Reliability and Availability Analysis of a Single Unit System with Varying Demand

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Abstract   This paper investigates the reliability and availability analysis of a single unit system in an industry, depending on the variation in demand. Variation in demand affects the production also and hence the system may go to down state when the units produced are in excess. The system in the down state is made operative as soon as the produced units are less in number than those demanded. Using regenerative point technique, Mean Time to System Failure, the Availabilities in both the cases i.e. when demand is greater than or equal to production and when demand is less than production have been obtained.

Keywords: Variation in demand, Mean Time to System Failure, Availability.

1. INTRODUCTION

Literature of Reliability contains lot of studies on the reliability and availability analysis of various systems. These studies are contributed by various researchers including Kumar et al. (1995), Attahiru and Zhao (1998), Taneja and Naveen (2003), Taneja and Nanda (2003), Taneja *et al.* (2004), Parashar and Taneja (2007), Mathew *et al.* (2011) wherein the concepts of instruction, warm standby subject to degradation, three units and repair facilities, patience time with chances of non-availability of expert repairman, one of the two repair policies adopted by the expert repairman after having tried by the ordinary repairman, master slave concept for a PLC system, two unit parallel CC plant system respectively have been taken up. These studies have considered the demand as fixed. However, there exist many practical situations where the demand of the units produced is not fixed. Such a situation may be seen in **General Cable Energy System.**

A General Cable System working in a company was observed wherein a rod of Cu or Al 10 mm comes as a raw material, then drawing operation is performed on this rod, Series of dies are used to draw the rod into wire

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of 2 mm size or required size. Then the output diameter of the wire from the drawing machine is sent to extrusion machine. Extrusion is a process of coating the wires with polyvinyl chloride(PVC) or cross-linked polymers layer. In extrusion, PVC is fed onto the hooper, where it melts inside the extruder & fire is fed from the other side to make a coat of PVC onto the naked wire. Same step is being followed for 2-3 wires. After extrusion, 2-3 PVC coated wires are sent to drum twister, the function of drum twister is to twist all the 2-3 coated cores & make a bunch of it, workers put somefibre fillers also to fill the gap in the twists. Then this twisted cable is sent to final extruder for coating the final layer of PVC onto it. After this, testing is carried out .This is the whole process for cable manufacturing. Also, demand for the cable so produced was observed to be varied from time to time. Hence the authors realized the need of studying reliability and availability analysis of a system with varying demand for such systems and hence the present paper.

The present paper investigates the reliability and availability analysis of a single unit system depending on the variation in demand. As variation in demand affects the production and hence the system is required to be put to down state when the units produced are already in excess. The system in the down state is made operative as soon as the produced units are less in number than those demanded. Using regenerative point technique, the mean time to system failure, the availabilities in both the cases i.e. when demand is greater than or equal to production and when demand is less than production have been obtained.

2. MATERIALS AND METHODS

The present study gives the probabilistic analysis of a single unit system by making use of regenerative point technique and obtains various measures of system effectiveness such as mean time to system failure, availability when demand is greater than or equal to production and availability when demand is less than the production.

3. NOMENCLATURE

○	Operative state for the system
⊟	Failed state for the system
⬡	Down state for the system
λ	Failure rate of the operative unit

