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Bènard-Marangoni Convection with Free Slip Bottom and Mixed Thermal Boundary Conditions

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Abstract The onset of cellular convection induced by surface tension gradients in a horizontal liquid layer heated from below is examined by making use of linear stability analysis for mixed thermal boundary conditions with free-slip condition at the lower boundary. We use a combination of analytical and numerical techniques to obtain a detailed description of marginal stability curves. It is established numerically that 'the principle of exchange of stabilities' is valid. The numerical results are presented for a wide range of the values of the parameters characterizing the nature of thermal boundary conditions. We investigate for the first time, a situation wherein value of the parameter characterizing the thermal condition at the upper boundary varies inversely to that characterizing the thermal condition at the lower boundary, and obtained distinct ranges in which increasing values of the parameter of the lower boundary lead to formation of convection cells of increasing or decreasing size at the onset of convection.

Keywords: Surface tension, convection, conducting, insulating, linear stability.

1. INTRODUCTION

The phenomenon of the onset of surface tension induced convection in a thin horizontal fluid layer heated from below with free upper surface was first established experimentally by Block (1956) and theoretically by Pearson (1958). They established that the patterned hexagonal cells observed by B`enard (1900, 1901) and explained by Rayleigh (1916) in terms of buoyancy, were in fact due to temperature dependent surface tension. Convection driven by surface tension gradients is now commonly known as B`enard-Marangoni convection (after an earlier observation by Italian physicist Carlo Giuseppe Matteo Marangoni (1840-1925)), in contrast to buoyancy driven Rayleigh-B`Marangoni convection has received a great deal of research activities because it has many applications in geophysics, oceanography, atmospheric sciences,

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chemical engineering of paints and detergents. Quantitative disagreement between experiment and theory has indicated that gravity was present in Bènard's experiments as well as in other experiments involving convection in a fluid layer with free surface in a laboratory on the earth, therefore, Nield (1964) considered the combined effects of surface tension and buoyancy (Bènard-Marangoni convection) on the onset of convection in a fluid layer heated from below with free upper surface and found that the two effects causing instability reinforce one another and that as the depth of the fluid layer decreases, the surface tension effects become more dominant. Further contributions made by many researchers, namely Scriven and Sternling (1964), Smith (1966), Davis (1969) and Takashima (1981a,b) have refined Pearson's model by incorporating more realistic conditions. For a detail study of Marangoni convection one may be referred to the work of Normand et al. (1977), Koschmieder (1993), and Schatz et al. (1995).

We consider the problem of the onset of cellular convection as considered by Pearson (1958), with no-slip condition replaced by the free-slip condition at the lower boundary and with mixed thermal boundary conditions on both the lower and upper boundaries. Although a free boundary at the bottom may seem artificial, it can be simulated by replacing the bottom plate by a layer of a much less viscous liquid (Goldstein and Graham, 1969). Further, the solution is also useful in explaining the structure of the solutions to the problem with other boundary conditions. The mixed thermal conditions are more realistic and have several physical justifications that arise from a more accurate description of heat transfer phenomenon in the environment surrounding the fluid (Sparrow et al., 1964; Nield, 1967; Proctor, 1981). The limiting cases of parameters describing the mixed thermal boundary conditions include various combination of boundary conditions as special cases, namely when both upper and lower boundary surfaces are either thermally conducting or insulating and either one of them is thermally conducting while the other one is thermally insulating. A Fourier series method is used to obtain the characteristic value equation. It is found, by solving the characteristic value problem numerically, that 'the principle of exchange of stabilities' is valid. The numerical results are obtained for a wide range of values of the parameters characterizing the nature of thermal boundary conditions. We investigate for the first time, a situation where in value of the parameter characterizing the thermal condition at the upper boundary varies inversely to that of characterizing the thermal condition at the lower boundary and obtained distinct ranges where the increasing values of the parameter of the lower boundary lead to formation of convection cells of increasing or decreasing size at the onset of convection.

