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Mahgoub Deterioration Method and its Application in Solving Duo-combination of Nonlinear PDE's

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ABSTRACT

This paper aims to solve Duo-combination of non linear partial differential equations by a latest approach called Mahgoub deterioration method (MDM). The latest technique is mix of the Mahgoub transform furthermore the, Adomian deterioration method. The generalized solution has been proved. Mahgoub deterioration method (MDM) is a very successful tool for finding the correct solution of linear and non linear partial differential equations. The continuance and uniqueness of solution is based on MDM.

1. Introduction

Multiple problems in Mathematics are carved by nonlinear partial differential equation. Various researchers are putting efforts to go through these problems finding the correct or almost accurate solutions using diverse procedure. A thousand and one researchers were keen in solving differential equations as well as paid immersion in going through the solution of nonlinear partial differential equations by several approaches. In the past few years, a number of integral transforms have been introduced which help us in solving ODEs and PDEs. We have applied Mahgoub deterioration method to find the exact solution to solve duo-combination of nonlinear partial differential equations (CSNLPDEs). A new Mahgoub Transform are introduced by [6] Mohand Mahgoub (2016). Dualism between Mahgoub integral transform and some integral transforms have been found [8]. The utility of Mahgoub integral transform method [7-10] exists in the literature to solve partial differential equations, ordinary differential equations, fractional ordinary differential equation and integral equations. We can see that several problems in the field of Physics and Engineering have been found to show the accuracy of the MDM.

2. Preliminaries & Definitions of Mahgoub Transform

2.1 Mahgoub transform

The Mahgoub transform is denoted by operator $\mathbb{M}$ (.) and Mahgoub transform of $\omega(t)$ is defined by the integral equation:

$$\mathbb{M}(\omega(t)) = (v) = v\int_0^\infty \omega(t)e^{-vt}dt, t \geq 0, \text{ and } \rho_1 \leq v \leq \rho_2. \quad (2.1)$$

In a set A the function is defined in the form

$$A = \left\{ \omega(t) : \exists \mathbb{M}, \rho_1, \rho_2 > 0. |\omega(t)| < \mathbb{M}e^{\frac{|t|}{\rho_j}} \right\}, \quad (2.2)$$

where ρ_1 and ρ_2 may be finite or infinite and the constant must be finite number. Mahgoub transform is defined for function of exponential order .

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NOTE: The lector can refer to more about the Mahgoubl transform in[6] .

2.2 Derivative of Mahgoub transform

Let function $\omega(t)$ be the derivative of $\omega(t)$ as for "t" further more n^{th} order derivative of the same as for "t", then Mahgoub transform of derivative is given by:

$$\mathbb{M}\left[\omega^{n}(t)\right]=v^{n}\mathbb{H}(v)-\sum_{k=0}^{n-1}\omega^{k}(0). \tag{2.3}$$

If we put $n=1,2,3\ldots$ in equation (2.3), then we get Mahgoub transform of first and second derivative of $\omega(t)$ with respect to "t":

$$\mathbb{M}\left[\omega'(t)\right]=v\,\mathbb{H}(v)-v\,\omega(0),$$
$$\mathbb{M}\left[\omega''(t)\right]=v^{2}\mathbb{H}(v)-v\omega'(0)-v^{2}\omega(0).$$

2.3 Adomian deterioration method

Adomian deterioration method is a semi analytical method for solving varied types of differential and integral equation, both linear and non-linear, and including partial differential equations. This method was developed from 1970s to 1990s by George Adomian[1-3]. The preferred standpoint of this method is that it diminishes the size of computation work and maintains the high accuracy of the analytical solution in terms of a rapidly convergence series [4]. In Adomian deterioration method, a solution can be decomposed into an infinite series that converges rapidly into the exact solution. The linear and non-linear portion of the equation can be separated by Adomian deterioration method. The inversion of linear operator can be represented by the linear operator if any given condition is taken into consideration. The deterioration of a series is obtained by non linear portion which is called Adomian polynomials. By the using Adomian polynomials we can find a solution in the form of a series which can be determined by the recursive relationship.