λ_1	Rate of decrease of demand so as to become less than production
λ_2	Rate of increase of demand so as to become not less than production
λ_3	Rate of going from upstate to downstate (reason behind this is that the demand is less than production & production goes on increasing and as a result we have lot of produces in the stock. This production needs to be stopped.
λ_4	Rate of change of state from down to up when there is no produce with the system and demand is there
$Op_{d \geq p}$	Operative state for the system when demand is not less than production
$Op_{d < p}$	Operative state for the system when demand is less than production
D	Notation for down state
p_1	Probability that during the repair time the demand is greater than or equal to production
p_2	Probability that during the repair time the demand is less than production
F_r	Failed unit under repair
Ao^d	Steady state availability of the system when demand is not less than production
Ao^P	Steady state availability of the system when demand is less than production
$M_i(t)$	Probability that system up initially in regenerative state i is up at time t without passing through any other regenerative state
p_{ij}	Transition probability from a regenerative state i to regenerative state j
m_{ij}	Contribution to mean sojourn time in regenerative state i before transiting to regenerative state j without visiting to any other state
$\mu_i(t)$	Mean sojourn time in regenerative state i before transiting to any other state
*	Symbol for Laplace transform
**	Symbol for Laplace Stieltjes transform
©	Symbol for Laplace convolution
s	Symbol for Stieltjes convolution
g(t),G(t)	p.d.f. and c.d.f. of repair time for the unit

$q_{ij}(t), Q_{ij}(t)$	p.d.f. and c.d.f. of first passage time from a regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0,t]

4. MODEL DESCRIPTION AND ASSUMPTIONS

1 As soon as the unit fails, it is undertaken for repair
2 Completion of the repair process puts the unit back into operation.
3 All the random variables are independent.
4 The breakdown times are assumed to be exponentially distributed whereas the repair time distributions are arbitrary.

5. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The transition diagram showing the various states of the system is shown as in Fig 1.The epochs of entry into states S_0, S_1 and S_3 are regeneration points and thus are regenerative states. States S_2 and S_4 are failed states. The transition probabilities are:

$$q_{01}(t) = \lambda_1 e^{-(\lambda+\lambda_1)t} \quad ; \quad q_{02}(t) = \lambda e^{-(\lambda+\lambda_1)t}$$
$$q_{10}(t) = \lambda_2 e^{-(\lambda_2+\lambda_3+\lambda)t} \quad ; \quad q_{13}(t) = \lambda_3 e^{-(\lambda_2+\lambda_3+\lambda)t}$$
$$q_{14}(t) = \lambda_1 e^{-(\lambda_2+\lambda_3+\lambda)t} \quad ; \quad q_{20}(t) = g(t)$$
$$q_{30}(t) = \lambda_4 e^{-\lambda t} \quad ; \quad q_{40}(t) = p_1 g(t)$$
$$q_{41}(t) = p_2 g(t)$$

The non-zero elements $p_{ij} = \lim_{s \to 0} q_{ij}^*(s)$ are:

$$q_{01} = \lambda_1/(\lambda+\lambda_1) \quad ; \quad q_{02} = \lambda/(\lambda+\lambda_1)$$
$$q_{10} = \lambda_2/(\lambda+\lambda_2+\lambda_3) \quad ; \quad q_{13} = \lambda_3/(\lambda+\lambda_2+\lambda_3)$$
$$q_{14} = \lambda/(\lambda+\lambda_2+\lambda_3) \quad ; \quad q_{20} = 1$$
$$q_{30} = 1 \quad ; \quad q_{40} = p_1$$
$$q_{41} = p_2$$

By these transition probabilities, it can be verified that

$$p_{01} + p_{02} = 1 \quad ; \quad p_{10} + p_{13} + p_{14} = 1$$
$$p_{20} = 1 \quad ; \quad p_{30} = 1;$$
$$p_{40} + p_{41} = p_1 + p_2 = 1$$

The mean sojourn time (μ_i) in state i, are

$$\mu_0 = 1/(\lambda+\lambda_1);$$
$$\mu_1 = 1/(\lambda+\lambda_2+\lambda_3)$$

$$\mu_2 = -g*'(0);$$
$$\mu_3 = 1/\lambda_4;$$
$$\mu_4 = -g*'(0) = \mu_2$$

The unconditional mean time taken by the system to transit for any state j when it is counted from epoch of entrance into state i is mathematically stated as:

$$m_{ij} = \int_0^\infty t\,q_{ij}(t)\,dt = -q_{ij}*'(0)$$

Thus,

$$m_{01} + m_{02} = \mu_0;\ m_{10} + m_{13} + m_{14} = \mu_1;\ m_{20} = \mu_2;\ m_{30} = \mu_3;$$