2. THE PERTURBATION EQUATIONS AND BOUNDARY CONDITIONS

We consider an infinite horizontal layer of viscous fluid of uniform thickness d at rest, whose upper boundary surface is free, where surface tension gradients arise due to temperature perturbations. We choose a Cartesian coordinate system of axes with the x and y axes in the plane of the lower surface and the z axis along the vertically upward direction so that the fluid is confined between the planes at z =0 and $z = d$. A temperature gradient is maintained across the layer by maintaining the lower boundary at a constant temperature T_0 and the upper boundary at $T_1 < (T_0)$. It is assumed that surface tension is given by the simple linear law $\tau = \tau_1 - \sigma(T - T_1)$ where the constant τ_1 is the unperturbed value of τ at the unperturbed surface temperature $T = T_1$ and $-\sigma = \left(\frac{\partial \tau}{\partial T}\right)_{T=T_1}$ represents the rate of change of surface tension with temperature, evaluated at temperature T_1, and surface tension being a monotonically decreasing function of temperature, σ is positive.

The Boussinesq approximation enables one, to write the simplified governing equation of continuity, motion and heat conduction in the relevant context (neglecting buoyancy) as

$$\nabla.\mathbf{u} = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = -\frac{1}{\rho}\nabla P - gk + \nu\nabla^2\mathbf{u}, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u}.\nabla\right)T = k\nabla^2 T \tag{3}$$

where $\mathbf{u}$ =(u, v, w) is the velocity, P is the pressure and T is the temperature.

The gravitational acceleration represented by g, the gradient vector by ∇ and the unit vector in the z-direction by $\mathbf{k}$. The density ρ, the kinematic viscosity ν, the thermal diffusivity κ are each assumed to be constant. $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and t represents time.

The initial stationary state solution with temperature governed by conduction and pressure in hydrostatic balance is given as

$$\mathbf{u}_* = (0,0,0), T_* = T_0 - \beta z,\ P_* = P_0 - gp(z + \frac{1}{2}\alpha\beta z^2)$$

for all $0 \le z \le d$, where $\beta = \frac{(T_0 - T_1)}{d} > 0$ is the uniform temperature gradient and α is the coefficient of volume expansion of liquid. Let the initial steady state be slightly perturbed, and write

$$\mathbf{u} = (0+u, 0+v, 0+w),\ T = T_* + \theta,\ P = P_* + \delta P$$

where **u** =(u, v, w) now represent perturbation velocity, θ the perturbation temperature and δP the perturbation pressure. Following Chandrasekhar (1961) the linearised equations governing small perturbations are given by

$$\left(\frac{\partial}{\partial t} - v\nabla^2\right)\nabla^2 w = 0 \tag{4}$$

$$\left(\frac{\partial \theta}{\partial t} - \beta w\right) = k\nabla^2\theta \tag{5}$$

At the lower free surface z = 0, the free-slip conditions require that

$$w = \frac{\partial^2 w}{\partial z^2} = 0$$

Since the tangential viscous stress experienced by the liquid at the upper free surface is balanced by the tractions due variation with temperature of surface tension (Pearson, 1958), we have

$$\rho v \frac{\partial^2 w}{\partial z^2} = \sigma \nabla_1{}^2\theta$$

where use is being made of the continuity equation and $\nabla_1{}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. If the temperature at a boundary is kept constant, then θ = 0, while if the heat flux across the boundary is kept constant, then $\frac{\partial \theta}{\partial z} = 0$. We apply the mixed thermal boundary conditions of the form $\frac{\partial \theta}{\partial z} + L\theta = 0$, where the sign of the parameter L (Biot number) must be chosen to ensure that the perturbation heat transfer is out of the liquid layer. Thus, our boundary conditions at z = 0 are

$$w = 0, \quad \frac{\partial^2 w}{\partial z^2} = 0, \quad d\frac{\partial \theta}{\partial z} - L_0\theta = 0 \tag{6a,b,c}$$

and at $z = d$ are

$$w = 0, \quad \rho v \frac{\partial^2 w}{\partial z^2} - \sigma\nabla_1{}^2\theta = 0, \quad d\frac{\partial \theta}{\partial z} + L_1\theta = 0 \tag{7a,b,c}$$

where the parameters L_0 and L_1 (both positive) represent the Biot numbers referred to the lower and upper boundary respectively.

