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Some Applications of The New Integral Transform For Partial Differential Equations

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ABSTRACT

In this paper we have derived Sadik transform of the partial derivatives of a function of two variables. We have demonstrated the applicability of the Sadik transform by solving some examples of partial differential equations. We have verified solutions of partial differential equations by Sadik transform with the Laplace transform and the Sumudu transform.

1. Introduction

There are a lot of methods to solve partial differential equations, but for linear partial differential equations the most powerful method is an integral transformation method. Laplace transform is the most effective tool to solve some kinds of ordinary and partial differential equations. Actually an electric engineer Oliver Heaviside made Laplace transform popular by developing its operational calculus. After Laplace transform, in 1993 again an electrical engineer Watugula in [1] proposed a new integral transform named the Sumudu transform and used it for solving problems in control engineering, it is similar to the Laplace transform having the preservation property of unit and change of scale. After that, T. Elzaki [6] introduced a new integral transform named Elzaki transform and applied it for solving partial differential equations, Shaikh Sadikali has been applied Elzaki transform for solving integral equations of convolution type see in [5]. Likewise many integral transforms have been proposed which are similar to the Laplace transform, and each new transform claimed its own superiority over the Laplace transform. In this paper we considered a new integral transform named the Sadik transform [3], [4]. It is similar to the Laplace transform but the Laplace transform, the Sumudu transform, Elzaki transform and all integral transforms with kernel of an exponential type are particular cases of the Sadik transform. Due to the very general and unified nature of the Sadik transform, we can transport a problem of partial differential equations into the known transformation technique which is available in the literature through the Sadik transform.

2. Preliminaries

In this section we demonstrated basic definitions and some operational properties of Sadik transform.

2.1 Definition [2]

If,

i) $f(t)$ is piecewise continuous on the interval $0 \le t \le A$ for any $A > 0$.

ii) $|f(t)| \le K.e^{at}$ when $t \ge M$, for any real constant a. and some positive constant K and M.

Then, the Laplace transform of $f(t)$ is defined by

$$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt \;\; for\; Re(s) > a \qquad (1)$$

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2.2 Definition [1]

If

$$f(t)\in\left\{f(t):M,k_1,k_2>0,\left|f(t)\right|<M.\exp\left(t/k_j\right)\right\}$$

Then the Sumudu transform of $f(t)$ is defined by,

$$F(u)=S\left[f(t)\right]=\int_0^\infty e^{-t}f(u.t)\,dt \qquad (2)$$

When the integral of right side of (2) is converges.

2.3 Definition [3]

If,

i) $f(t)$ is piecewise continuous on the interval $0\le t\le A$ for any $A>0$.

ii) $\left|f(t)\right|\le K.e^{w^a.t}$ when $t\ge M$, for any real constant w^a . and some positive constant K and M. Then the Sadik transform of $f(t)$ is defined by

$$F\left(v^\alpha,\beta\right)=S\left[f(t)\right]=\frac{1}{v^\beta}\int_0^\infty e^{-v^\alpha t}f(t)\,dt,\ for Re\left(v^\alpha\right)>w^a \qquad (3)$$

Where,
v is complex variable,
α is any non zero real numbers, and
β is any real number.
The beauty of this transform is that by changing the values of α and β we can convert a considered problem into the Laplace transform, the Sumudu transform, Elzaki transform and all other transforms whose kernels are of an exponential type. For instant suppose that $\alpha=1$ and $\beta=0$ then it will be the Laplace transform, if $\alpha=-1$ and $\beta=1$ then it will be the Sumudu transform, and so on.

3. Operational Properties of the Sadik transform

If the Sadik transform of $f(t)$ is denoted by $F\left(v^\alpha,\beta\right)$ then the followings are the operational properties of the Sadik transform of some standard functions, see in [3].