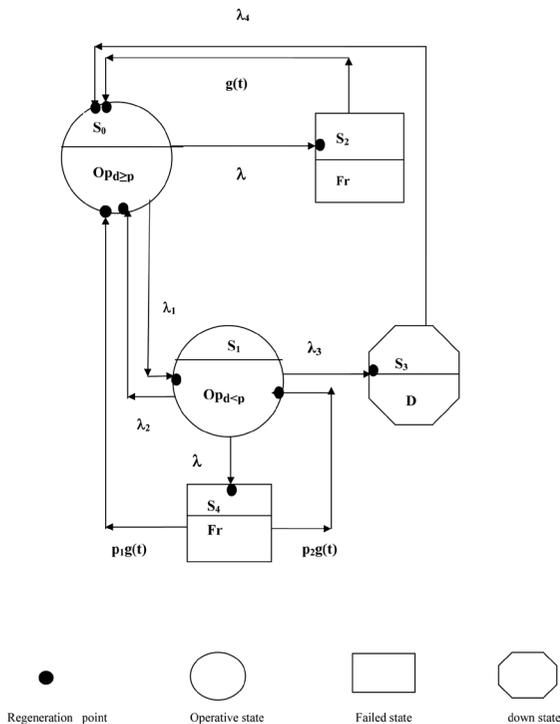$$m_{40} + m_{41} = \int_0^\infty \bar{G}(t)\,dt = \mu_4$$

Figure 1: State Transition Diagram

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6 MATHEMATICAL ANALYSIS:

6.1 Mean Time to System Failure

To determine the mean time to system failure (MTSF) of the given system, we regard the failed states as absorbing states. By probabilistic arguments, we obtain the following recursive relations for $\phi_i(t)$:

$$\phi_0(t) = Q_{01}(t)s\phi_1(t) + Q_{02}(t)$$
$$\phi_1(t) = Q_{10}(t)s\phi_0(t) + Q_{13}(t)s\phi_1(t) + Q_{14}(t)$$
$$\phi_3(t) = Q_{30}(t)s\phi_0(t)$$

Taking Laplace – Steltjes Transform (L.S.T.) of these relations and solving them for $\phi_0{}^{**}(s)$, we obtain

$$\phi_0{}^{**}(s) = N(s)/D(s)$$

where

$$N(s) = q_{01}{}^*(s)q_{14}{}^*(s) + q_{02}{}^*(s)$$
$$D(s) = 1 - q_{01}{}^*(s)q_{10}{}^*(s) - q_{01}{}^*(s)q_{13}{}^*(s)q_{30}{}^*(s)$$

As Reliability of a device in time 't' is the probability that the system does not fail during time (0,t) when initially system starts from up state S_i (i=0,1,3). If T is a random variable representing the time till failure of a device starting with an initial operable condition at t = 0, then the reliability $R(t)$ of the device is given by

$$R(t) = P[T > t] = 1\text{-}P[T \leq t] = 1 - \Phi(t).$$

Taking Laplace transform of above relation
We get,

$$R^*(s) = (1/s) - (\phi_0{}^{**}(s)/s)$$
$$= (1 - \phi_0{}^{**}(s))/s$$
$$= \frac{1 - \Phi_0{}^{**}(s)}{s}$$

where $R*(s)$ is the Laplace transform of R(t). The Reliability R(t) of the system at time t can be obtained taking inverse Laplace transform of $R*(s)$.