3. NORMAL MODE ANALYSIS

We now analyse an arbitrary disturbance in terms of normal modes, supposing that the perturbations w and θ have the forms

$$w(x, y, z) = W(z)\exp(\iota(a_x + a_y) + st)$$

$$\theta(x,y,z) = \Theta(z)\exp(\iota(a_x + a_y) + st)$$

where $a = \sqrt{a_x{}^2 + a_y{}^2}$ the horizontal wave number of the disturbance and s is a time constant (a complex number in general). We take d as unit of length and write $D = \frac{d}{dz}$ with z now expressed in terms of this new unit. We also let $W = \frac{wd}{v}$, $\Theta = \frac{k\theta}{\beta dv}$ and $p = \frac{sd^2}{v}$, then perturbation equations (4) and (5) and can be reduced to the following non-dimensional form

$$(D^2 - a^2)(D^2 - a^2 - p)W = 0 \tag{8}$$

$$(D^2 - a^2 - pP_r)\Theta = -W \tag{9}$$

where $M = \frac{\sigma\beta d^2}{\rho k v}$ is the Marangoni number and $P_r = \frac{v}{k}$ is the Prandtl number.
In terms of new variables and units the boundary conditions (6a, b, c) and (7a,b,c) are given by

$$W = 0, \quad D^2W = 0, \quad [D - L_0]\Theta = 0, \quad at\, z = 0 \qquad \text{(10a,b,c)}$$

$$W = 0, \quad D^2W + a^2M\Theta = 0, \quad [D + L_1]\Theta = 0, \quad at\, z = 1 \qquad \text{(11a,b,c)}$$

The differential equation (8)-(9) together with boundary conditions (10a, b, c) (11a, b, c) form an eigenvalue system of the six order.

4. SOLUTION OF THE PROBLEM

The Fourier series method presented by Nield (1964) is convenient for the present problem. The constants to be eliminated are denoted by

$$\lambda_1 = D^2W(0), \ \lambda_2 = D^2W(1), \ \lambda_3 = \Theta(0), \ \lambda_4 = \Theta(1)$$

We let

$$W = \sum_{n=1}^{\infty}\left[A_n - \frac{2}{n^3\pi^3}\{\lambda_1 - (-1)^n\lambda_2\}\right] sin n\pi z \tag{12}$$

where the boundary conditions (10a) and (11a) have already been used while writing (12), and

$$\Theta = \sum_{n=1}^{\infty}\left[B_n + \frac{2}{n\pi}\{\lambda_3 - (-1)^n\lambda_4\}\right] sin n\pi z \tag{13}$$

Then, we have

$$D^2W = \sum_{n=1}^{\infty}\left[A_n(-n^2\pi^2) + \frac{2}{n\pi}\{\lambda_1 - (-1)^n\lambda_2\}\right] sin n\pi z \tag{14}$$

$$D^4W=\sum_{n=1}^{\infty}A_n(n^4\pi^4)sinn\pi z \tag{15}$$

$$D^2\Theta=\sum_{n=1}^{\infty}B_n(-n^2\pi^2)sinn\pi z \tag{16}$$

The differential equations (8) and (9) are satisfied by substituting the complete Fourier expansions for W, Θ and their derivatives of even order (12)-(16) and equating the coefficients of $sinn\pi z$, we obtain

$$(n^2\pi^2+a^2)(n^2\pi^2+a^2+p)A_n=\frac{2a^2(2n^2\pi^2+a^2)+2p(n^2\pi^2+a^2)}{n^3\pi^3}\times[\lambda_1-(-1)^n\lambda_2] \tag{17}$$

$$A_n-(n^2\pi^2+a^2+pP_r)B_n=\frac{2}{n^3\pi^3}[\lambda_1-(-1)^n\lambda_2]+\frac{2(a^2+pP_r)}{n\pi}[\lambda_3-(-1)^n\lambda_4] \tag{18}$$

The remaining boundary conditions require that

$$\lambda_1=0 \tag{19}$$

$$\sum_{n=1}^{\infty}n\pi B_n-(1+L_0)\lambda_3+\lambda_4=0 \tag{20}$$

$$\lambda_2+a^2M\lambda_4=0 \tag{21}$$

$$\sum_{n=1}^{\infty}(-1)^n n\pi B_n-\lambda_3+(1+L_1)\lambda_4=0 \tag{22}$$

From equations (17)-(18), A_n and B_n can be expressed in terms of $\lambda_1,\lambda_2,\lambda_3,\lambda_4$. Substitution in (19), (20) and (22) and using (21) then yields three homogeneous equations in λ_1,λ_2 and λ_3. Elimination of these constants gives the required eigenvalue equation:

$$\begin{vmatrix} 1 & 0 & 0 \\ \sum_{n=1}^{\infty}\frac{(-1)^n En}{H_n} & \sum_{n=1}^{\infty}\frac{En}{H_n}-\frac{1}{a^2M}\left(\sum_{n=1}^{\infty}\frac{G_n}{H_n}+\frac{L_1+1}{2}\right) & \sum_{n=1}^{\infty}\frac{(-1)^n G_n}{H_n}+\frac{1}{2} \\ \sum_{n=1}^{\infty}\frac{E_n}{H_n} & \sum_{n=1}^{\infty}\frac{(-1)^n E_n}{H_n}-\frac{1}{a^2M}\left(\sum_{n=1}^{\infty}\frac{(-1)^n}{H_n}+\frac{1}{2}\right) & \sum_{n=1}^{\infty}\frac{G_n}{H_n}+\frac{L_0+1}{2} \end{vmatrix}=0 \tag{23}$$

where
$$\left.\begin{aligned} E_n &= n^2\pi^2, \\ F_n &= E_n(E_n + a^2 + pP_r) \\ G_n &= (E_n + a^2)(E_n + a^2 + p)(a^2 + pP_r) \\ H_n &= (E_n + a^2)(E_n + a^2 + p)(E_n + a^2 + pP_r) \end{aligned}\right\} \tag{24}$$

From equation (23), we obtain M in terms of a, P_r, L_0, L_1 and p as the ratio of two determinants given by

$$M(a,P_r,L_0,L_1,p) = \frac{1}{a^2}\frac{\begin{vmatrix} \sum_{n=1}^{\infty}\frac{G_n}{H_n} + \frac{L_1+1}{2} & \sum_{n=1}^{\infty}\frac{(-1)^n G_n}{H_n} + \frac{1}{2} \\ \sum_{n=1}^{\infty}\frac{(-1)^n G_n}{H_n} + \frac{1}{2} & \sum_{n=1}^{\infty}\frac{G_n}{H_n} + \frac{L_0+1}{2} \end{vmatrix}}{\begin{vmatrix} \sum_{n=1}^{\infty}\frac{E_n}{H_n} & \sum_{n=1}^{\infty}\frac{(-1)^n G_n}{H_n} + \frac{1}{2} \\ \sum_{n=1}^{\infty}\frac{(-1)^n E_n}{H_n} & \sum_{n=1}^{\infty}\frac{G_n}{H_n} + \frac{L_0+1}{2} \end{vmatrix}} \tag{25}$$

4.1. Characterization of the Marginal State

It is difficult to prove analytically 'the principle of exchange of stabilities' owing to the peculiar nature of the boundary conditions for the present problem, therefore, it is desirable to settle the question numerically. For a non-stationary neutral state we put $p = \iota pi$ with p_i real in (25) which after separating into the real and imaginary parts may be rewritten in the form

$$M = M_r(a,P_r,L_0,L_1,p_i) + \iota M_i(a,P_r,L_0,L_1,p_i) \tag{26}$$

where M_r and M_i are real valued functions of the parameters in the parentheses.

Since M must clearly be real, this requires that

$$M_i = 0 \tag{27}$$

The function Mi was computed numerically in detail for the ranges $0 \le a \le 20$, $0 \le P_r \le 10^3$, $0 \le L_0 \le 10^{10}$, $0 \le L_1 \le 10^{10}$, $0 \le p_i \le 10^4$; (Note that these ranges of parameters cover usual laboratory conditions). In all cases it was found that the only real value of pi that satisfied (27) was $p_i = 0$, thus indicating that the marginal state is indeed stationary and the possibility of over stability has been excluded.