3. Investigation of Mahgoub Deterioration Method (MDM)

In this section we explain the Mahgoub deterioration method (MDM) for non linear non-homogeneous duo-combination of PDEs of the model:

$$\begin{aligned}&H_t u+H_{\epsilon}w+L_1(u,w)=p_1(\epsilon,t)\\&H_t w+H_{\epsilon}u+L_2(u,w)=p_2(\epsilon,t),\end{aligned} \tag{3.1}$$

with subject to initial conditions

$$\begin{aligned}&u(\epsilon,0)=r_1(\epsilon)\\&w(\epsilon,0)=r_2(\epsilon).\end{aligned} \tag{3.2}$$

where $p_1(\epsilon,t),p_2(\epsilon,t)$ are the non- homogeneous terms (source tem). H_t and H_{ϵ} are first differential operators. and $L_1(u,w)$ and $L_2(u,w)$ are the non linear operators we carry out the Mahgoub transform to Eq.(3.1) and Eq.(3.2) to get :

$$\begin{aligned}&\vartheta\,u(\epsilon,v)-u(\epsilon,0)+\mathbb{M}\left[w_{\epsilon}\right]\\&\qquad+\mathbb{M}\left[L_1(u,w)\right]=\mathbb{M}\left[p_1(\epsilon,t)\right]\\&\vartheta\,w(\epsilon,v)-w(,\epsilon,0)+\mathbb{M}\left[u_{\epsilon}\right]\\&\qquad+\mathbb{M}\left[L_2(u,w)\right]=\mathbb{M}\left[p_2(\epsilon,t)\right]\end{aligned} \tag{3.3}$$

By substituting the given initial condition in Eq.(3.2) in to Eq.(3.3), we obtain

$$\begin{aligned}&u(\epsilon,v)=\frac{1}{\vartheta}\,r_1(\epsilon)+\frac{1}{\vartheta}\mathbb{M}\left[p_1(\epsilon,t)\right]\\&\qquad-\frac{1}{\vartheta}\mathbb{M}\left[w_{\epsilon}+L_1(u,w)\right]\\&u(\epsilon,v)=\frac{1}{\vartheta}\,r_2(\epsilon)+\frac{1}{\vartheta}\mathbb{M}\left[p_2(\epsilon,t)\right]\\&\qquad-\frac{1}{\vartheta}\mathbb{M}\left[w_{\epsilon}+L_2(u,w)\right]\end{aligned} \tag{3.4}$$

taking the inverse Mahgoub transform of Eq.(3.4) we get:

$$\begin{aligned}&u(\epsilon,t)=P_1(\epsilon,t)-\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[w_{\epsilon}+L_1(u,w)\right]\right],\\&w(\epsilon,t)=P_2(\epsilon,t)-\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_{\epsilon}+L_2(u,w)\right]\right],\end{aligned} \tag{3.5}$$

where the terms $P_1(\epsilon,t)$ and $P_2(\epsilon,t)$ comes from the source terms.

We have function $u(\epsilon,t)$ and $w(\epsilon,t)$ which is unknown functions. For these functions, we adopt infinite series solution of the form:

$$\begin{aligned}&u(\epsilon,t)=\sum_{n=0}u_n(\epsilon,t)\\&w(\epsilon,t)=\sum_{n=0}w_n(\epsilon,t)\end{aligned} \tag{3.6}$$

Now , we can easily decompose the non linear terms $L_1(u,w)$ and $L_2(u,w)$ and can write as:

$$\begin{aligned}&L_1(u,w)=\sum_{n=0}D_n(\epsilon,t)\\&L_2(u,w)=\sum_{n=0}E_n(\epsilon,t)\end{aligned} \tag{3.7}$$

where D_n and E_n are Adomian polynomials which is given by :

$$A_n = \frac{1}{n!}\frac{d^n}{d\text{€}^n}\left[F\left(\sum_{i=0}^{n}\mu^i u_i\right)\right]_{\mu=0}$$
$$B_n = \frac{1}{n!}\frac{d^n}{d\text{€}^n}\left[F\left(\sum_{i=0}^{n}\mu^i w_i\right)\right]_{\mu=0} \tag{3.8}$$

where $n = 0,1,2,3,\ldots$
Using Eq. (3.8) and Eq.(3.7), we get :

$$\sum_{n=0} u_n(\text{€},\mathrm{t}) = P_1(\text{€},\mathrm{t}) - \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\sum_{n=0} w_{n\text{€}} + \sum_{n=0} D_n\right]\right]$$
$$\sum_{n=0} w_n(\text{€},\mathrm{t}) = P_2(\text{€},\mathrm{t}) - \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\sum_{n=0} u_{n\text{€}} + \sum_{n=0} E_n\right]\right] \tag{3.9}$$