As the mean time to system failure (MTSF) of the system is given by

$$\text{MTSF} = E(t)$$

$$= \int_0^\infty t\phi(t)dt$$

$$= \int_0^\infty t(dR(t)/dt)dt$$

$$= \int_0^\infty R(t)dt$$

$$= \lim_{s\to 0} \int_0^\infty e^{-st}R(t)dt$$

$$= \lim_{s\to 0} R^*(s)$$

Putting the value of $R*(s)$, we get mean time to system failure (MTSF) when the system starts from the state '0'is

$$\text{MTSF} == \lim_{s\to 0} \quad R^*(s) = \lim_{s\to 0} \frac{1 - \Phi_0^{**}(s)}{s}$$

Using L'Hospital rule and putting the value of Φ_0**(s) from eq^n , we have
MTSF= N/ D,

where N =μ_0+ p_{01} μ_1+p_{01} p_{13} μ_3
D = p_{01} p_{14}+ p_{02}

7. AVAILABILITY ANALYSIS WHEN DEMAND IS NOT LESS THAN PRODUCTION

Using the arguments of the theory of the regeneration process, the availability $A_i^d(t)$ is seen to satisfy the following recursive relations :

$$A_0^d(t) = M_0(t) + q_{01}(t) \copyright A_1^d(t) + q_{02}(t) \copyright A_2^d(t)$$
$$A_1^d(t) = q_{10}(t) \copyright A_0^d(t) + q_{13}(t) \copyright A_3^d(t) + q_{14}(t) \copyright A_4^d(t)$$
$$A_2^d(t) = q_{20}(t) \copyright A_0^d(t)$$
$$A_3^d(t) = q_{30}(t) \copyright A_0^d(t)$$
$$A_4^d(t) = q_{40}(t) \copyright A_0^d(t) + q_{41}(t) \copyright A_1^d(t)$$

where $M_0(t) = e^{-(\lambda+\lambda_1)t}$

Taking Laplace transforms of the above equations and solving them for $A_0^*(s)$, we get

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$$A_0^{d*}(s) = N_1(s) / D_1(s)$$

where $N_1(s) = M_0^*(s)[1 - q_{14}^*(s)q_{41}^*(s)]$

$$D_1(s) = 1 + [q_{02}^*(s)q_{14}^*(s)q_{20}^*(s)\, q_{41}^*(s) - q_{02}^*(s)\, q_{20}^*(s) - q_{14}^*(s)q_{41}^*(s)$$
$$- q_{01}^*(s)q_{13}^*(s)q_{30}^*(s) - q_{01}^{(3)*}(s)q_{14}^*(s)q_{40}^*(s) - q_{01}^*(s)q_{10}^*(s)]$$

In steady- state, the availability of the system is given by

$$A_0^d = \lim_{s \to 0}(sA_0^*(s)) = N_1 / D_1$$

where $N_1 = (1 - p_{14}p_{41})\mu_0$ and

$$D_1 = \mu_0(1 - p_{14}p_{41}) + \mu_1 p_{01} + \mu_2 p_{02}(1 - p_{14}p_{41}) + \mu_3(p_{01}p_{13}) + \mu_4(p_{01}p_{14})$$

8. AVAILABILITY ANALYSIS WHEN DEMAND IS LESS THAN PRODUCTION

Using the arguments of the theory of the regeneration process, the availability $A_i^p(t)$ is seen to satisfy the following recursive relations :

$$A_0^P(t) = q_{01}(t)©A_1^P(t) + q_{02}(t)©A_2^P(t)$$
$$A_1^P(t) = M_1(t) + q_{10}(t)©A_0^P(t) + q_{13}(t)©A_3^P(t) + q_{14}(t)©A_4^P(t)$$
$$A_2^P(t) = q_{20}(t)©A_0^P(t)$$
$$A_3^P(t) = q_{30}(t)©A_0^P(t)$$
$$A_4^P(t) = q_{40}(t)©A_0^P(t) + q_{41}(t)©A_1^P(t)$$

where $M_1(t) = e^{-(\lambda_2 + \lambda_3 + \lambda)t}$

Taking Laplace transforms of the above equations and solving them for $A_0^*(s)$, we get

$$A_0^{P*}(s) = N_2(s) / D_1(s)$$

Where $N_2(s) = M_1^*(s)q_{01}^*(s)$

In steady- state, the availability of the system is given by

$$A_0 = \lim_{s \to 0}(sA_0^*(s)) = N_2 / D_1$$

where $N_2 = p_{01}\mu_1$ and D_1 is already specified.