1) If $f(t)=t^n$ then, Sadik transform of $f(t)=t^n$ is

$$S\left[t^n\right]=F\left(v^\alpha,\beta\right)=\frac{n!}{v^{(n+1)\alpha+\beta}} \qquad (4)$$

2) If $f(t)=\sin(at)$ then, Sadik transform of $f(t)=\sin(at)$ is

$$S\left[\sin(at)\right]=F\left(v^\alpha,\beta\right)=\frac{av^{-\beta}}{v^{2\alpha}+a^2} \qquad (5)$$

3) If $f(t)=cos(at)$ then, Sadik transform of $f(t)=cos(at)$ is

$$S\left[\cos(at)\right]=F\left(v^\alpha,\beta\right)=\frac{v^{\alpha-\beta}}{v^{2\alpha}+a^2} \qquad (6)$$

4) If $f(t)=e^{at}$ then, Sadik transform of $f(t)=e^{at}$ is

$$S\left[e^{at}\right]=F\left(v^\alpha,\beta\right)=\frac{v^{-\beta}}{v^\alpha-a} \qquad (7)$$

5) Sadik transform of hyperbolic functions

$$S\left[\sinh(at)\right]=F\left(v^\alpha,\beta\right)=\frac{av^{-\beta}}{v^{2\alpha}-a^2} \qquad (8)$$

$$S\left[\cosh(at)\right]=F\left(v^\alpha,\beta\right)=\frac{v^{\alpha-\beta}}{v^{2\alpha}-a^2} \qquad (9)$$

6) *Sadik transform of derivatives:*
If $F(v)$ is Sadik transform of *f*(*t*) then,

$$S\left[f'(t)\right]=v^\alpha F\left(v^\alpha,\beta\right)-v^{-\beta}f(0) \qquad (10)$$

$$S\left[f^{(n)}(t)\right]=v^{n\alpha}F\left(v^\alpha,\beta\right)-\sum_{k=0}^{n-1}v^{k\alpha-\beta}f^{(n-1)-k} \qquad (11)$$

Now suppose that $\varphi(x,t)$ is a piecewise continuous function with exponential order such that their derivatives are also a piecewise continuous functions with exponential order then we can find its Sadik transform and we will use these properties for solving some partial differential equations in next section.

3.1 Proposition

If $G\left(x,v^\alpha,\beta\right)$ is a Sadik transform of $\varphi(x,t)$ and $\varphi_t(x,t)$ is a first partial derivative of $\varphi(x,t)$ with respect to variable t then

$$S[\varphi_t(x,t)] = v^{\alpha}G(x,v^{\alpha},\beta) - v^{-\beta}\varphi(x,0) \qquad (12)$$

Proof: Starting with the definition of Sadik transform,

$$S[\varphi_t(x,t)] = v^{-\beta}\int_0^{\infty} e^{-tv^{\alpha}}\varphi_t(x,t)\,dt$$

By using integration by part, we get

$$= v^{-\beta}\left\{\left[e^{-tv^{\alpha}}.\varphi(x,t)\right]_0^{\infty} - \int_0^{\infty} e^{-tv^{\alpha}}\left(-v^{\alpha}\right)\varphi(x,t)\,dt\right\}$$

Assuming $Re(v^{\alpha}) > 0$, we get

$$= -v^{-\beta}\varphi(x,0) + v^{\alpha}v^{-\beta}\int_0^{\infty} e^{-tv^{\alpha}}\varphi(x,t)\,dt$$

Hence,

$$S[\varphi_t(x,t)] = v^{\alpha}G(x,v^{\alpha},\beta) - v^{-\beta}\varphi(x,0)$$

3.2 Proposition

If $G(x,v^{\alpha},\beta)$ is a Sadik transform of $\varphi(x,t)$ and $\varphi_{tt}(x,t)$ is a second order partial derivative of $\varphi(x,t)$ with respect to variable 't' then

$$S[\varphi_{tt}(x,t)] = v^{2\alpha}G(x,v^{\alpha},\beta) - v^{\alpha-\beta}\varphi(x,0) - v^{-\beta}\varphi_t(x,0) \qquad (13)$$

Proof:

Integrating by parts twice we can get the required result easily, therefore we omit the proof.

3.3 Proposition

If $G(x,v^{\alpha},\beta)$ is a Sadik transform of $\varphi(x,t)$ then

$$S[\varphi_x(x,t)] = G_x(x,v^{\alpha},\beta) \qquad (14)$$

Where $\varphi_x(x,t)$ and $G_x(x,v^{\alpha},\beta)$ are partial derivatives of $\varphi(x,t)$ and $G(x,v^{\alpha},\beta)$ with respect to the variable x.

Proof

Using differentiation under integral sign, we can prove this result easily. In general we can easily established the following

$$S[\varphi_{x^n}(x,t)] = G_{x^n}(x,v^{\alpha},\beta)$$

4. Applications of Sadik Transform to Partial Differential Equations

In this section we demonstrate the applicability of new integral transform named Sadik transform to solve some examples of partial differential equations.