Therefore, from Eq.(3.9). we find the recursive relation which is given by

$$u_0(\text{€},\mathrm{t}) = P_1(\text{€},\mathrm{t})$$
$$u_1(\text{€},\mathrm{t}) = -\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\sum_{n=0} w_{0\text{€}} + \sum_{n=0} D_0\right]\right]$$
$$u_2(\text{€},\mathrm{t}) = -\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\sum_{n=0} w_{1\text{€}} + \sum_{n=0} D_1\right]\right] \tag{3.10}$$

thus,

$$u_{n+1}(\text{€},\mathrm{t}) = -\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[w_{n\text{€}} + D_0\right]\right] \; n \geq 0. \tag{3.11}$$

Similarly,

$$w_0(\text{€},\mathrm{t}) = P_2(\text{€},\mathrm{t})$$
$$w_1(\text{€},\mathrm{t}) = -\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\sum_{n=0} u_{0\text{€}} + \sum_{n=0} E_0\right]\right]$$
$$w_2(\text{€},\mathrm{t}) = -\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\sum_{n=0} u_{1\text{€}} + \sum_{n=0} E_1\right]\right] \tag{3.12}$$

Similarly, we arrive at

$$w_{n+1}(\text{€},\mathrm{t}) = -\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_{n\text{€}} + E_0\right]\right] \quad n \geq 0.$$

The exact solutions of the non linear system are given by:

$$u(\text{€},\mathrm{t}) = \sum_{n=0} u_n(\text{€},\mathrm{t})$$

$$w(\text{€},\mathrm{t}) = \sum_{n=0} w_n(\text{€},\mathrm{t})$$

Thus, the Adomian deterioration method gives a convergent series solution which is absolute and uniformly convergent.

4. Relevance of Mahgoub Deterioration Method

In this section, the Mahgoub deterioration method (MDM) is applied for two coupled systems and is compared to our solutions with that of the existing exact solutions.

Example 4.1. Examine the duo-combination of nonlinear PDEs of the form:

$$u_{\mathrm{t}} - u_{\text{€€}} - 2uu_{\text{€}} + (uw)_{\text{€}} = 0$$
$$w_{\mathrm{t}} - w_{\text{€€}} - 2ww_{\text{€}} + (uw)_{\text{€}} = 0, \tag{4.1}$$

with subject to initial conditions,

$$u(\text{€},0) = sin\text{€}$$
$$w(\text{€},0) = sin\text{€} \tag{4.2}$$

By taking Mahgoub transform of derivatives on both hands of Eq.(4.1), we get

$$\vartheta u(\text{€},v) - u(\text{€},0) - \mathbb{M}[u_{\text{€€}}]$$
$$-2\,\mathbb{M}[uu_{\text{€}}] + \mathbb{M}[(uw)_{\text{€}}] = 0$$
$$\vartheta w(\text{€},v) - w(\text{€},0) - \mathbb{M}[w_{\text{€€}}]$$
$$-2\,\mathbb{M}[ww_{\text{€}}] + \mathbb{M}[(uw)_{\text{€}}] = 0. \tag{4.3}$$

now using initial conditions, we arrive

$$u(\text{€},\vartheta) = \frac{1}{\vartheta}\sin\text{€} + \frac{1}{\vartheta}\mathbb{M}\left[u_{\text{€€}} + 2uu_{\text{€}} - (uw)_{\text{€}}\right]$$
$$w(\text{€},\vartheta) = \frac{1}{\vartheta}\sin\text{€} + \frac{1}{\vartheta}\mathbb{M}\left[w_{\text{€€}} + 2ww_{\text{€}} - (uw)_{\text{€}}\right] \tag{4.4}$$

proceeds, the inverse Mahgoub transform of Eq.(4.4), we obtain

$$u(\text{€},\mathrm{t}) = \sin\text{€} + \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_{\text{€€}} + 2uu_{\text{€}} - (uw)_{\text{€}}\right]\right]$$
$$w(\text{€},\mathrm{t}) = \sin\text{€} + \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[w_{\text{€€}} + 2ww_{\text{€}} - (uw)_{\text{€}}\right]\right] \tag{4.5}$$

now, we assume a series solution for the unknown function $u(\text{€},\mathrm{t})$ and $w(\text{€},\mathrm{t})$ of the form:

$$u(\text{€},\mathrm{t}) = \sum_{n=0} u_n(\text{€},\mathrm{t})$$
$$w(\text{€},\mathrm{t}) = \sum_{n=0} w_n(\text{€},\mathrm{t}) \tag{4.6}$$