9. GRAPHICAL ANALYSIS

For the particular case, the rate of repair (α) is assumed to be exponentially distributed. Let us take g (t) = $\alpha\, e^{-\alpha t}$.

Using the above particular case and the values $\lambda_1 = 0.005$, $\lambda_2 = 1$, $\lambda_3 = 0.1$, $\lambda_4 = 2$, $\alpha = 2$,

$$P_{01} = \lambda_1 / (\lambda + \lambda_1); \quad p_{02} = \lambda / (\lambda + \lambda_1)$$

$$P_{10} = \lambda_2 / (\lambda + \lambda_2 + \lambda_3); \quad p_{13} = \lambda_3 / (\lambda + \lambda_2 + \lambda_3); \quad p_{14} = \lambda / (\lambda + \lambda_2 + \lambda_3)$$

$$P_{20} = 1; p_{30} = 1; P_{40} = P_1 = 0.665; P_{41} = P_2 = 0.335$$

$$\mu_0 = 1 / (\lambda + \lambda_1); \quad \mu_1 = 1 / (\lambda + \lambda_2 + \lambda_3); \quad \mu_2 = -g^{*\prime}(0) = \mu_4; \mu_3 = 1 / \lambda_4;$$

estimated from a single unit system, graphical study has been made which enables us to draw various conclusions.

It is observed from the graph that

1   For $\lambda = 0.2$, value of MTSF is 5.01, availability when demand is greater than or equal to production is 0.82 where as availabity when demand is less than production is .0003

2   For $\lambda = 0.4$, value of MTSF is 2.5, availability when demand is greater than or equal to production is 0.7 where as availabity when demand is less than production is .0026

3   For $\lambda = 0.6$, value of MTSF is1.67, availability when demand is greater than or equal to production is 0.61 where as availabity when demand is less than production is .002

4   For $\lambda = 0.8$, value of MTSF is 1.25, availability when demand is greater than or equal to production is 0.53 where as availabity when demand is less than production is .0016

5   For $\lambda = 1$, value of MTSF is 1, availability when demand is greater than or equal to production is 0.48 where as availabity when demand is less than production is .0032
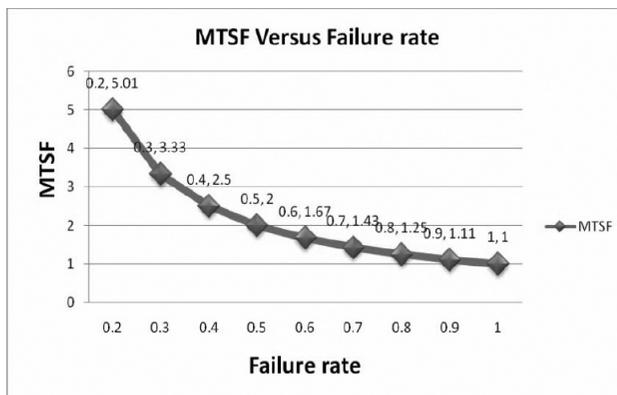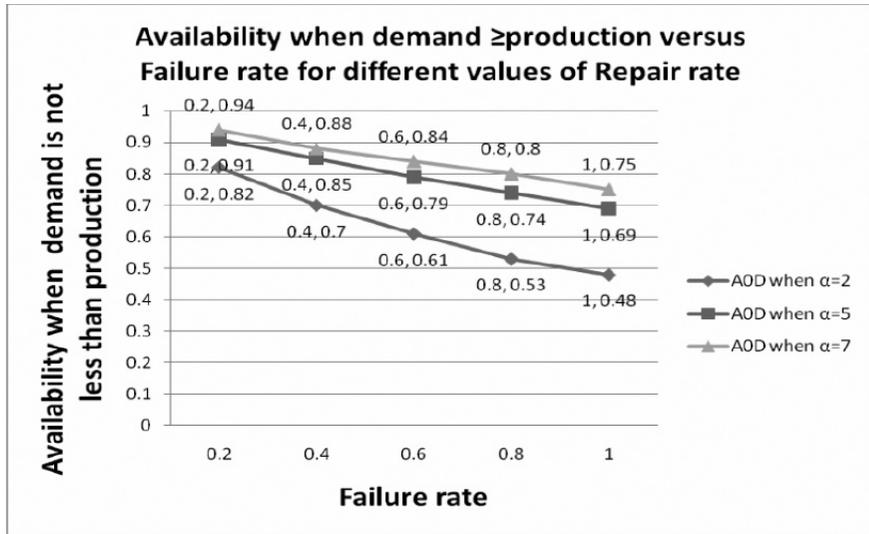

