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An Optimum Inventory Policy for Exponentially Deteriorating Items, Considering Multi Variate Consumption Rate with Partial Backlogging

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Abstract Customer purchasing deeds may be affected by factors such as selling price and inventory level instead of demand which is considered either constant or function of single variable which is not feasible. Consequently in the present study we have considered demand rate as a function of stock-level and selling price both. In present study, in order to develop this model it has been assumed that items are exponentially decaying and shortages are partially backlogged and most realistic backlogging rate is considered. In this research, we proposed a partial backlogging inventory model for exponentially decaying items considering stock and selling price dependent demand rate in fuzzy environment. In developing the model demand rate, ordering cost, purchasing cost, holding cost, backordering cost and opportunity cost are considered as triangular fuzzy numbers. Graded mean integration representation method is used for defuzzification. A numerical example is provided to illustrate the problem. Sensitivity analysis of the optimal solution with respect to the changes in the value of system parameters is also discussed.

Keywords: Inventory model, triangular fuzzy numbers, signed distance, stock and price dependent Demand, exponential distribution.

1. INTRODUCTION

Several items such as food items, vegetables and pharmaceuticals decay with time when kept in inventory. Therefore it is important to study the inventory of the items which deteriorate with time. Several authors have presented the inventory models of decaying items. Ghare & Schrader (1963) initially considered the impact of deterioration for a constant demand. Later on Shah & Jaiswal (1977), Aggarwal (1978), Covert & Philips (1973) and Goyal & Giri (2001) developed the inventory models of deteriorating item. Thus it is necessary to consider backlogging rate. Researchers, such as **Park (1982)**, **Hollier** and **Mak** (**1983**) and **Wee** (**1995**) developed inventory models with partial backorders. **Goyal** and **Giri** (**2003**) developed production inventory model with shortages partially backlogged. **Wu** *et al*. (**2006**) developed a

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replenishment policy for deteriorating items with stock-dependent demand and partial backlogging. **Singh** (**2008**) presented a perishable inventory model with quadratic demand and partial backlogging. **Skouri** *et al*. (**2009**) presented an inventory model with ramp type demand rate, partial backlogging and Weibull deterioration rate.

The first model for decaying items was presented by **Ghare** and **Schrader** (**1963**). It was extended by **Covert** and **Philip** (**1973**) considering Weibull distribution deterioration. **Goswami** *et al*. (**1991**) developed an inventory model for deteriorating items with shortages and a linear trend in demand also a complete survey for deteriorating inventory models was presented by **Raafat** (**1991**). Some other papers relevant to this topic are **Teng**, *et al*., (**2002**), **Chang** *et al*.(**2009**) and **Abdul** and **Murata**, (**2011**).

Many classical research articles assumed that the demand is constant during the sales period. In real life, the demand may be inspired if there is a large pile of goods displayed on shelf. **Levin** *et al*. (**1972**) termed that the more goods displayed on shelf, the more customer's demand will be generated. **Gupta** and **Vrat** (**1986**) were the first to build up models for stock-dependent consumption rate. **Baker** and **Urban** (**1988**) established an EOQ model for a power-form inventory-level-dependent demand pattern. **Mandal** and **Phaujdar** **(1989)** then developed a production inventory model for deteriorating items with uniform rate of production and linearly stock-dependent demand. Other papers related to this research area are **Giri and Chaudhuri** (**1998**), **Chung** (**2003**), **Chang** (**2004**), **Alfares** (**2007**), **Goyal** and **Chang** (**2009**) and **Chang** *et al*. (**2010**).

In real life situations, due to impreciseness of parameters in inventory, it is important to consider them as fuzzy numbers. The concept of fuzzy set theory first introduced by **Zadeh L**. (**1965**), after that Park (**1987**) extended the classical EOQ model by introducing the fuzziness of ordering cost and holding cost. A fuzzy model for inventory with backorder, where the backorder quantity was fuzzified as the triangular fuzzy number was presented by Chang *et al.* (**1998**). Recently a supply chain inventory model under fuzzy demand was established by **Ruoning Xu, Xiaoyan Zhai** (**2010**).

Above cited papers reveals that many research articles are developed in which demand is considered as the function of stock level or selling price, shortages are allowed and partially backlogged, but there is no such research paper which is partially backlogged assuming demand rate as the function of selling price with exponentially decaying and inventory level in fuzzy environment. In lots of business practices it is observed that several parameters in inventory system are imprecise. Therefore, it is necessary to consider them as fuzzy numbers while developing the inventory model.

In the present study we have developed a partial backlogging inventory model for exponential deteriorating items considering stock and price sensitive demand rate in fuzzy surroundings. A numerical example to prove that the optimal solution exists and is unique is provided and the sensitivity analysis with respect to system parameters is discussed. The concavity is also shown through the figure.

II. ASSUMPTIONS AND NOTATIONS

The basic assumptions of the model are as follows:

(1) The demand rate is a function of stock and selling price considered as $f(t) = (a+bQ(t) - p)$ where $a > 0$, $0 < b < 1$, $a > b$ and p is selling price.

(2) Holding cost $h(t)$ per item per time-unit is time dependent and is assumed to be $h(t) = h + \delta t$ where $\delta > 0$, $h > 0$.

(3) Shortages are allowed and partially backlogged and rate is assumed to be $1/(1+\eta t)$ which is a decreasing function of time.

(4) The deterioration rate is time dependent.

(5) T is the length of the cycle.