then Eq.(4.5) becomes

$$u(\epsilon,t)=\sin\epsilon +\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\sum_{n=0}u_{n\epsilon\epsilon}+2\,\epsilon A_n(u)-\left(\sum_{n=0}B_n(u,w)\right)_\epsilon\right]\right]$$
$$w(\epsilon,t)=\sin\epsilon +\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\sum_{n=0}w_{n\epsilon\epsilon}+2C_n(w)-\left(\sum_{n=0}B_n(u,w)\right)_\epsilon\right]\right], \tag{4.7}$$

where A_n, B_n and C_n and are Adomian polynomials. which characterizes the non linear terms $uu_\epsilon,(uw)_\epsilon$ and ww_ϵ relatively . Now, using aforementioned techniques, we reach the following recursive relation as:

$$\begin{aligned}u_0(\epsilon,t)&=\sin\epsilon\\ u_1(\epsilon,t)&=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_{0\epsilon\epsilon}+2A_0(u)-\left(B_0(u,w)\right)_\epsilon\right]\right]\\ u_2(\epsilon,t)&=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_{1\epsilon\epsilon}+2A_1(u)-\left(B_1(u,w)\right)_\epsilon\right]\right]\end{aligned} \tag{4.8}$$

thus,

$$u_{n+1}(\epsilon,t)=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_{n\epsilon\epsilon}+2A_n(u)-\left(B_n(u,w)\right)_\epsilon\right]\right],n\geq 0 \tag{4.9}$$

similarly,

$$\begin{aligned}w_0(\epsilon,t)&=\sin\epsilon\\ w_1(\epsilon,t)&=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[w_{0\epsilon\epsilon}+2C_0(w)-\left(B_0(u,w)\right)_\epsilon\right]\right]\\ w_2(\epsilon,t)&=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[w_{1\epsilon\epsilon}+2C_1(w)-\left(B_1(u,w)\right)_\epsilon\right]\right]\end{aligned} \tag{4.10}$$

Finally,

$$w_{n+1}(\epsilon,t)=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[w_{n\epsilon\epsilon}+2C_n(w)-\left(B_n(u,w)\right)_\epsilon\right]\right],n\geq 0\,. \tag{4.11}$$

Consequently, from the recurrence relation in Eq.(4.9) and Eq.(4.11), we can determine remaining units of the solution as go with:

$$\begin{aligned}u_1(\epsilon,t)&=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_{0\epsilon\epsilon}+2A_0(u)-\left(B_0(u,w)\right)_\epsilon\right]\right]\\ &=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_{0\epsilon\epsilon}+2u_0u_{0\epsilon}-\left(u_0w_0\right)_\epsilon\right]\right]\\ &=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[-sin\epsilon+2sin\epsilon cos\ \epsilon-2sin\epsilon cos\epsilon\right]\right]\\ &=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[-sin\epsilon\right]\right]\\ &=-sin\epsilon\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}[1]\right]\\ &=-t\,sin\epsilon\end{aligned} \tag{4.12}$$

$$\begin{aligned}w_1(\epsilon,t)&=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[w_{0\epsilon\epsilon}+2C_0(u)-\left(B_0(u,w)\right)_\epsilon\right]\right]\\ &=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[w_{0\epsilon\epsilon}+2w_0w_{0\epsilon}-\left(u_0w_0\right)_\epsilon\right]\right]\\ &=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[-sin\epsilon+2sin\epsilon cos\epsilon-2sin\epsilon cos\epsilon\right]\right]\\ &=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[-sin\epsilon\right]\right]\\ &=-sin\epsilon\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}[1]\right]\\ &=-t sin\epsilon\,.\end{aligned} \tag{4.13}$$

and

$$\begin{aligned}u_2(\epsilon\,,t)&=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_{1\epsilon\epsilon}+2A_1(u)-\left(B_1(u,w)\right)_\epsilon\right]\right]\\ &=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\begin{matrix}u_{1\epsilon\epsilon}+2\left(u_1u_{0\epsilon}+u_0u_{1\epsilon}\right)-\\ \left(u_1w_0+u_0w_{1\epsilon}+w_1u_{\epsilon0}+w_0u_{1\epsilon}\right)\end{matrix}\right]\right]\\ &=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[t\,sin\epsilon-4tsin\epsilon cos\epsilon+4tsin\epsilon cos\epsilon\right]\right]\\ &=sin\epsilon\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}[t]\right]\\ &=sin\epsilon\mathbb{M}^{-1}\left[\frac{1}{\vartheta^3}\right]\\ &=\frac{t^2}{2!}sin\epsilon.\end{aligned} \tag{4.14}$$

