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Bayes estimation of change point in the count data model: a Particular case of Discrete Burr Type III Distribution

MAYURI PANDYA AND SMITA PANDYA

Department of Statistics, Maharaja Krishnkumarsinhiji Bhavnagar University
University campus, Near Gymkhana Bhavnagar-364002, India

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Abstract: A sequence of independent count data $X_1, X_{2........} X_m, X_{m+1}, \ldots.., X_n$ where observations from a particular case of discrete Burr family type III distribution with distribution function $F_1(t)$ at time t later it was found that there was change in the system at some point of time m and it is a reflected in the sequence X_m by change in distribution function $F_2(t)$ at time t. The Bayes estimates of change point and parameters of Particular case of Bur Type III Distribution are derived under Linex and General Entropy loss functions.

Keywords: Bayes estimate, Change point, discrete Burr type III distribution.

1. Introduction

A survey of the literature concerning the Burr family of distributions is given in Burr [1] and Fry [2]. Nair, and Asha [3] and sreehari M. [4] attempted to obtain discrete analogues of Burr's family. We consider Discrete Burr Type III distribution with change point

A countdata model based on discrete burr type III distribution is specified to represent the distribution of a count data set under dispersion and using this model statistical inferences are made. Countdata systems are often subject to random changes. It may happen that at some point of time instability in the sequence of count data is observed. The problem of study to estimate the time when this change has started occurring This is called change point inference problem. Bayesian ideas has been often proposed as veiled alternative to classical estimation procedure in the study of such change point problem. The monograph of Broemeling and Tsurumi [5] on structural change, Jani P.N. and Pandya M.[6], Pandya M. .[7] , Pandya, M. Pandya, S and Andharia, P. [8] are

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useful reference. In this paper, we develop probability models that account for changing a Particular case of Bur Type III distribution and have obtained Bayes estimators of change point m $\theta_{1,}\theta_2$.

2. Proposed Change Point Model

Let $X_1, X_{2,\ldots\ldots\ldots} X_n (n \geq 3)$ be a sequences of observed count data. Let first m observations $X_1, X_{2,\ldots\ldots\ldots} X_m$ have come from Particular case of Bur Type III Distribution with probability mass function as,

$$p_x = \frac{(x_i + 1) - \theta_1^{x_i}}{(x_i + 1)!} \qquad 0 < \theta_1 < 1, \mathrm{x} = 0,1,2\ldots$$

$$\mathrm{i} = 1,2\ldots\mathrm{m}$$

With $F_1(t) = 1 - \dfrac{\theta_1^{t+1}}{(t+1)!}$

And later n-m observations $X_{m+1}, \ldots\ldots, X_n$ have come from Particular case of Bur Type III Distribution function with probability mass function as,

$$p_x = \frac{(x_i + 1 - \theta_2) - \theta_2^{x_i}}{(x_i + 1)!} \qquad 0 < \theta_2 < 1, \mathrm{x} = 0,1,2\ldots$$

$$\mathrm{i} = \mathrm{m} + 1\ldots\mathrm{n}$$

with $F_2(t) = 1 - \dfrac{\theta_2^{t+1}}{(t+1)!}$

Where the change point m is unknown parameter

The likelihood function, given the sample information $\underline{X} = (X_1, X_{2,\ldots\ldots\ldots} X_m, X_{m+1}, \ldots.., X_n)$ is

$$\mathrm{L}(\theta_1, \theta_2, m \mid \underline{X}) = \frac{(x_i + 1 - \theta_1)^m \theta_1^{s_m} (x_i + 1 - \theta_2)^{n-m} \theta_2^{s_n - s_m}}{k_3} \qquad (1)$$

Where $s_m = \Sigma_{i=1}^m x_i$

$s_n = \Sigma_{i=1}^n x_i$

$$k_3 = \Pi_{i=1}^n (x_i + 1)! \qquad (2)$$

3 Posterior Distribution Functions Using Informative Prior

As in Broemeling et al. [5], we suppose the marginal prior distribution of m to be discrete uniform over the set {1, 2, …..n – 1}

$$g_1(m) = \frac{1}{n-1}$$

We consider the ratios ψ_{1t} and ψ_{2t} depending on the distribution at time t and given as,

$$\begin{aligned} \psi_{it} &= 1 - F_{it}, \quad \mathrm{i} = 1,2 \\ &= \frac{\theta_i^{t+1}}{(t+1)!} \quad \mathrm{i} = 1,2 \end{aligned} \tag{3}$$