(6) Replenishment is instantaneous and lead time is zero.

(7) The order quantity in one cycle is Q.

(8) The selling price per unit item is p.

(9) A is the cost of placing an order.

(10) c_1 is the purchasing cost per unit per unit.

(11) c_2 the backorder cost per unit per unit time.

(12) c_3 the opportunity cost (i.e., goodwill cost) per unit.

(13) $P(T,t_1,p)$ the total profit per unit time.

(14) The deterioration of units follows the exponential distribution (say)

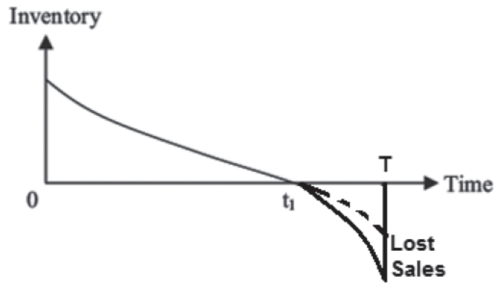
$$f(t) = e^{-\lambda(t-\gamma)}, \lambda > 0, t > \gamma$$

So $F(t) = 1 - e^{-\lambda(x-y)}$

Hence $\theta(t) = \dfrac{f(t)}{1-F(t)} = \lambda,$

Where F(t) is distribution function of exponential distribution.

(15) During time t1, inventory is depleted due to deterioration and demand of the item. At time t1 the inventory becomes zero and shortages start going on.



MATHEMATICAL FORMULATION AND SOLUTION

Let Q (t) be the inventory level at time t ($0 \leq t \leq T$). During the time interval [0, t_1] inventory level decreases due to the combined effect of demand and deterioration both and at t_1 inventory level depletes up to zero. The differential equation to describe immediate state over [0, t_1] is given by

$$Q(t) + \lambda Q(t) = -(a + bQ(t) - p \quad 0 \leq t \leq t_1 \tag{1}$$

Again, during time interval [t_1,T] shortages stars occurring and at T there are maximum shortages, due to partial backordering some sales are lost. The differential equation to describe instant state over [t_1,T] is given by

$$Q(t) = -\frac{(a-p)}{[1+\eta(T-t)]} \qquad t_1 \leq t \leq T \tag{2}$$

With condition

Solving equation (1) and equation (2) and neglecting higher powers of α, we get

$$Q(t) = (a-p)\left[t_1 - t + \frac{b}{2}(t_1{}^2 - t^2) + \frac{\lambda}{2}(t_1^2 - t^2)\right]e^{-bt-\lambda t} \quad 0 \leq t \leq t_1 \tag{3}$$

And

$$Q(t) = \frac{(a-p)}{\eta}\left[\log\{1+\eta(T-t)\} - \log\{1+\eta(T-t_1)\}\right] \quad t_1 \leq t \leq T \tag{4}$$

At time 0 inventory level is Q(0) and is given by

$$Q(0) = (a-p)\left(t_1 + \frac{bt_1^2}{2} + \frac{\lambda t_1^2}{2}\right)$$

At time T maximum shortages (Q1) occurs and is given by

$$Q_1 = \frac{(a-p)}{\eta}[\log\{1+\eta(T-t_1)\}]$$

The order quantity is Q and is given by

$$Q=(a-p)\left(t_1+\frac{bt_1^2}{2}+\frac{\lambda t_1^2}{2}+\frac{1}{\eta}\log(1-\eta)(T-t)\right)$$

The purchasing cost is

$$PC=c_1(a-p)\left(t_1+\frac{bt_1^2}{2}+\frac{\lambda t_1^2}{2}+\frac{1}{\eta}\log(1-\eta)(T-t)\right) \tag{5}$$

Ordering cost is

$$OC=A \tag{6}$$

Holding cost is

$$HC=\int_0^{t_1}(h+\delta t)Q(t)dt$$

$$=\int_0^{t_1}(h+\delta t)\left[(a-p)\left[t_1-t+\frac{b}{2}(t_1{}^2-t^2)+\frac{\lambda}{2}(t_1^2-t^2)\right]e^{-bt-\lambda t}\right]dt$$

$$=(a-p)\left[\frac{ht_1^2}{2}+\frac{\delta t_1^3}{6}+\frac{bht_1^3}{6}+\frac{b\delta t_1^4}{24}-\frac{b^2ht_1^4}{8}-\frac{b^2\delta t_1^5}{15}+\frac{\lambda ht_1^3}{3}-\frac{\lambda ht_1^3}{6}\right.$$
$$\left.+\frac{3\lambda\delta t_1^3}{8}-\frac{\lambda bht_1^4}{4}-\frac{\lambda\delta t_1^4}{3}+\frac{\lambda\delta bt_1^5}{30}-\frac{\lambda\delta bt_1^5}{6}-\frac{\lambda^2ht_1^4}{8}-\frac{\lambda^2\delta t_1^5}{15}\right] \tag{7}$$