$$\begin{aligned}w_2(\epsilon,t)&=\mathbb{M}\left[\frac{1}{\vartheta}\mathbb{M}\left[w_{1\epsilon\epsilon}+2C_1(u)-\left(B_1(u,w)\right)_\epsilon\right]\right]\\ &=\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\begin{matrix}w_{1\epsilon\epsilon}+2\left(w_1w_{0\epsilon}+w_0w_{1\epsilon}\right)-\\ \left(u_1w_0+u_0w_{1\epsilon}+w_1u_{0\epsilon}+w_0u_{1\epsilon}\right)\end{matrix}\right]\right]\\ &=M^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[t\,sin\epsilon-4tsin\epsilon cos\epsilon+4tsin\epsilon cos\epsilon\right]\right]\\ &=sin\epsilon\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}[t]\right]\end{aligned}$$

$$= sin€\mathbb{M}^{-1}\left[\frac{1}{\vartheta^3}\right]$$
$$= \frac{t^2}{2!}sin€. \tag{4.15}$$

finally, the approximate solution of the known functions $u(€,t)$and $w(€,t)$ are given by:

$$\begin{aligned} w(€,t) &= \sum_{n=0} w_n(€,t) \\ &= w_0(€,t) + w_1(€,t) + w_2(€,t) + \dots \\ &= sin€ - tsin€ + \frac{t^2}{2!}sin€ + \dots \\ &= sin€\left(1 - t + \frac{t^2}{2!} + \dots\right) \\ &= e^{-t}\ sin€ \end{aligned} \tag{4.16}$$

and

$$\begin{aligned} w(€,t) &= \sum_{n=0} w_n(€,t) \\ &= w_0(€,t) + w_1(€,t) + w_2(€,t) + \dots \\ &= sin€ - tsin€ + \frac{t^2}{2!}sin€ + \dots \\ &= sin€\left(1 - t + \frac{t^2}{2!} + \dots\right) \\ &= e^{-t}\ sin€ \end{aligned} \tag{4.17}$$

thus, we get exact solution of the given non linear coupled system :

$$\begin{aligned} u(€,t) &= e^{-t}\ sin€ \\ w(€,t) &= e^{-t}\ sin€ \end{aligned} \tag{4.18}$$

Example 4.2. We think about the duo-combination of nonlinear PDE of the model:

$$\begin{aligned} & p_t + u_\eta \varnothing_\psi - u_\psi \varnothing_\eta = -p \\ & u_t + w_\eta p_\psi + p_\eta \varnothing_\psi = u \\ & \varnothing_t + p_\eta u_\psi + p_\psi u_\eta = \varnothing\ , \end{aligned} \tag{4.19}$$

subject to the initial conditions

$$\begin{aligned} p(\eta,\psi,0) &= e^{\eta+\psi} \\ u(\eta,\psi,0) &= e^{\eta-\psi} \\ \varnothing(\eta,\psi,0) &= e^{\psi-\eta}. \end{aligned} \tag{4.20}$$

Proceeds the Mahgoub transform derivatives both sides of Eq.(4.19). we get ,

$$\begin{aligned} & p(\eta,\psi,\vartheta) = \frac{1}{\vartheta}e^{\eta+\psi} + \frac{1}{\vartheta}\mathbb{M}\left[u_\psi \varnothing_\eta - u_\eta \varnothing_\psi - p\right] \\ & \vartheta u(\eta,\psi,v) - u(\eta,\psi,0) + \mathbb{M}\left[\varnothing_\eta p_\psi\right] + \mathbb{M}\left[p_\eta \varnothing_\psi\right] = \mathbb{M}[u] \\ & \vartheta\varnothing(\eta,\psi,v) - \varnothing(\eta,\psi,0) + \mathbb{M}\left[p_\eta u_\psi\right] + \mathbb{M}\left[p_\psi u_\eta\right] = \mathbb{M}[\varnothing] \end{aligned} \tag{4.21}$$

Then using the initial conditions of Eq.(4.20) into Eq.(4.21). We arrive to:

$$\begin{aligned} p(\eta,\psi,\vartheta) &= \frac{1}{\vartheta}e^{\eta+\psi} + \frac{1}{\vartheta}\mathbb{M}\left[u_\psi \varnothing_\eta - u_\eta \varnothing_\psi - p\right] \\ u(\eta,\psi,\vartheta) &= \frac{1}{\vartheta}e^{\eta-\psi} + \frac{1}{\vartheta}\mathbb{M}\left[u - \varnothing_\eta p_\psi - p_\eta \varnothing_\psi\right] \\ w(\eta,\psi,\vartheta) &= \frac{1}{\vartheta}e^{\psi-\eta} + \frac{1}{\vartheta}\mathbb{M}\left[\varnothing - p_\eta u_\psi - p_\psi u_\eta\right] \end{aligned} \tag{4.22}$$