We also suppose that some information on these ratios is available, and can be known in terms of prior mean value μ_{ψ_1} and μ_{ψ_2} we suppose the Independent log inverse gamma (LIG) priors on ψ_{1t} and ψ_{2t} with respective means μ_{ψ_1} and μ_{ψ_2} and standard deviation σ_ψ viz**.**

$$g_1(\psi_{it}) = \frac{b_i^{a_i}}{\Gamma a_i} \psi_{it}^{b_i - 1} \left[\mathrm{In}\left(1/\psi_{it}\right)\right]^{a_i - 1} \quad a_i, b_i > 0 \ \mathrm{i} = 1,2 \qquad 0 \le \psi_{it} \le 1 \tag{4}$$

If the prior means μ_{ψ_1} and μ_{ψ_2} and a common standard deviation σ_ψ are known, Then the hyper parameters can be obtain by solving

$$1 + \frac{2}{b_i} == \left(1 + \frac{1}{b_i}\right)^{k_i} \quad \mathrm{i} = 1,2 \tag{5}$$

$$\mathrm{a}_i = \frac{\mathrm{In}\left(\mu_{\psi_i}\right)}{\mathrm{In}\left(\frac{b_i}{b_i + 1}\right)} \quad \mathrm{i} = 1,2 \tag{6}$$

Where

$$\mathrm{k}_i = \frac{\mathrm{In}\left[\left(\mu_{\psi_i}\right)^2 + \left(\sigma_{\psi_i}\right)^2\right]}{\mathrm{In}\left(\mu_{\psi_i}\right)} \quad \mathrm{i} = 1,2 \tag{7}$$

We assume that ψ_{1t}, ψ_{2t} and *m* are priori independent. The joint prior density is say,

$$g_1\left(\psi_{1t}, \psi_{2t}, m\right) = k_1 \psi_{1t}^{b_1 - 1} \psi_{2t}^{b_2 - 1} \left[\mathrm{In}\left(1/\psi_{1t}\right)\right]^{a_1 - 1} \left[\mathrm{In}\left(1/\psi_{2t}\right)\right]^{a_2 - 1} \tag{8}$$

$$\text{where } k_1 = \frac{1}{n-1}\frac{b_1^{a_1}}{\Gamma a_1}\frac{b_2^{a_2}}{\Gamma a_2} \tag{9}$$

The likelihood function (1) has been reparameterized in term of m and $\psi_{it} = \frac{\theta_i^{t+1}}{(t+1)!}$, i = 1,2

$$L(\psi_{1t},\psi_{2t},m \mid \underline{X}) = \frac{1}{3}\left[x_i + 1 - k_2(\psi_{1t})^{\frac{1}{(t+1)}}\right]^m (\psi_{1t})^{\frac{s_m}{(t+1)}} k_{k_2}^{s_m} \left[x_i + 1 - k_2(\psi_{2t})^{\frac{1}{(t+1)}}\right]^{n-m} (\psi_{2t})^{\frac{s_n - s_m}{(t+1)}} k_2^{s_{n-s_m}} \tag{10}$$

$$\text{where } k_2 = \left[(t+1)!\right]^{\frac{1}{t+1}} \tag{11}$$

The Joint posterior density of ψ_{1t},ψ_{2t}, and m say, $g_1(\psi_{1t},\psi_{2t},m \mid \underline{X})$, results in

$$\begin{aligned} g_1(\psi_{1t},\psi_{2t},m \mid \underline{X}) &= \frac{L(\psi_{1t},\psi_{2t},m \mid \underline{X})\, g_1(\psi_{1t},\psi_{2t},m)}{h_1(\underline{X})} \\ &= k_4 (\psi_{1t})^{\frac{s_m}{(t+1)}+b_1-1}\left[\text{In}(1/\psi_{1t})\right]^{a_1-1}\left[x_i + 1 - k_2(\psi_{1t})^{\frac{1}{(t+1)}}\right]^m \\ &(\psi_{2t})^{\frac{s_n-s_m}{(t+1)}+b_2-1}\left[\text{In}(1/\psi_{2t})\right]^{a_2-1}\left[x_i + 1 - k_2(\psi_{2t})^{\frac{1}{(t+1)}}\right]^{n-m} h_1^{-1}(\underline{X}) \end{aligned} \tag{12}$$