Shortage cost due to backordered is

$$BC=c_2\int_{t_1}^{T}[-Q(t)]dt=c_2\int_{t_1}^{T}\frac{(a-p)}{\eta}[\log\{1+\eta(T-t)\}-\log\{1+\eta(T-t_1)\}]dt$$

$$=\frac{c_2(a-p)}{\eta^2}[\eta(T-t_1)-\log\{1+\eta(T-t_1)\}] \tag{8}$$

Lost sales cost due to lost sales is

$$LS=c_3(a-p)\int_{t_1}^{T}\left[1-\frac{1}{(1+\eta(T-t)}\right]$$

$$=\frac{c_3(a-p)}{\eta}[\eta(T-t_1)-\log\{1+\eta(T-t_1)\}] \tag{9}$$

Sales revenue is given by

$$SR=p\int_0^{t_1}(a+bQ(t)-p)dt+p\int_{t_1}^{T}(a-p)dt$$

$$=p(a-p)T+b(a-p)p\left(\frac{t_1{}^2}{2}+\frac{bt_1{}^3}{3}+\frac{\lambda t_1{}^3}{3}\right) \tag{10}$$

From (6), (7), (8), (9) and (10) total profit per unit time is given by

$$p(T,t_1,p)=\frac{1}{T}(SR-OC-PC-HC-BC-LS)$$

$$=\frac{1}{T}\left[\begin{array}{l} p(a-p)T+b(a-p)p\left(\dfrac{t_1^{\,2}}{2}+\dfrac{bt_1^{\,3}}{3}+\dfrac{\lambda t_1^{\,3}}{3}\right) \\ -A-c_1(a-p)\left(t_1+\dfrac{bt_1^{\,2}}{2}+\dfrac{\lambda t_1^{\,2}}{2}+\dfrac{1}{\eta}\log(1+\eta(T-t_1))\right) \\ -\dfrac{c_2(a-p)}{\eta^2}\{\eta(T-t_1)-\log(1+\eta(T-t_1))\}-\dfrac{c_3(a-p)}{\eta}\{\eta(t-t_1) \\ -\log(1+\eta(T-t_1))\} \\ -(a-p)\left\{\begin{array}{l} \dfrac{ht_1^{\,2}}{2}+\dfrac{\delta t_1^{\,3}}{6}+\dfrac{bht_1^{\,3}}{6}+\dfrac{b\delta t_1^{\,4}}{24}-\dfrac{b^2ht_1^{\,4}}{8}-\dfrac{b^2\delta t_1^{\,5}}{15} \\ +\dfrac{\lambda ht_1^{\,3}}{3}-\dfrac{\lambda ht_1^{\,3}}{6}+\dfrac{3\lambda\delta t_1^{\,4}}{8}-\dfrac{\lambda bht_1^{\,4}}{4}-\dfrac{\lambda\delta t_1^{\,4}}{3}+\dfrac{\lambda\delta bt_1^{\,5}}{30} \\ -\dfrac{\lambda\delta bt_1^{\,5}}{6}-\dfrac{\lambda^2ht_1^{\,4}}{8}-\dfrac{\lambda^2\delta t_1^{\,5}}{15} \end{array}\right\} \end{array}\right] \tag{11}$$

Let $t_1 = \gamma T$, $0 < \gamma < 1$

Hence we get the profit function

$$P(T,p)=\frac{1}{T}\left[\begin{array}{l} p(a-p)T+b(a-p)p\left(\dfrac{\gamma^2T^2}{2}+\dfrac{b\gamma' T^3}{3}+\dfrac{\lambda\gamma^3T^3}{3}\right)-A \\ -c_1(a-p)\left(\begin{array}{l}\gamma T+\dfrac{b\gamma^2T^2}{2}+\dfrac{\lambda\gamma^2T^2}{2} \\ +\dfrac{1}{\eta}\log(1+\eta(T-\gamma T))\end{array}\right) \\ -\dfrac{c_2(a-p)}{\eta^2}\{\eta(T-\gamma T)-\log(1+\eta(T-\gamma T))\} \\ -\dfrac{c_3(a-p)}{\eta}\{\eta(T-\gamma T)-\log(1+\eta(T-t_1))\} \\ -(a-p)\left\{\begin{array}{l} \dfrac{h\gamma^2T^2}{2}+\dfrac{\lambda\gamma^3T^3}{6}+\dfrac{bh\gamma^3T^3}{6}+\dfrac{b\delta\gamma^4T^4}{24}-\dfrac{b^2h\gamma^4T^4}{8}-\dfrac{b^2\delta\gamma^5T^5}{15} \\ +\dfrac{\lambda h\gamma^3T^3}{3}-\dfrac{\lambda h\gamma^3T^3}{6}+\dfrac{3\lambda\delta\gamma^4T^4}{8}-\dfrac{\lambda bh\gamma^4T^4}{4}-\dfrac{\lambda\delta\gamma^4T^4}{3} \\ +\dfrac{\lambda\delta b\gamma^5T^5}{30}-\dfrac{\lambda\delta b\gamma^5T^5}{6}-\dfrac{\lambda^2h\gamma^4T^4}{8}-\dfrac{\lambda^2\delta\gamma^5T^5}{15} \end{array}\right\} \end{array}\right] \tag{12}$$

Our objective is to maximize the profit function $P(T, p)$ .The necessary conditions for maximizing the profit are

$$\frac{\partial P(T,P)}{\partial T}=0 \text{ And } \frac{\partial P(T,P)}{\partial P}=0$$