Example1. [6]

If $y(x,t)$ is a solution of the following first order partial differential equation

$$y_x + 7y_t = y$$

With initial condition,

$$y(x,0) = 5e^x$$

Then find the solution using Sadik transform.

Solution:

Let $Y(x,v^{\alpha},\beta)$ be Sadik transform of $y(x,t)$.
Now applying Sadik transform, we get

$$Y_x(x,v^{\alpha},\beta) + 7.\left[v^{\alpha}Y(x,v^{\alpha},\beta) - v^{-\beta}y(x,0)\right] = Y(x,v^{\alpha},\beta)$$

Using initial condition, we can rewrite this equation as

$$Y_x(x,v^{\alpha},\beta) + \left[7v^{\alpha}-1\right]Y(x,v^{\alpha},\beta) = 35e^x v^{-\beta}$$

It is linear ordinary differential equation in $Y_x(x,v^{\alpha},\beta)$
integrating factor $= \exp(7v^{\alpha}-1)x$.
Hence required solution is

$$Y(x,v^{\alpha},\beta)\exp(7v^{\alpha}-1)x = \int 35v^{-\beta}\exp(7v^{\alpha}-1)x\,dx + c$$

$$Y(x,v^{\alpha},\beta)e^{(7v^{\alpha}-1)x} = 5v^{-\alpha-\beta}e^{7v^{\alpha}x} + c$$

Since y is bounded therefore c=0.

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$$Y\left(x,v^{\alpha},\beta\right)=5v^{-\alpha-\beta}e^{x}=\frac{5e^{x}}{v^{\alpha+\beta}}$$

Taking inverse Sadik transform, we get

$$y(x,t)=5e^{x}S^{-1}\left[\frac{1}{v^{\alpha+\beta}}\right]$$

Therefore,

$$y(x,t)=5e^{x}$$

Example2. [6]

Solve

$$y_{xx}(x,t)+y_{tt}(x,t)=0$$

Subject to the condition that

$$y(x,0)=6sin^{2}(x),\, y_{t}(x,0)=\cos(6x)$$

Solution:

Applying the Sadik transform, we get

$$Y_{xx}\left(x,v^{\alpha},\beta\right)+v^{2\alpha}Y\left(x,v^{\alpha},\beta\right)-v^{\alpha-\beta}y(x,0)-v^{-\beta}y_{t}(x,0)=0$$

Therefore,

$$Y_{xx}\left(x,v^{\alpha},\beta\right)+v^{2\alpha}Y\left(x,v^{\alpha},\beta\right)=6v^{\alpha-\beta}sin^{2}(x)-v^{-\beta}\cos(6x)$$

It is second order ordinary differential equation, hence its particular integral is

$$Y\left(x,v^{\alpha},\beta\right)=\frac{1}{D^{2}+v^{2\alpha}}\left[3v^{\alpha-\beta}-3v^{\alpha-\beta}\cos(2x)-v^{-\beta}\cos(6x)\right]$$

Hence,

$$Y\left(x,v^{\alpha},\beta\right)=\frac{3}{v^{\alpha+\beta}}-\frac{3v^{\alpha-\beta}\cos(2x)}{v^{2\alpha}-4}-\frac{v^{-\beta}\cos(6x)}{v^{2\alpha}-36}$$

Taking inverse Sadik transform, we get

$$y(x,t)=3-3\cosh(2t)\cos(2x)-\frac{1}{6}\sinh(6t)\cos(6x)$$

It is required solution.

Anytime we can trans figurate above problem to any other known transform, which is available in the literature by fixing values of alpha and beta. Suppose that $\alpha=1\,\text{and}\,\beta=0$, then above problem is ready to solve by the Laplace transform.

$$Y\left(x,v^{1},0\right)=\frac{3}{v}-\frac{3v\cos(2x)}{v^{2}-4}-\frac{\cos(6x)}{v^{2}-36}$$

Taking inverse Laplace transform, we get

$$y(x,t)=3-3\cosh(2t)\cos(2x)-\frac{1}{6}\sinh(6t)\cos(6x)$$

It is the same as above answer which is obtained by the Sadik transform.