When the onset of instability sets in as stationary convection, the margional state is obtained by setting $p = 0$ in equation (25) with

$$\left.\begin{aligned} E_n &= n^2\pi^2 \\ F_n &= E_n(E_n + a^2) \\ G_n &= a^2(E_n + a^2)^2 \\ H_n &= (E_n + a^2)^3 \end{aligned}\right\} \tag{28}$$

Since each series in the eigenvalue equation (25) with relation (28) can be summed in terms of hyperbolic functions which after simplification gives

$$M(a, L_0, L_1) = \frac{8aS_a^{\,2}[a(aS_a + (L_0 + L_1)C_a) + L_0L_1S_a]}{aS_a(S_a^{\,2} - a^2) + L_0(C_a^{\,3} + aS_a - (2a^2 + 1)C_a)} \tag{29}$$

For fixed value of each pair (L_0,L_1), equation (29) gives M as a function of the wave number a. The minimum of M as a varies is the critical Marangoni number M_c and the value of wave number at which M attains the minimum is the critical wave number a_c.

Remark 1: When $L_0 \to \infty$ in this case, we find from equation (29) that

$$M = \frac{8aS_a^{\,2}(aC_a + L_1S_a)}{C_a^{\,3} + aS_a - (2a^2 + 1)C_a} \tag{30}$$

and this expression for the Marangoni number is identical with that obtained by Boeck and Thess (1997) at for the conducting case of the lower boundary (when the changes in the notation are allowed).

Remark 2: When $L_0 \to 0$ in this case, we find from equation (29) that

$$M = \frac{8aS_a(aS_a + L_1C_a)}{S_a^{\,2} - a^2} \tag{31}$$

and this expression for the Marangoni number is identical with that obtained by Gupta and Surya (2012) for the insulating case of the lower boundary.

5 NUMERICAL RESULTS AND DISCUSSION

A symbolic algebra package is used to compute numerically the minimum value of the Marangoni number M with respect to wave number a for given values of L_0 and L_1 using relation (29). The numerical values of M_c and a_c for various values of L_0 and L_1 are presented in Table 1.

In figure 1(a), the neutral stability curves are plotted (using relation (29)) for various values of L_0 when L_1 = 0 and L_1 = 5. For fixed value of each pair (L_0, L_1), the lowest point on each curve gives the critical Marangoni number M_c and the corresponding critical wave number a_c. The region below each

Table 1: Values of M_c and a_c for various values of L_0 when $L_1 = 0$, 1, 5 and 10^{10}

	$L_1 = 0$		$L_1 = 1$		$L_1 = 5$		$L_1 = 10^{10}$	
L_0	M_c	a_c	M_c	a_c	M_c	a_c	M_c	a_c
0	24	0.008	61.633	1.373	162.923	1.733	23.800×10^{10}	2.098
10^{-3}	24.68	0.265	61.650	1.374	162.943	1.773	23.802×10^{10}	2.099
10^{-2}	26.150	0.470	61.806	1.378	163.127	1.776	23.817×10^{10}	2.101
10^{-1}	30.701	0.820	63.242	1.417	164.852	1.810	23.959×10^{10}	2.121
1	42.122	1.311	71.521	1.619	175.780	1.940	24.892×10^{10}	2.248
2	46.503	1.444	75.769	1.707	181.896	2.011	25.435×10^{10}	2.316
5	50.398	1.546	79.833	1.783	188.002	2.076	26.152×10^{10}	2.399
10	54.050	1.630	83.832	1.850	194.223	2.137	26.567×10^{10}	2.444
10^2	57.183	1.693	87.380	1.904	199.909	2.186	27.108×10^{10}	2.498
10^3	57.556	1.700	87.810	1.910	200.608	2.192	27.176×10^{10}	2.505
10^{10}	57.598	1.700	87.858	1.910	200.687	2.193	27.183×10^{10}	2.505

curve represents the stable state. From figure 1(a), we observed that the neutral stability curves move upwards for increasing values of L_0 (or L_1), for a fixed value of L_1 (or L_0), clearly showing the stabilizing effect of both L_0 and L_1. Figure 1(b) illustrates the variation of the critical wave numbers a_c at the marginal stability. It clearly shows that for a given value of L_1 value of a_c increases for increasing values of L_0. Further, it is interesting to note that $a_c \to 0$ when both L_0 and L_1 tends to zero.