We get

$$\frac{\partial P(T,p)}{\partial T}=\left[\begin{array}{l} b(a-p)p\left(\frac{\gamma^{2}}{2}+\frac{2b\gamma^{3}T^{3}}{3}+\frac{2\lambda\gamma^{3}T}{3}\right)+\frac{A}{T^{2}}+(a-p)\left(\frac{c_{2}}{\eta^{2}}+\frac{c_{3}}{\eta}\right) \\ \left\{\begin{array}{l} +\frac{\eta(1-\gamma)}{T(1+\eta(T-\gamma T))} \\ -\frac{1}{T^{2}}\log(1+\eta(T-\gamma T)) \end{array}\right\}-c_{1}(a-p)\left(\begin{array}{l} \frac{b\gamma^{2}}{2}+\frac{\lambda\gamma^{2}}{2}+\frac{(1-\gamma)}{T(1+\eta(T-\gamma T))} \\ -\frac{1}{\eta T^{2}}\log(1+\eta(T-\gamma T)) \end{array}\right) \\ -(a-p)\left\{\begin{array}{l} \frac{h\gamma^{2}}{2}+\frac{\delta\gamma^{3}T}{3}+\frac{bh\gamma^{3}T}{3}+\frac{b\delta\gamma^{4}T^{2}}{8}-\frac{3b^{2}h\gamma^{4}T^{2}}{8}-\frac{4b^{2}\delta\gamma^{5}T^{3}}{15} \\ +\frac{2\lambda h\gamma^{3}T}{3}-\frac{\lambda h\gamma^{3}T}{3}+\frac{9\lambda\delta\gamma^{4}T^{2}}{8}-\lambda\delta\gamma^{4}T^{2}-\frac{3\lambda bh\gamma^{4}T^{2}}{4} \\ +\frac{4\lambda\delta b\gamma^{5}T^{3}}{30}-\frac{4\lambda\delta b\gamma^{5}T^{3}}{6}-\frac{3\lambda^{2}h\gamma^{4}T^{2}}{8}-\frac{4\lambda^{2}\delta\gamma^{5}T^{3}}{15} \end{array}\right\} \end{array}\right]=0 \tag{13}$$

And

$$\frac{\partial P(T,p)}{\partial p}=\left[\begin{array}{l} (a-2p)+b(a-2p)\left(\frac{\gamma^{2}T}{2}+\frac{b\gamma^{3}T^{2}}{3}+\frac{\lambda\gamma^{3}T^{2}}{3}\right) \\ \frac{1}{T}\left\{\begin{array}{l} c_{1}\left(\gamma T+\frac{b\gamma^{2}T^{2}}{2}+\frac{\lambda\gamma^{2}T^{2}}{2}+\frac{1}{\eta}\log(1+\eta(T-\gamma T))\right) \\ +\left(\frac{c_{2}}{\eta^{2}}+\frac{c_{3}}{\eta}\right)\left\{\begin{array}{l} \eta(T-\gamma T)- \\ \log(1+\eta(T-\gamma T)) \end{array}\right\} \\ +\left\{\begin{array}{l} \frac{h\gamma^{2}T^{2}}{2}+\frac{\delta\gamma^{3}T^{3}}{6}+\frac{bh\gamma^{3}T^{3}}{6}+\frac{b\delta\gamma^{4}T^{4}}{24}-\frac{b^{2}h\gamma^{4}T^{4}}{8}-\frac{b^{2}\delta\gamma^{5}T^{5}}{15} \\ +\frac{\lambda h\gamma^{3}T^{3}}{3}-\frac{\lambda h\gamma^{3}T^{3}}{6}+\frac{3\lambda\delta\gamma^{4}T^{4}}{8}-\frac{\lambda bh\gamma^{4}T^{4}}{4}-\frac{\lambda\delta b\gamma^{4}T^{4}}{3} \\ +\frac{\lambda\delta b\gamma^{5}T^{5}}{3}-\frac{\lambda\delta b\gamma^{5}T^{5}}{6}-\frac{\lambda^{2}h\gamma^{4}T^{4}}{8}-\frac{\lambda^{2}\delta\gamma^{5}T^{5}}{15} \end{array}\right\} \end{array}\right\} \end{array}\right]=0 \tag{14}$$

Using the software Mathematica-8.1, from equation (13) and (14) we can determine the optimum values of T^* and p^* simultaneously and the optimal value $P^*(T, P)$ of the average net profit is determined by (12) provided they satisfy the sufficiency conditions for maximizing $P^*(T, p)$ are

$$\frac{\partial^2 P(T,p)}{\partial T^2} < 0, \frac{\partial^2 P(T,p)}{\partial p^2} < 0 \text{ and}$$

$$\frac{\partial^2 P(T,p)}{\partial T^2}\frac{\partial^2 P(T,p)}{\partial p^2} - \left(\frac{\partial^2 P(T,p)}{\partial T \partial p}\right)^2 > 0$$

III. NUMERICAL EXAMPLE

Let A = 250, a = 180, b = 0.015, c_1 = 20, c_2 = 5, c_3 = 25, λ = 0.124, γ = 0.5, h = 0.6, η = 0.5, δ = 0.04

Based on these input data, the findings are as follows:

p^*=102.259, t_1^*=0.96327, Q^*=159.338, T^*=1.92654 and P^*(T,p)=5943.42.

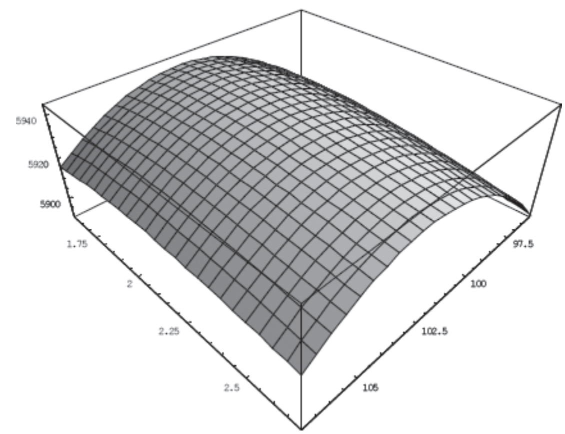


Figure 1: Concavity of the profit function.