Also, Suppose that $\alpha=-1\,and\,\beta=1$, then above problem is ready to solve by the Sumudu transform.

$$Y\left(x,v^{-1},1\right)=\frac{3}{1}-\frac{3\cos(2x)}{v^{-2}-4}-\frac{v^{-1}\cos(6x)}{v^{-2}-36}$$

It can be further written as

$$Y\left(x,v^{-1},1\right)=\frac{3}{1}-\frac{3v^{2}\cos(2x)}{1-4v^{2}}-\frac{\text{v.}\cos(6x)}{1-36v^{2}}$$

Taking inverse Sumudu transform, we get

$$y(x,t)=3-3\cosh(2t)\cos(2x)-\frac{1}{6}\sinh(6t)\cos(6x)$$

Now it's time to check the beauty of this Sadik transform, Suppose that $\alpha=2\,and\,\beta=-2$, then above problem is ready to solve by such a integral transform particularly which is yet not exist in the literature or yet anybody wouldn't proposed.

Therefore,

$$Y\left(x,v^{2},-2\right)=\frac{3}{1}-\frac{3v^{4}\cos(2x)}{v^{4}-4}-\frac{v^{2}\cos(6x)}{v^{4}-36}$$

Since $\alpha=2\,and\,\beta=-2$, by inspection we can easily identify each term in R.H.S. for taking inverse Sadik transform.

$$Y\left(x,v^{2},-2\right)=3\left(\frac{1}{v}\right)^{0}-\frac{3v^{4}\cos(2x)}{v^{4}-4}-\frac{v^{2}\cos(6x)}{v^{4}-36}$$

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If we got any power of $\frac{1}{v}$ then we can consider its inverse Sadik transform is t^n, now we will find the value of n. Since the power of $\frac{1}{v}$ is zero therefore,

$(n+1)\alpha+\beta=0$

Put $\alpha=2\, and\, \beta=-2$, we get $n=0$, hence

$$S^{-1}\left[\left(\frac{1}{v}\right)^0\right]=t^0=1$$

Now we will find

$$S^{-1}\left[\frac{3v^4\cos(2x)}{v^4-4}\right]=3\cos(2x)\,S^{-1}\left[\frac{v^4}{v^4-4}\right]$$

Since the denominator is v^4-4 and $\alpha=2$, the power of is nothing but 2α so by inspection, its inverse Sadik transform would be either (2*t*) *or* cosh (2*t*) but numerator is $v^{\alpha-\beta}$

Therefore,

$$S^{-1}\left[\frac{v^4}{v^4-4}\right]=\cosh(2t)$$

Hence,

$$S^{-1}\left[\frac{3v^4\cos(2x)}{v^4-4}\right]=3\cos(2x)\cosh(2t)$$

In the third term of R.H.S. numerator having power 2 (i.e. $-\beta$) of v so adjust numerator by multiplying and dividing 6, we must have

$$S^{-1}\left[\frac{v^2\cos(6x)}{v^4-36}\right]=\frac{1}{6}\cos(6x)\,S^{-1}\left[\frac{6v^2}{v^4-36}\right]$$

$$S^{-1}\left[\frac{v^2\cos(6x)}{v^4-36}\right]=\frac{1}{6}\cos(6x).\sinh(6t)$$

Hence the final solution of example 2. is

$$S^{-1}\left[\frac{v^2\cos(6x)}{v^4-36}\right]=\frac{1}{6}\cos(6x).\sinh(6t)$$

Conclusions

Sadik transform has been successfully applied to solve partial differential equations in a simple manner.

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References

[1] G. K. Watugula, A New Integral Transform to solve differential equations and control engineering problems, *International Journal of Mathematical Education in sxience 7 Technology*, **24,** 409–421 (1993).

[2] Lokenath Debnath and D, Bhatta, Integral Transform and Their Application, Second Edition, Champman & Hall/CRC (2006).

[3] Sadikali Latif Shaikh, Introducing a New Integral Transform: SADIK Transform, American International Journal of Research in Science, Technology, Engineering & Mathematics, **22(1),** 100–103 (2018).

[4] Sadikali latif Shaikh, "Sadik Transform In Control Theory", International journal of Innovative Sciences & Research Technology, **3,** Issue 5,396–398, (2018).

[5] Shaikh Sadikali and M.S. Chaudhary, On A new Integral Transform and Solution of Some Integral Equations, *International Journal of Pure and Applied Mathematics*, **73,** 299–308, (2011).

[6] M. E. Tariff and M. E. Salih, "Application of New Transform Elzaki Transform to Partial Differential Equations", *Global Journal of pure and applied Mathematics*, **7(1),** pp. 65–70, (2011).