by taking inverse Mahgoub transform of Eq. (4.22), we achieve

$$\begin{aligned} p(\eta,\psi,t) &= e^{\eta+\psi} + \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_\psi \varnothing_\eta - u_\eta \varnothing_\psi - p\right]\right] \\ u(\eta,\psi,t) &= e^{\eta-\psi} + \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u - \varnothing_\eta p_\psi - p_\eta \varnothing_\psi\right]\right] \\ \varnothing(\eta,\psi,t) &= e^{\psi-\eta} + \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\varnothing - p_\eta u_\psi - p_\psi u_\eta\right]\right] \end{aligned} \tag{4.23}$$

we have functions $p(\eta,\psi,t), u(\eta,\psi,t)$ and $\varnothing(\eta,\psi,t)$ which are unknown functions, for these functions, we adopt infinite series solutions of the form :

$$\begin{aligned} p(\eta,\psi,t) &= \sum_{n=0} p_n(\eta,\psi,t) \\ u(\eta,\psi,t) &= \sum_{n=0} u_n(\eta,\psi,t) \\ w(\eta,\psi,t) &= \sum_{n=0} \varnothing_n(\eta,\psi,t). \end{aligned} \tag{4.24}$$

From Eq. (4.23) can be re-written in the form:

$$\begin{aligned} p(\eta,\psi,t) &= e^{\eta+\psi} \\ &\quad + \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\sum_{n=0} A_n(u,\varnothing) - \sum_{n=0} B_n(u,\varnothing) - \sum_{n=0} p_n\right]\right] \\ u(\eta,\psi,t) &= e^{\eta-\psi} \\ &\quad + \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\sum_{n=0} u_n - \sum_{n=0} C_n(\varnothing,p) - \sum_{n=0} D_n(\varnothing,p)\right]\right] \\ \varnothing(\eta,\psi,t) &= e^{\psi-\eta} \\ &\quad + \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\sum_{n=0} \varnothing_n - \sum_{n=0} E_n(p,u) - \sum_{n=0} F_n(p,u)\right]\right] \end{aligned} \tag{4.25}$$

where A_n, B_n, C_n, D_n, E_n and F_n are Adomian polynomials. It is characterized by the nonlinear terms $u_{a\psi}\varnothing_\eta, u_\eta\varnothing_\psi$, $\varnothing_\eta p_\psi, p_\eta\varnothing_\psi, p_\eta u_\psi$ and $p_\psi u_\eta$ appropriately. Now we can obtain the recursive relation by Eq. (4.25) as follows:

$$\begin{aligned} p_1(\eta,\psi,\mathrm{t}) &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[A_0(u,\varnothing)-B_0(u,\varnothing)-p_0\right]\right] \\ p_2(\eta,\psi,\mathrm{t}) &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[A_1(u,\varnothing)-B_1(u,\varnothing)-p_1\right]\right] \end{aligned} \quad (4.26)$$

similarly, we can obtain $p_{n+1}(\eta,\psi,\mathrm{t})$. Which is given by,

$$p_{n+1}(\eta,\psi,\mathrm{t}) = \mathbb{M}^{-1}\left[\frac{1}{\theta}\mathbb{M}\left[A_n(u,\varnothing)-B_n(u,\varnothing)-p_n\right]\right] n \geq 0. \quad (4.27)$$

Again, we continue in the same manner for term $u_{n+1}(\eta,\psi,\mathrm{t})$ and $\varnothing_{n+1}(\eta,\psi,\mathrm{t})$ which can be obtained by Eq. easily. We will eventually have

$$u_{n+1}(\eta,\psi,\mathrm{t}) = \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_n - C_n(\varnothing,p) - D_n(\varnothing,p)\right]\right], n \geq 0. \quad (4.28)$$

$$\varnothing_{n+1}(\eta,\psi,\mathrm{t}) = \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\varnothing_n - E_n(p,u) - F_n(p,u)\right]\right], n \geq 0. \quad (4.29)$$