IV. OBSERVATIONS

It is observe from the table that optimal replenishment quantity and total profit increases as the parameters **a**, **b** increases. As the parameter c_1 increases the order quantity increases but the total profit slightly decreases. The optimal replenishment quantity and total profit decreases as the parameters c_2, λ, η and h increases. The optimal order quantity and total profit very slightly decreases as the parameter δ increases.

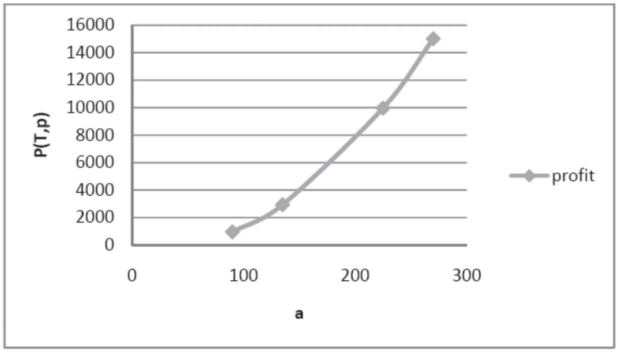


Figure 2: Net Profit v/s change in parameter a.

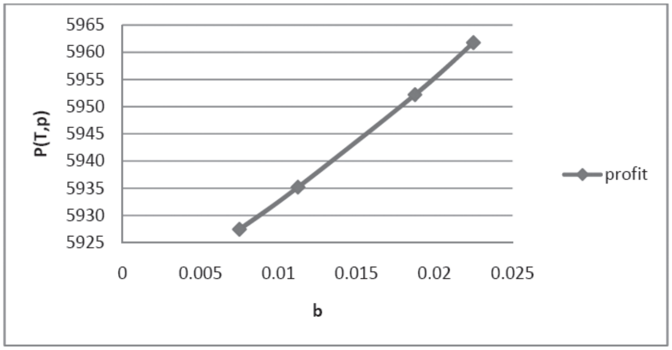


Figure 3: Net Profit v/s change in parameter b.

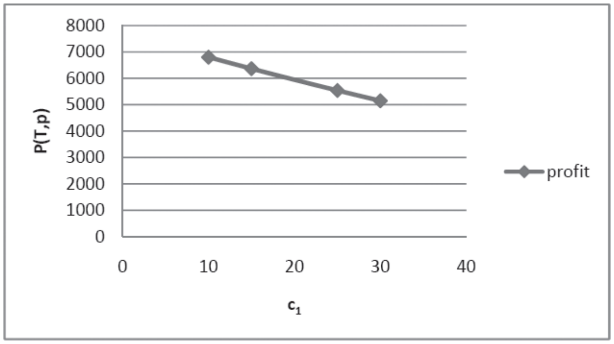


Figure 4: Net Profit v/s change in parameter c_1.

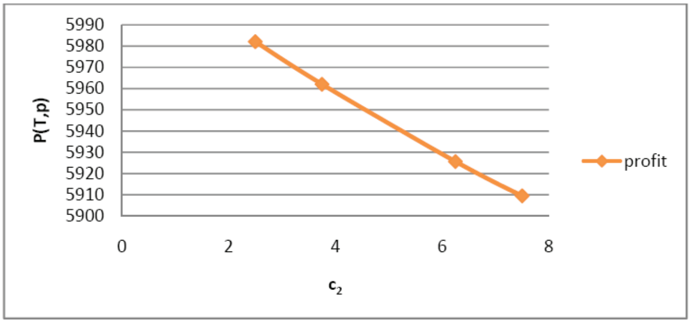


Figure 5: Net Profit v/s change in parameter c_2.

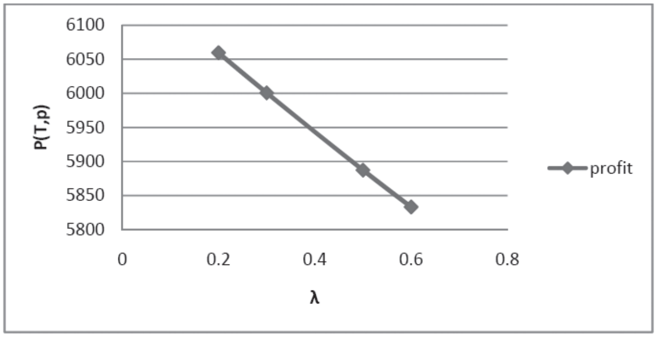


Figure 6: Net Profit v/s change in parameter λ.

