th return spread between 40-th and 5-th percentiles. Finally, we assume that d_j is asset *j*-th return spread between 95-th and 60-th percentiles. We report the mean and standard deviation and range of U_k as 0.005, 0.0028 and 0.01. This shows that there is no change point in the mean

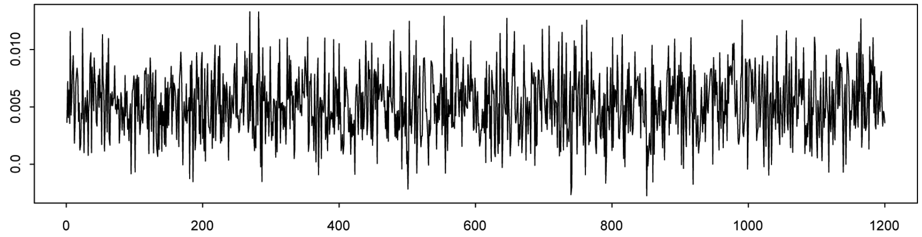

Figure 1: TEPIX Series

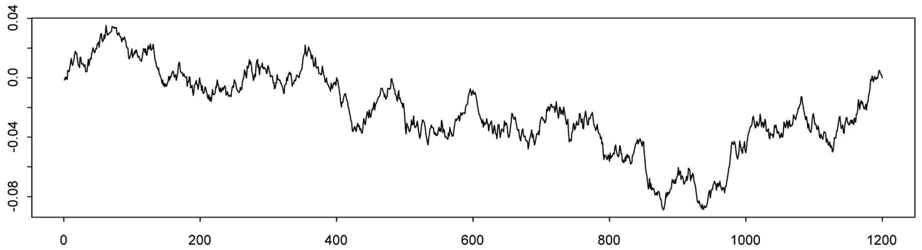

Figure 2: The usual CUSUM of TEPIX Series

of data. Our studies also shows that a GARCH model is fitted to the above mentioned returns and also there is no change point in parameters of GARCH series.

5 CONCLUSIONS

In this paper, we proposed two methods for change point detection in mean of rate of returns of a specified stock market. We proposed two methods cusum procedure and rolling analysis for change point detection. The returns are approximated by fuzzy numbers.

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REFERENCES

Bermudez, J. D., Segura, J.V. and Vercher, E. (2005). Fuzzy portfolio optimization under downside risk measures. *Working paper.* Universitat de Valencia, Spain. http://dx.doi.org/10.1016/j.fss.2006.10.026

Csorgo, M., and Horvath, L., (1997). *Limit Theorems in Change Point Analysis*, Wiley. UK.

Halunga, A. G., Osborn, D. R. and Sensier, M. (2009). Changes in the order of integration of US and UK inflation. *Economics Letters* **102**, 30-32. http://dx.doi.org/10.1016/j.econlet.2008.10.008

Hawkins, D. M. (1977). Testing a sequence of observations for a shift in location. *Journal of American Statistical Association* **72**, 180–186. http://dx.doi.org/10.1080/01621459.1977.10479935

Hillebrand, E. and Schnabl, G. (2003). The effects of Japanese foreign exchange intervention: GARCH estimation and change point detection. *Dis-cussion Paper,* No. **6**. Japan Bank for International Cooperation (JBIC). http://dx.doi.org/10.2139/ssrn.464104

Inclan, C. and Tiao, G. (1994). Use of Cumulative Sum of Square for Retrospective Detection of Changes of Variance. *J. Amer. Statist. Assoc*. **89**. 913-923. http://dx.doi.org/10.1080/01621459.1994.10476824

Leon, T., Liern, V., and Vercher, E. (2002). Viability of infeasible port-folio selection problems: A fuzzy approach. *European Journal of Operational Research* **139**, 178-189. http://dx.doi.org/10.1016/S0377-2217(01)00175-8

Zulkifli, M., Daud, M. and Omar, S. (2009). A fuzzy approach to portfolio selection. *Sains Malaysiana* **38**, 895-899.