Hence using the Eq. (4.27), Eq.(4.28), and Eq. (4.29). we can obtain the remaining components of the functions $p(\eta,\psi,\mathrm{t}), u(\eta,\psi,\mathrm{t})$ and $\varnothing(\eta,\psi,\mathrm{t})$. Which are unknown functions :

$$\begin{aligned} p_1(\eta,\psi,\mathrm{t}) &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[A_0(u,\varnothing)-B_0(u,\varnothing)-p_0\right]\right] \\ &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_{0\psi}\varnothing_{0x} - u_{0x}\varnothing_{0\psi} - p_0\right]\right] \\ &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[e^{\eta-\psi}e^{\psi-\eta} - e^{\eta-\psi}e^{\psi-\eta} - e^{\eta+\psi}\right]\right] \\ &= -e^{\eta+\psi}\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}[1]\right] \\ &= -e^{\eta+\psi}\mathrm{t}. \end{aligned} \quad (4.30)$$

And

$$\begin{aligned} u_1(\eta,\psi,\mathrm{t}) &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_0 - C_0(\varnothing,p) - D_0(\varnothing,p)\right]\right] \\ &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_0 - \varnothing_{0x}p_{0\psi} - p_{0\eta}\varnothing_{0\psi}\right]\right] \\ &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[e^{\eta-\psi} + e^{\psi-\eta}e^{\eta+\psi} - e^{\eta+\psi}e^{\psi-\eta}\right]\right] \\ &= e^{\eta-\psi}\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}[1]\right] \\ &= \mathrm{t}e^{\eta-\psi}. \end{aligned} \quad (4.31)$$

$$\begin{aligned} \varnothing_1(\eta,\psi,\mathrm{t}) &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\varnothing_0 - E_0(p,u) - F_0(p,u)\right]\right] \\ &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\varnothing_0 - p_{0\eta}u_{0\psi} - p_{0\psi}u_{0\eta}\right]\right] \\ &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[e^{-\eta+\psi} + e^{\eta+\psi}e^{\eta-\psi} - e^{\eta+\psi}e^{\eta-\psi}\right]\right] \\ &= e^{-\eta+\psi}\mathbb{M}^{-1}\left[\frac{1}{\vartheta}M[1]\right] \\ &= \mathrm{t}e^{\psi-\eta}. \end{aligned} \quad (4.32)$$

And

$$\begin{aligned} p_2(\eta,\psi,\mathrm{t}) &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[A_1(u,\varnothing)-B_1(u,\varnothing)-p_1\right]\right] \\ &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\left(u_{1\psi}\varnothing_{0\eta} + u_{0\psi}\varnothing_{1\eta}\right) - \left(u_{1\eta}\varnothing_{0\eta} + u_{0\eta}\varnothing_{1\psi}\right) - p_1\right]\right] \\ &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\begin{array}{c}\left(\mathrm{t}e^{\eta-\psi}e^{-\eta+\psi} + e^{\eta-\psi}\mathrm{t}e^{-\eta+\psi}\right) - \\ \left(\mathrm{t}e^{\eta-\psi}e^{-\eta+\psi} + e^{\eta-\psi}\mathrm{t}e^{-\eta+\psi}\right) + te^{\eta+\psi}\end{array}\right]\right] \end{aligned}$$

$$
\begin{aligned}
&= e^{\eta+\psi}\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}[t]\right] \\
&= e^{\eta+\psi}\mathbb{M}^{-1}\left[\frac{1}{\vartheta^3}\right] \\
&= \frac{\mathrm{t}^2 e^{\eta+\psi}}{2!}.
\end{aligned}
\tag{4.33}
$$

$$
\begin{aligned}
u_2(\eta,\psi,\mathrm{t}) &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_1 - C_1(\varnothing,p) - D_1(\varnothing,p)\right]\right] \\
&= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[u_1 - \left(\varnothing_{1\eta}p_{0\psi} + \varnothing_{0\eta}p_{1\psi}\right) - \left(p_{1\eta}\varnothing_{0\psi} + p_{0\eta}\varnothing_{1\psi}\right)\right]\right] \\
&= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}M\left[\begin{array}{c}\mathrm{t}e^{\eta-\psi} - (\mathrm{t}e^{-\eta+\psi}e^{\eta+\psi} - e^{-\eta+\psi}\mathrm{t}e^{\eta+\psi}) - \\ \left(-\mathrm{t}e^{\eta+\psi}e^{-\eta+\psi} + e^{\eta+\psi}\mathrm{t}e^{-\eta+\psi}\right)\end{array}\right.\right] \\
&= e^{\eta-\psi}\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}[\mathrm{t}]\right] \\
&= e^{\eta-\psi}\mathbb{M}^{-1}\left[\frac{1}{\vartheta^3}\right] \\
&= \frac{\mathrm{t}^2 e^{\eta-\psi}}{2!}.
\end{aligned}
\tag{4.34}
$$