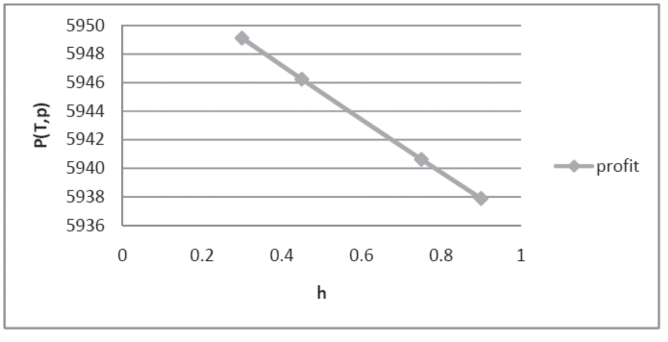


Figure 7: Net Profit v/s change in parameter h

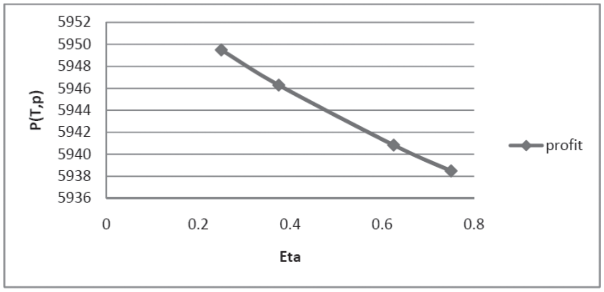


Figure 8: Net Profit v/s change in parameter η.

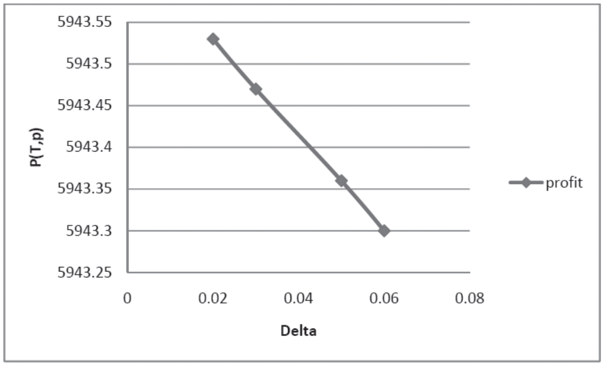


Figure 9: Net Profit v/s change in parameter δ.

V. FUZZY MATHEMATICAL MODEL

In this study we consider a, A, c_1, c_2, c_3, h and δ as fuzzy numbers i.e, as Then $\hat{a},\hat{A},\hat{c}_1,\hat{c}_2,\hat{c}_3,\hat{h}$ and $\hat{\delta}$ $P^*(T,P)$ is regarded as the estimate of total profit per unit time in the fuzzy sense.

$\hat{a}=(a-\Delta_1, a, a+\Delta_2)$ Where $0<\Delta_1<a$ and $\Delta_1\Delta_2>0$

$\hat{A}=(A-\Delta_3, A, A+\Delta_4)$ Where $0<\Delta_3<A$ and $\Delta_3\Delta_4>0$

$\hat{c}_1=(c_1-\Delta_5, c_1, c_1+\Delta_6)$ Where $0<\Delta_5<c_1$ and $\Delta_5\Delta_6>0$

$\hat{c}_2=(c_2-\Delta_7, c_2, c_2+\Delta_8)$ Where $0<\Delta_7<c_2$ and $\Delta_7\Delta_8>0$

$\hat{c}_3=(c_3-\Delta_9, c_3, c_3+\Delta_{10})$ Where $0<\Delta_9<c_3$ and $\Delta_9\Delta_{10}>0$

$\hat{h}=(h-\Delta_{11}, h, h+\Delta_{12})$ Where $0<\Delta_{11}<h$ and $\Delta_{11}\Delta_{12}>0$

$\hat{\delta}=(\delta-\Delta_{13}, \delta, \delta+\Delta_{14})$ Where $0<\Delta_{13}<\delta$ and $\Delta_{13}\Delta_{14}>0$

And the signed distance of $\hat{a}$ to $\hat{0}$ is given by the relation

$$\mathrm{d}(a,0) = a + \tfrac{1}{4}(\Delta_2 - \Delta_1) \text{ where } \mathrm{d}(a,0) > 0 \text{ and } \mathrm{d}(a,0) \in [\text{a-}\Delta_1, \text{a+}\Delta_2]$$

Similarly, the signed distance of other parameters to $\hat{0}$ is given by the relations

$$d(\hat{A},\hat{0}) = A + \tfrac{1}{4}(\Delta_4 - \Delta_3), \text{Where d } (\hat{A},\hat{0}) > 0 \text{ and } \mathrm{d}(\hat{A},\hat{0}) \in [\text{A-}\Delta_3, \text{A+}\Delta_4]$$

$$d(\hat{c}_1,\hat{0}) = c_1 + \tfrac{1}{4}(\Delta_6 - \Delta_5), \text{Where d } (\hat{c}_1,\hat{0}) > 0 \text{ and } \mathrm{d}(\hat{c}_1,\hat{0}) \in [\mathrm{c}_1\text{-}\Delta_5, \mathrm{c}_1\text{+}\Delta_6]$$

$$d(\hat{c}_2,\hat{0}) = c_2 + \tfrac{1}{4}(\Delta_8 - \Delta_7), \text{Where d } (\hat{c}_2,\hat{0}) > 0 \text{ and } \mathrm{d}(\hat{c}_2,\hat{0}) \in [\mathrm{c}_2\text{-}\Delta_7, \mathrm{c}_2\text{+}\Delta_8]$$

$$d(\hat{c}_3,\hat{0}) = c_3 + \tfrac{1}{4}(\Delta_{10} - \Delta_9), \text{Where d } (\hat{c}_3,\hat{0}) > 0 \text{ and } \mathrm{d}(\hat{c}_3,\hat{0}) \in [\mathrm{c}_3\text{-}\Delta_9, \mathrm{c}_3\text{+}\Delta_{10}]$$

$$d(\hat{h},\hat{0}) = h + \tfrac{1}{4}(\Delta_{12} - \Delta_{11}), \text{Where d } (\hat{h},\hat{0}) > 0 \text{ and } \mathrm{d}(\hat{h},\hat{0}) \in [\text{h-}\Delta_{11}, \text{h+}\Delta_{12}]$$

$$d(\hat{\delta},\hat{0}) = \delta + \tfrac{1}{4}(\Delta_{14} - \Delta_{13}), \text{Where d } (\hat{\delta},\hat{0}) > 0 \text{ and } \mathrm{d}(\hat{\delta},\hat{0}) \in [\text{h-}\Delta_{13}, \text{h+}\Delta_{14}]$$