Where

$$K_4 = \frac{1}{k_3} K_1 K_2^{s_n} \tag{13}$$

$$\begin{aligned} h_1(\underline{X}) &= \Sigma_{m=1}^{n-1}\Sigma_{j_1=0}^{m}\Sigma_{j_2=0}^{n-m} K^{**}(m) \int_0^1 (\psi_{1t})^{\frac{s_m+j_1}{(t+1)}+b_1-1}\left[\text{In}(1/\psi_{1t})\right]^{a_1-1} d\psi_{1t} \\ &\int_0^1 (\psi_{2t})^{\frac{s_n-s_m+j_2}{(t+2)}+b_2-1}\left[\text{In}(1/\psi_{2t})\right]^{a_2-1} dr_{2t} \\ &= \Sigma_{m-1}^{n-1} T_1(m) \end{aligned} \tag{14}$$

Where

$$T_1(\text{m}) = \Sigma_{j_1=0}^{m}\Sigma_{j_2=0}^{n-m} K^{**}(m)\Gamma a_1\left[\frac{s_m + j_1}{(t+1)} + b_1\right]^{-a_1} \Gamma a_2\left[\frac{s_n - s_m + j_2}{(t+1)} + b_2\right]^{-a_2} \tag{15}$$

$$K^{**}(m) = K_4 \left(x_i+1\right)^n \left(-1\right)^{j_1} \binom{m}{j_1} \frac{k_2 j_1}{\left(x_i+1\right) j_1} \left(-1\right) j_2 \binom{n-m}{j_2} \frac{k_2 j_2}{\left(x_i+1\right)^{j_2}}$$

$$\left[x_i+1-k_2\left(\psi_{1t}\right)^{\frac{1}{(t+1)}}\right]^m = \left(x_i+1\right)^m \Sigma_{j_1=0}^{m} (-1)^{j_1} \binom{m}{j_1} \frac{k_2 j_1}{\left(x_i+1\right) j_1} \left(\psi_{1t}\right)^{\frac{j_1}{(t+1)}} \quad \text{and}$$

$$\left[x_i+1-k_2(\psi_{2t})^{\frac{1}{(t+1)}}\right]^{n-m} = \left(x_i+1\right)^{n-m} \Sigma_{j_2=0}^{n-m} \left(-1\right)^{j_2} \binom{n-m}{j_2} \frac{k_2^{j_2}}{\left(x_i+1\right) j_2} \left(\psi_{2t}\right)^{\frac{j_2}{(t+1)}}$$

K_4 is given in (13)

Marginal Posterior Density of ψ_{1t} and of ψ_{2t} are obtained by integrating the joint posterior density of ψ_{1t} and ψ_{2t}, m given in (12) with respect to ψ_{2t} and with respect to ψ_{1t} respectively and summing over m

$$g_1\left(\psi_{1t} \mid \underline{X}\right) = \Sigma_{m=1}^{n-1} \Sigma_{j_1=0}^{m} \Sigma_{j_2=0}^{n-m} K^{**}(\mathrm{m}) \left(\psi_{1t}\right)^{\frac{s_m+j_1}{(t+1)}+b_1-1} \left[\mathrm{In}\left(1/\psi_{1t}\right)\right]^{a_1-1} \Gamma a_2 \left[\frac{s_n-s_m+j_2}{(t+1)}+b_2\right]^{-a_2} h_1^{-1}(\underline{X}) \tag{16}$$

$$g_1\left(\psi_{2t} \mid \underline{X}\right) == \Sigma_{m=1}^{n-1} \Sigma_{j_1=0}^{m} \Sigma_{j_2=0}^{n-m} K^{**}(\mathrm{m}) \left(\psi_{2t}\right)^{\frac{s_{n-s_m}+j_2}{(t+1)}+b_2-1} \left[\mathrm{In}\left(1/\psi_{2t}\right)\right]^{a_2-1} \Gamma a_1 \left[\frac{s_m+j_1}{(t+1)}+b_1\right]^{-a_1} h_1^{-1}(\underline{X}) \tag{17}$$

Now change of variables, ψ_{it} i=1,2 in (16) and (17) respectively, we get marginal posterior density of θ_1 and θ_2 as,

$$g_1\left(\theta_1 \mid \underline{X}\right) = \Sigma_{m=1}^{n-1} \Sigma_{j_1=0}^{m} \Sigma_{j_2=0}^{n-m} K^{**}(\mathrm{m}) \left(\frac{\theta_1^{t+1}}{(t+1)!}\right)^{\left[\frac{s_m+j_1}{(t+1)}+b_1-1\right]} \left[\mathrm{In}\left(\frac{(t+1)!}{\theta_1^{t+1}}\right)\right]^{a_1-1} a_2 \left[\frac{s_n-s_m+j_2}{(t+1)}+b_2\right]^{-a_2} h_1^{-1}(\underline{X}) \tag{18}$$