$$
\begin{aligned}
\varnothing_2(\eta,\psi,\mathrm{t}) &= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\varnothing_1 - E_1(p,u) - F_1(p,u)\right]\right] \\
&= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\varnothing_1 - \left(p_{1\eta}u_{0\psi} + p_{0\eta}u_{1\psi}\right) - \left(p_{1\psi}u_{0\eta} + p_{0\psi}u_{1\eta}\right)\right]\right] \\
&= \mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}\left[\begin{array}{c}te^{\psi-n} - \left(-te^{m+\psi}e^{\eta-\psi} - e^{\eta-\psi}\right) - \\ \left(-te^{\eta+\psi}e^{\eta-\psi} + e^{\eta+\psi}te^{\eta-\psi}\right)\end{array}\right]\right]. \\
&= e^{-\eta+\psi}\mathbb{M}^{-1}\left[\frac{1}{\vartheta}\mathbb{M}[\mathrm{t}]\right] \\
&= e^{\psi-\eta}\mathbb{M}^{-1}\left[\frac{1}{\vartheta^3}\right] \\
&= \frac{\mathrm{t}^2 e^{\psi-\eta}}{2!}.
\end{aligned}
\tag{4.35}
$$

$$
\begin{aligned}
p(\eta,\psi,\mathrm{t}) &= \sum_{n=0} p_n(\eta,\psi,\mathrm{t}) \\
&= p_0(\eta,\psi,\mathrm{t}) + p_1(\eta,\psi,\mathrm{t}) + p_2(\eta,\psi,\mathrm{t}) + \dots \\
&= e^{\eta+\psi} - \mathrm{t}e^{\eta+\psi} + \frac{\mathrm{t}^2 e^{\eta+\psi}}{2!} + \dots \\
&= e^{\eta+\psi}\left[1 - \mathrm{t} + \frac{\mathrm{t}^2}{2!} + \dots\right] \\
&= e^{\eta+\psi-\mathrm{t}}.
\end{aligned}
\tag{4.36}
$$

And

$$
\begin{aligned}
u(\eta,\psi,\mathrm{t}) &= \sum_{n=0} u_n(\eta,\psi,\mathrm{t}) \\
&= u_0(\eta,\psi,\mathrm{t}) + u_1(\eta,\psi,\mathrm{t}) + u_2(\eta,\psi,\mathrm{t}) + \dots \\
&= e^{\eta-\psi} + \mathrm{t}e^{\eta-\psi} + \frac{\mathrm{t}^2 e^{\eta-\psi}}{2!} + \dots \\
&= e^{\eta-\psi}\left[1 + \mathrm{t} + \frac{\mathrm{t}^2}{2!} + \dots\right] \\
&= e^{\eta-\psi+\mathrm{t}}.
\end{aligned}
\tag{4.37}
$$

Subsequently,

$$\begin{aligned}\varnothing(\eta,\psi,\mathrm{t}) &= \sum_{n=0}\varnothing_n(\eta,\psi,\mathrm{t})\\ &= \varnothing_0(\eta,\psi,\mathrm{t})+\varnothing_1(\eta,\psi,\mathrm{t})+\varnothing_2(\eta,\psi,\mathrm{t})+\ldots\\ &= e^{\psi-\eta}+\mathrm{t}e^{\psi-\eta}+\frac{\mathrm{t}^2e^{\psi-\eta}}{2!}+\ldots\\ &= e^{\psi-\eta}\left[1+\mathrm{t}+\frac{\mathrm{t}^2}{2!}+\ldots\right]\\ &= e^{\psi-\eta+\mathrm{t}}.\end{aligned} \tag{4.38}$$

On that account, the explicit solution of the unknown functions which is shown by:

$$\begin{aligned}p(\eta,\psi,\mathrm{t}) &= e^{\eta+\psi-\mathrm{t}}\\ u(\eta,\psi,\mathrm{t}) &= e^{\eta-\psi+\mathrm{t}}\\ \varnothing(\eta,\psi,\mathrm{t}) &= e^{\psi-\eta+\mathrm{t}}.\end{aligned} \tag{4.39}$$

Hence, we find the same result as obtained by Natural Deterioration Method (NDM) [5].

5. Conclusion

The Mahgoub deterioration method (MDM) is used for solving the combination of non linear duo partial differential equation with initial conditions. We found MDM is powerful and easy – to- use analytic tool for PDE's and thus, the present study highlights the efficiency of the method. Also, we get the exact solution when compared to the result with NDM [5]. This clearly shows that Mahgoub deterioration method can play an important role in future for solving nonlinear PDE's.

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