Now, by the fuzzy triangular rule fuzzy total profit per unit is

$$\mathrm{FP}\ (\hat{a},\hat{A},\hat{c}_1,\hat{c}_2,\hat{c}_3,\hat{h},\hat{\delta}) = (\mathrm{F}_1,\mathrm{F}_2,\mathrm{F}_3)$$

And F_1, F_2, F_3 are obtained as

$$F_1 = \frac{1}{T}\left[\begin{array}{l} p(a-\Delta_1-p)T + b(a-\Delta_1-p)p\left(\dfrac{\gamma^2T^2}{2} + \dfrac{b\gamma^3T^3}{3} + \dfrac{\alpha\gamma^3T^3}{3}\right) - (A+\Delta_4) \\ -(c_1+\Delta_6)(a-\Delta_1-p)\left(\begin{array}{l}\gamma T + \dfrac{b\gamma^2T^2}{2} + \dfrac{\alpha\gamma^2T^2}{2} \\ +\dfrac{1}{\eta}\log(1+\eta(T-\gamma T))\end{array}\right) - \dfrac{(c_2+\Delta_8)(a-\Delta_1-p)}{\eta^2} \\ \{\eta(T-\gamma T) - \log(1+\eta(T-\gamma T))\} - \dfrac{(c_3+\Delta_{10})(a-\Delta_1-p)}{\eta}\{\eta(T-\gamma T) - \\ \log(1+\eta(T-\gamma T))\} \\ \left\{\begin{array}{l} \dfrac{(h-\Delta_{11})\gamma^2T^2}{2} + \dfrac{(\delta-\Delta_{13})\gamma^3T^3}{6} + \dfrac{b(h-\Delta_{11})\gamma^3T^3}{6} + \dfrac{b(\delta-\Delta_{13})\gamma^4T^4}{24} \\ -\dfrac{b^2(h-\Delta_{11})\gamma^4T^4}{8} - \dfrac{b^2(\delta-\Delta_{13})\gamma^5T^5}{15} + \dfrac{\lambda(h-\Delta_{11})\gamma^3T^3}{3} - \dfrac{\lambda(h-\Delta_{11})\gamma^3T^3}{6} \\ +\dfrac{3\lambda(\delta-\Delta_{13})\gamma^4T^4}{8} - \dfrac{\lambda b(h-\Delta_{11})\gamma^4T^4}{4} - \dfrac{\lambda\delta\gamma^4T^4}{3} + \dfrac{b\lambda(\delta-\Delta_{13})\gamma^5T^5}{30} \\ -\dfrac{\lambda(\delta-\Delta_{13})b\gamma^5T^5}{6} - \dfrac{\lambda^2(h-\Delta_{11})\gamma^4T^4}{8} - \dfrac{\lambda^2(\delta-\Delta_{13})\gamma^5T^5}{15} \end{array}\right\} \end{array}\right] \tag{15}$$

$$F_2=\frac{1}{T}\left[\begin{array}{l} p(a-p)T+b(a-p)p\left(\frac{\gamma^2T^2}{2}+\frac{b\gamma^3T^3}{3}+\frac{\lambda\gamma^3T^3}{3}\right)-A \\ -c_1(a-p)\left(\begin{array}{l}\gamma T+\frac{b\gamma^2T^2}{2}+\frac{\lambda\gamma^2T^2}{2}\\ +\frac{1}{\eta}\log(1+\eta(T-\gamma T))\end{array}\right)-\frac{c_2(a-p)}{\eta^2}\{\eta(T-\gamma T)-\log(1+\eta(T-\gamma T))\} \\ -\frac{c_3(a-p)}{\eta}\{\eta(T-\gamma T)-\log(1+\eta(T-\gamma T))\} \\ -(a-p)\left\{\begin{array}{l}\frac{h\gamma^2T^2}{2}+\frac{\delta\gamma^3T^3}{6}+\frac{bh\gamma^3T^3}{6}+\frac{b\delta\gamma^4T^4}{24}-\frac{b^2h\gamma^4T^4}{8}-\frac{b^2\delta\gamma^5T^5}{15}\\ +\frac{\lambda h\gamma^3T^3}{3}-\frac{\lambda h\gamma^3T^3}{6}+\frac{3\lambda\delta\gamma^4T^4}{8}-\frac{\lambda bh\gamma^4T^4}{4}-\frac{\lambda\delta\gamma^4T^4}{3}\\ +\frac{\lambda\delta b\gamma^5T^5}{30}-\frac{\lambda\delta b\gamma^5T^5}{6}-\frac{\lambda^2h\gamma^4T^4}{8}-\frac{\lambda^2\delta\gamma^5T^5}{15}\end{array}\right\}\end{array}\right] \tag{16}$$