$$g_1\left(\theta_2 \mid \underline{X}\right) = \Sigma_{m=1}^{n-1} \Sigma_{j_1=0}^{m} \Sigma_{j_2=0}^{n-m} K^{**}(\mathrm{m}) \left(\frac{\theta_2^{t+1}}{(t+1)!}\right)^{\left[\frac{s_{n-s_m}+j_2}{(t+1)}+b_2-1\right]} \left[\mathrm{In}\left(\frac{(t+1)!}{\theta_2^{t+1}}\right)\right]^{a_2-1} \Gamma a_1 \left[\frac{s_m+j_1}{(t+1)}+b_1\right]^{-a_1} h_1^{-1}(\underline{X}) \tag{19}$$

The marginal posterior density of change point m is say $g_1\left(m \mid \underline{X}\right)$ is obtained as

$$g_1\left(m \mid \underline{X}\right) = \int_0^1 \int_0^1 g_2\left(\theta_1, \theta_2 \mid \underline{\mathrm{X}}\right) d\theta_1 d\theta_2 = \frac{T_1(\mathrm{m})}{\sum_{m=1}^{n-1} T_1(\mathrm{m})} \tag{20}$$

Where $T_1(\mathrm{m})$ same as in (15)

4. Bayes Estimates of Change Point & Other Perameters Under Asymmetric Loss Functions

In this section, we derive Bayes estimator of change point m under different asymmetric loss function using both prior considerations explained in section 3. A useful asymmetric loss, the Linex loss function was introduced by Varian [9] and expressed as,

$$\mathrm{L}_4\left(\alpha, \mathrm{d}\right) = \exp.\left[q_1\left(\mathrm{d} - \alpha\right)\right] - q_1(\mathrm{d} - \alpha) - \mathrm{I}, \mathrm{q}_1 \neq 0. \tag{21}$$

The sign of the shape parameter q_1 reflects the deviation of the asymmetry, $q_1 > 0$.

Minimizing expected loss function E_m [L_4 (m, d)] and using posterior distribution (20), we get the bayes estimates of m , using Linex loss function as,

$$m_L^* = -1/q_1 \cdot \ln\left[\sum_{m=1}^{n-1} \frac{e^{-q_1 m} T_1(m)}{\sum_{m=1}^{n-1} T_1(m)}\right], \tag{22}$$

Where $T_1(\mathrm{m})$ is same as in (14).

Minimizing expected loss function $E_{\theta_1}\left[\mathrm{L}_4\left(\theta_1, d\right)\right]$ and using posterior distribution (18) and we get the bayes estimates of θ_1, using Linex loss function as

$$\theta_{1L}^*(\mathrm{t}) = -\frac{1}{q_1} \ln\left[\sum_{m=1}^{n-m} \sum_{j_1=0}^{n-m} \sum_{j_2=0}^{n-m} K^{**}\left(m\right) \int_0^1 \left(\frac{\theta_1^{t+1}}{\left(t+1\right)!}\right)^{\left[\frac{s_m + j_1}{(t+1)} + b_1 - 1\right]} \right. \\ \left. \times \left[In\left(\frac{\left(t+1\right)!}{\theta_1^{t+1}}\right)\right]^{a_1 - 1} e^{-\theta_1 q_1} d\theta_1 \, \Gamma a_2 \left[\frac{s_n - s_m + j_2}{\left(t+1\right)} + b_2\right]^{-a_2} h_1^{-1}(\underline{\mathrm{X}})\right] \tag{23}$$

Minimizing expected loss function $E_{\theta_2}\left[L_4\left(\theta_2,d\right)\right]$ and using posterior distribution (19) and we get the bayes estimates of θ_2, using Linex loss function as

$$\theta_{2L}^*(t)=-\frac{1}{q_1}\ln\left[\Sigma_{m=1}^{n-m}\Sigma_{j_1=0}^{m}\Sigma_{j_2=0}^{n-m}K^{**}\left(m\right)\int_0^1\left(\frac{\theta_2^{t+1}}{(t+1)!}\right)^{\left[\frac{s_n-s_m+j_2}{(t+1)}+b_2-1\right]}\right. \tag{24}$$

$$\left.\times\left[In\left(\frac{(t+1)!}{\theta_2^{t+1}}\right)\right]^{a_2-1}e^{-\theta_2 q_1}d\theta_2\,\Gamma a_1\left[\frac{s_m+j_1}{(t+1)}+b_1\right]^{-a_1}h_1^{-1}(\underline{X})\right]$$