Figure 2: MTSF versus Failure rate

Figure 3: Availability when demand is not less than production versus Failure rate rate for different values of Repair rate
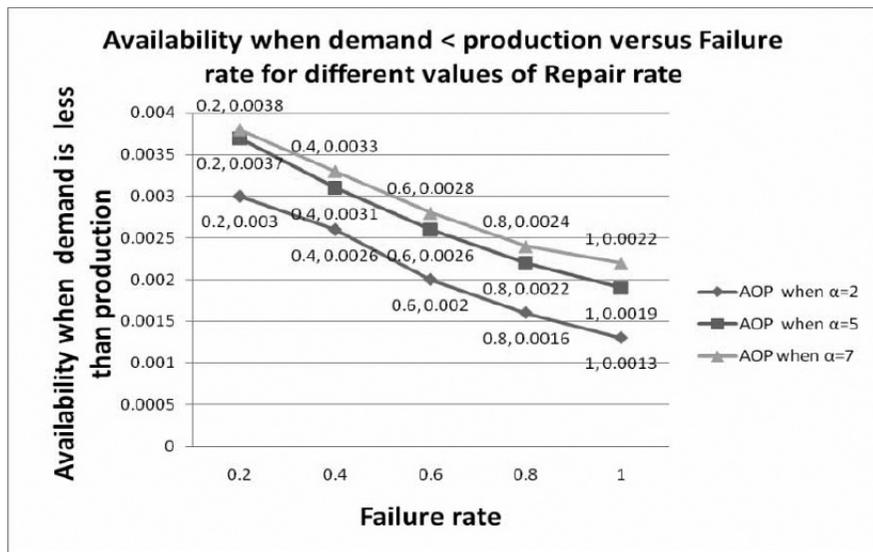
Figure 4: Availability when demand is less than production versus Failure rate for different values of Repair rate
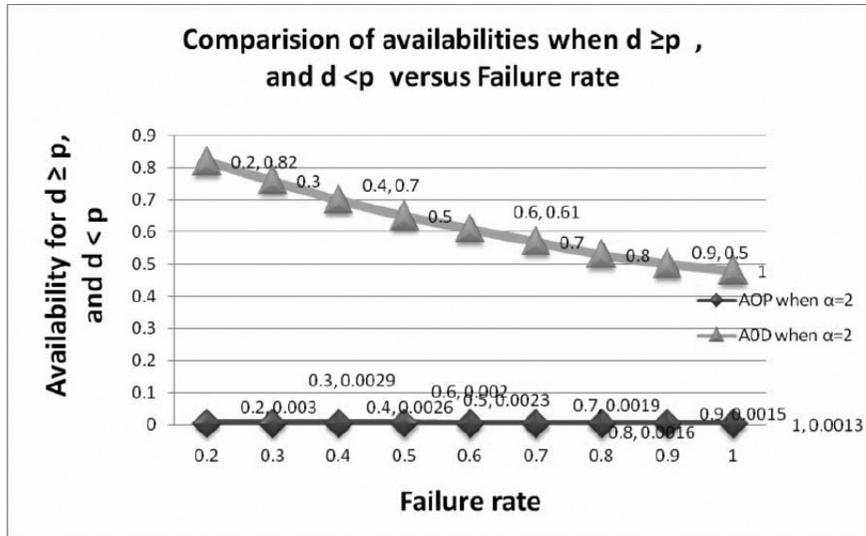
Figure 5: Comparsion of Availability when demand is less than production
and Availability when demand is not less than production for
different values of Failure rate