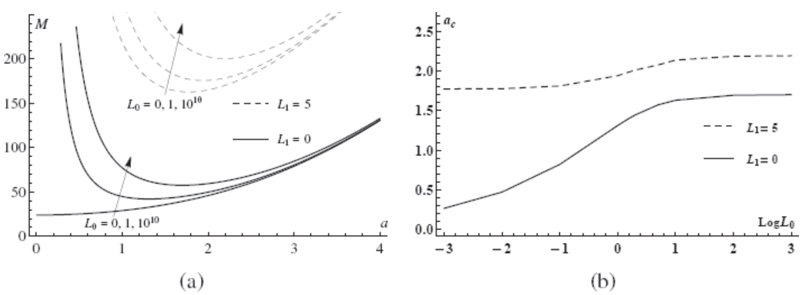

Figure 1: (a) Neutral stability curves for various L_0 when $L_1 = 0$ and 5. (b) Variation of a_c as a function of L_0 when $L_1 = 0$ and 5.

Case 1: When $L_0 = L_1 = L$ (say), we find from equation (29) that

$$M = \frac{8aS_a^{\,2}[a(aS_a + 2LCa) + L^2 S_a]}{a(aS_a(S_a^{\,2} - a^2) + L(C_a^{\,3} + aS_a - (2a^2 + 1)C_a} \tag{32}$$

The above expression for the Marangoni number is obtained for the case when both the lower and upper boundary surfaces are formally identical with regard to thermal boundary conditions. Values of M_c and corresponding a_c computed with the aid of (32) are listed in Table 2 for various fixed values of L.

Table 2: Values of M_c and a_c for various values of L

L	M_c	a_c
10^{-10}	24.008	0.006
10^{-3}	24.975	0.316
10^{-2}	27.186	0.558
10^{-1}	35.154	0.976
1	71.521	1.619
10	328.606	2.248
10^{2}	27.791×10^{2}	2.471
10^{3}	27.244×10^{3}	2.502
10^{10}	27.183×10^{10}	2.505

In the limiting case, when $L \to 0$, that is, when both lower and upper boundaries are insulating, value of $M_c \to 24$ and corresponding value of $a_c \to 0$ obtained by us agree precisely with those obtained by Gupta and Surya (2012) for the insulating case of the lower boundary. As $L \to \infty$, that is when both lower and upper boundaries are conducting, values of M_c become asymptotically proportional to L, however, values of corresponding wave number a_c remain finite (≈ 2.505). An increase in L from 0 to ∞ means a change in the thermal boundary condition from $D\Theta = 0$ (thermally insulating) to $\Theta = 0$ (thermally conducting). Therefore, when L is small it is easier for temperature perturbations to be set up, but when L is large any temperature perturbation decay rapidly. Hence, as L becomes large, the values of M_c tend to ∞ since it becomes difficult for the surface tension to be operative.

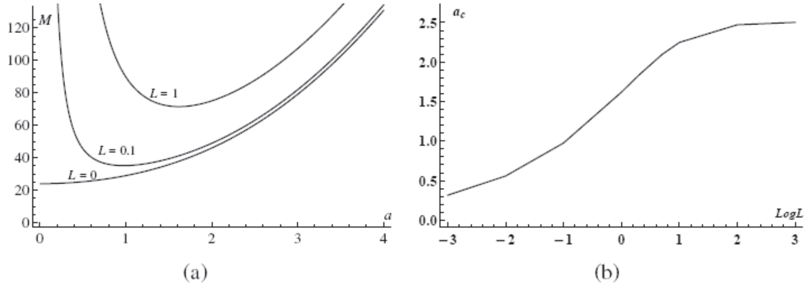

Figure 2: (a) Neutral stability curves for various values of $L_0 = L_1 = L$ (b) Variation of a_c as a function of L

In figure 2(a), the neutral stability curves are plotted (using (32)) for various values of L. From figure 2(a), we observe that the neutral stability curves move upwards for increasing values of L, clearly showing the stabilizing effect of L.

Figure 2(b) illustrates the variation of the critical wave number a_c at the marginal stability. It clearly shows that value of a_c increases for increasing values of L. In this case, we note that $a_c \to 0$ when $L = 0$.

Case 2: When $L_0 = L$ and $L_1 = L^{-1}$, we find from equation (29) that

$$M = \frac{8aS_a^{\,2}[a(aS_a + (L + L^{-1})Ca) + S_a]}{aS_a(S_a^{\,2} - a^2) + L(C_a^{\,3} + aS_a - (2a^2 + 1)C_a)} \tag{33}$$

The above expression for the Marangoni number is obtained for the case wherein value of the parameter L_1 characterizing the thermal nature of the upper boundary varies inversely to L_0 characterizing the thermal condition at the lower boundary. Values of M_c and corresponding a_c computed with the aid of (33) are listed in Table 3 for various values of L.