$$F_3=\frac{1}{T}\left[\begin{array}{l} p(a+\Delta_2-p)T+b(a+\Delta_2-p)p\left(\frac{\gamma^2T^2}{2}+\frac{b\gamma^3T^3}{3}+\frac{\lambda\gamma^3T^3}{3}\right)-(A-\Delta_3) \\ -(c_1-\Delta_5(a+\Delta_2-p)\left(\begin{array}{l}\gamma T+\frac{b\gamma^2T^2}{2}+\frac{\lambda\gamma^2T^2}{2}\\ +\frac{1}{\eta}\log(1+\eta(T-\gamma T))\end{array}\right)-\frac{(c_2-\Delta_7)(a+\Delta_2-p)}{\eta^2} \\ \{\eta(T-\gamma T)-\log(1+\eta(T-\gamma T))\}-\frac{(c_3-\Delta_9(a+\Delta_2-p)}{\eta}\{\eta(T-\gamma T)-\log(1+\eta(T-\gamma T))\} \\ -(a-\Delta_1-p)\left\{\begin{array}{l}\frac{(h+\Delta_{12})\gamma^2T^2}{2}+\frac{(\delta+\Delta_{14})\gamma^3T^3}{6}+\frac{b(h+\Delta_{12})\gamma^3T^3}{6}+\frac{b(\delta+\Delta_{14})\gamma^4T^4}{24}\\ -\frac{b^2(h+\Delta_{12})\gamma^4T^4}{8}-\frac{\lambda^2(\delta+\Delta_{14})\gamma^5T^5}{15}+\frac{\lambda(h+\Delta_{12})\gamma^3T^3}{3}-\frac{\lambda(h+\Delta_{12})\gamma^3T^3}{6}\\ +\frac{3\lambda(\delta+\Delta_{14})\gamma^3T^3}{8}-\frac{\lambda b(h+\Delta_{12})\gamma^4T^4}{4}-\frac{\lambda(\delta+\Delta_{14}\gamma^4T^4}{3}+\frac{b\lambda(\delta+\Delta_{14})b\gamma^5T^5}{30}\\ -\frac{\lambda(\delta+\Delta_{14})b\gamma^5T^5}{6}-\frac{\lambda^2(h+\Delta_{12})\gamma^4T^4}{8}-\frac{\lambda^2(\delta+\Delta_{14})\gamma^5T^5}{15}\end{array}\right\}\end{array}\right] \tag{17}$$

Now, defuzzified average profit is given by

$$\tilde{P}(T,p)=\frac{F_1+2F_2+F_3}{4} \tag{18}$$

Also,the defuzzified order quantity is Q and is given by

$$Q=\left(a+\frac{(\Delta_2-\Delta_1)}{4}-p\right)\left(t_1+\frac{bt_1^{\ 2}}{2}+\frac{\lambda t_1^{\ 2}}{2}+\frac{1}{\eta}\log(1+\eta(T-t_1))\right)$$

The necessary conditions for maximizing the average profit are

$$\frac{\partial\tilde{P}(T,P)}{\partial T}=0\,\text{and}\,\frac{\partial\tilde{P}(T,P)}{\partial p}=0$$

Using the software Mathematica-8.1, from the above two equations we can determine the optimum values of $\tilde{T}$ and $\tilde{p}$ simultaneously and the optimal value $\tilde{p}$ (T, p) of the average net profit is determined by (18).

VI. NUMERICAL EXAMPLE

Let A = 250, Δ_3 = 12.5, Δ_4 = 25, a = 180, Δ_1 = 9, Δ_2 = 18, b = 0.015, c_1 = 20, Δ_5 = 1, Δ_6 = 2, c_2 = 5, Δ_7 = 0.25, Δ_8 = 0.50, c_3 = 25, Δ_9 = 1.25, Δ_{10} = 2.5, λ = 0.124, γ = 0.5, h = 0.6, Δ_{11} = 0.03, Δ_{12} = 0.06, η = 0.5, δ = 0.04, Δ_{13} = 0.002, Δ_{14} = 0.004.

Based on these input data, the findings are as follows:

pf^*=103.545, $t_{1f}^{\ *}$=0.970155, Qf^*=162.444, Tf^*=1.94031 and $\tilde{p}$ (T,p)=6106.36.

VIII. SENSITIVITY ANALYSIS

Table 2

Optimal solution for the model with fuzzy demand rate, ordering cost, holding cost, purchasing cost, shortage cost, opportunity cost , It is observed from the above table that there is a very small increment in profit $\tilde{p}$ (pf*, Tf*) as (Δ_{11}, Δ_{12}) and (Δ_{13}, Δ_{14}) varies. When (Δ_1, Δ_2), varies, the optimal profit $\tilde{p}$(pf*, Tf*) decreases. When (Δ_5, Δ_6), varies, the optimal profit $\tilde{p}$ (pf*, Tf*) increases. The optimal profit $\tilde{p}$ (pf*, Tf*) slightly increases when (Δ_3, Δ_4), (Δ_7, Δ_8) and (Δ_9, Δ_{10}) varies.

VIIII. CONCLUSION

In the current study an inventory model is presented in which demand rate is considered as a function of price and stock both and exponentially decaying. In the development of model it is assumed that shortages are allowed and partially backlogged. The model is proposed in the following two senses: (1) crisp sense and (2) fuzzy sense. A numerical example to illustrate the problem in both the environments is provided and sensitivity analysis with respect to system parameters is also carried out.

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