Figure 2 shows the behavior of MTSF with respect to failure rate. It can be concluded from the graph that MTSF has lower value for higher value of failure rate.

Figure 3 shows that the behavior of availability when demand is not less than production with respect to failure rate for different values of repair rate. It can be concluded from the graph availability decreases with increase in failure rate. It can also observed that system has lower availability with low repair rate.

Figure 4 shows that the behavior of availability when demand is less than production with respect to failure rate for different values of repair rate. It can be concluded from the graph availability decreases with increase in failure rate. It can also observed that system has higher availability for higher values of repair rate.

Figure 5 shows the comparative study of two availabilities.it is clear that availability of the system when demand is not less than production is always more than the availability when demand is less than production for same value of the failure rate.

CONCLUSION

Availability of a system is typically measured as a factor of its reliability- as reliability increases, so does availability. The behavior of the mean time to

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system failure, availability when demand is not less than production, availability when demand is less than production with respect to failure rate is analyzed . It reveals from various graphs that all these get decreased with increase in the values of failure rates.However availability ha the higher values for higher values of repair rate.We also compare both availabilities i.e. availability when demand is not less than production and availability when demand is less than production by plotting them on the same graph for different values of failure rate. As, for repairable system, availability is a very meaningful measure, and achieving a high or required level of avail-ability is an essential requisite. Hence for increase in system availability, demand should be greater than or equal to production. By plotting other graphs, various suggestions can be given to the company.

REFERENCES

Attahiru Sule Alfa W.L. and Zhao, Y.Q. (1998); Stochastic analysis of a repairable system with three units and repair facilities. *Microelectron. Reliab.,* **38(4)**, 585-595. http://dx.doi.org/10.1016/S0026-2714(97)00204-7

Kumar, A., Gupta, S.K.   and Taneja, G. (1995); Probabilistic analysis of a two-unit cold standby system with instructions at need. *Microelectron. Reliab*., **35(5),** 829-832. http://dx.doi.org/10.1016/0026-2714(94)00099-A

Mathew, A.G., Rizwan, S.M., Majumder, M.C., Ramachandran, K.P. and Taneja, G.(2011); Reliability analysis of an identical two-unit parallel CC plant system operative with full installed capacity, *International Journal of Performability Engineering* **7(2),** 179-185.

Parashar, B. and Taneja, G. (2007); Reliability and profit evaluation of a PLC hot standby system based on a master slave concept and two types of repair facilities, *IEEE Transactions on Reliability*, **56(3)**, 534-539. http://dx.doi.org/10.1109/TR.2007.903151

Taneja, G. and Naveen V. (2003); Comparative study of two reliability models with patience time and chances of non-availability of expert repairman, *Pure and Applied Mathematika Sciences***, LVII (1-2)**, 23-35.

Taneja, G. and Nanda, J. (2003); Probabilistic analysis of a two-unit cold standby system with resume and repeat repair policies, *Pure and Applied Mathematika Sciences*, **LVII (1-2),** 37-49.

Taneja, G. Tyagi, V.K. and Bhardwaj, P. (2004); Profit analysis of a single unit programmable logic controller (PLC), *Pure and Applied Mathematika Sciences* **LX (1-2);** 55-71.