In the limiting case, when $L \to \infty$ ($L^{-1} \to 0$), that is, when the lower boundary is conducting and the upper one is insulating, value of $M_c = 57.598$ and $a_c = 1.7$ obtained by us agree precisely with those obtained by Boeck and Thess (1997) for the corresponding conducting case of the lower boundary. In the limiting case, when $L \to 0$ ($L^{-1} \to \infty$), that is, when the lower boundary is insulating and the upper one is conducting, values of M_c becomes asymptotically proportional to L and the corresponding a_c remains finite (≈ 2.098.)

In figure 3(a), the neutral stability curves are plotted (using relation (33)) for various values of L. From figure 3(a), we observe that the neutral stability curves move downwards for increasing values of L, clearly showing the destabilizing effect of L.

Table 3: Values of M_c and a_c for various values of L

L	M_c	a_c
10^{-10}	23.800×10^{10}	2.0983
10^{-3}	23.850×10^{3}	2.0958
10^{-2}	2429.94	2.0746
10^{-1}	286.414	1.9225
1	71.521	1.6186
1.72	63.004	1.6045
1.73	62.130	1.6046
2	61.565	1.6057
10^{2}	57.164	1.6614
10^{2}	57.499	1.6957
10^{3}	57.588	1.6999
10^{10}	57.598	1.7004

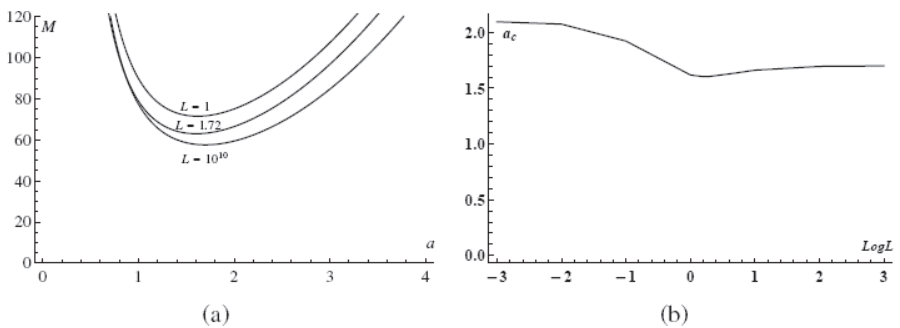

Figure 3: (a) Neutral stability curves for various values of $L_0 = L$ and $L_1 = L^{-1}$ (b) Variation of a_c as a function of L

Figure 3(b) illustrates the variation of the corresponding critical wave numbers at the marginal stability. It is interesting to note that a change from decreasing values to increasing values of a_c occurs for certain ranges of the values of the parameter L. The critical wave number a_c decreases in the range of values of L from 0 to 1.72 where as value of a_c increases for increasing values L which are more than 1.72.

6. CONCLUSIONS

The linear stability analysis of the Bènard-Marangoni Convection in a liquid layer with free slip bottom and mixed thermal boundary conditions has been studied theoretically. The following conclusions are drawn from the present study:

1. The validity of 'the principle of exchange of stabilities' is established numerically.
2. The threshold values, that is, the critical Marangoni number M_c and corresponding critical wave number a_c have been obtained for a wide variety of variations of parameters L_0 and L_1 characterizing respectively the thermal conditions of the lower and upper boundary.
3. It is established that when the thermal boundary conditions are identically same, that is, when $L_0 = L_1 = L$, increasing values of the parameter L have stabilizing effect on the onset of convection.
4. It is interesting to note that when the parameter characterizing the thermal condition of the upper boundary varies inversely to that of characterizing the thermal condition of the lower boundary, that is, when $L_0 = L$ and $L_1 = L^{-1}$, increasing values of L in the range from 0 to 1.72, values of ac decrease from 2.0983 to 1.6045 whereas for increasing values of L in the range from 1.73 to infinity, values of a_c increase from 1.6046 to 1.7004.
5. We also conclude that when the upper boundary is thermally conducting, the surface tension forces become inoperative, therefore, value of the critical Marangoni number tends to infinity for this case.

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