General Entropy loss function (GEL), proposed by Calabria and Pulcini [10] is given by,

$$L_5\left(\alpha,d\right)=\left(d/\alpha\right)^{q_3}-q_3\ln\left(d/\alpha\right)-1,$$

using posterior distributions (20), we get Bayes estimate of change point m under GEL ,say m_E^* *as*

$$m_E^*=\left[E_1\left[m^{-q_3}\right]\right]^{-1/q_3}=\left[\frac{\Sigma_{m=1}^{n-1}m^{-q_3}T_1(m)}{\Sigma_{m=1}^{n-1}T_1(m)}\right]^{-\frac{1}{q_3}}, \tag{25}$$

minimizing expectation $\left[E_{\theta_1}\left[L_5\left(\theta_1,d\right)\right]\right.$ and using posterior distributions (18), we get Bayes estimate of θ_1 using General Entropy loss function

$$\theta_{1E}^*=\left[E\left(\theta_1^{-q_3}\right)\right]^{-\frac{1}{q_3}}$$

$$=\left[\Sigma_{m=1}^{n-m}\Sigma_{j_1=0}^{m}\Sigma_{j_2=0}^{n-m}K^{**}(m)\int_0^1\left(\frac{\theta_1^{t+1}}{(t+1)!}\right)^{\left[\frac{s_m+j_1}{(t+1)}+b_1-1\right]}\right. \tag{26}$$

$$\left.\times\left[In\left(\frac{(t+1)!}{\theta_1^{t+1}}\right)\right]^{a_1-1}\theta_1^{-q_3}d\theta_1\,\Gamma a_2\left[\frac{s_n-s_m+j_2}{(t+1)}+b_2\right]^{-a_2}h_1^{-1}(\underline{X})\right]^{-\frac{1}{q_3}}$$

minimizing expectation $\left[E_{\theta_2}\left[L_5\left(\theta_2,d\right)\right]\right.$ and using posterior distributions (19), we get Bayes estimate of θ_2 using General Entropy loss function *as*

$$\theta_{2E}^{*} = \left[E\left(\theta_2^{-q_3}\right)\right]^{-\frac{1}{q_3}}$$

$$= \Sigma_{m=1}^{n-m}\Sigma_{j_1=0}^{m}\Sigma_{j_2=0}^{n-m}K^{**}(m)\int_0^1 \left(\frac{\theta_2^{t+1}}{(t+1)!}\right)^{\left[\frac{s_n - s_m + j_2}{(t+1)} + b_2 - 1\right]} \quad (27)$$

$$\times\left[In\left(\frac{(t+1)!}{\theta_2^{t+1}}\right)\right]^{a_2-1} \theta_2^{-q_3}\mathrm{d}\theta_2\ \Gamma a_1\left[\frac{s_m + j_1}{(t+1)} + b_1\right]^{-a_1} h_1^{-1}(\underline{X})\Bigg]^{-\frac{1}{q_3}}$$

5. Numerical Study

We have generated 30 random observation, the first 15 observations from discrete Burr type III distribution with $\psi_{1t} = 0.017$ at t=2 and $\psi_{2t} = 0.0085$ at t= 2 ψ_{1t} and ψ_{2t} themselves were random observations from log inverse gamma distributions with means $\mu_{\psi 1} = 0.017, \mu_{\psi 2} = 0.0085$, and standard deviation $\sigma_{\psi} = 0.01$ respectively, resulting in $a_1 = 0.01, b_1 = 10, a_2 = 0.009$ and $b_2 = 25$. These observations are given in Table 1 first row.

We have generated 6 random sample from proposed change point model discussed in section-2 with n=30, 50, 50 and m=15, 25, 35, θ_1=0.47, 0.2, $\psi_{1t} = 0.017$, 0.0013 at t=2 and $\psi_{2t} = 0.0085$, 0.0208 at t= 2 and θ_2= 0.8, 0.5. As explained in section 3, ψ_{1t} and ψ_{2t} themselves were random observation from LIG prior distributions with prior means $\mu_{\psi_1}, \mu_{\psi_2}$ respectively. These observations are given in Table 1 .We have calculated posterior means of m, $\theta_1$and θ_2 selected samples and the results are shown in Table 2

Table 1: Generated Samples of proposed change point model

Sample No.	n	m	Sample		Actual	
			$\theta_1 = 0.47$	$\theta_2 = 0.8$	ψ_{1t}	ψ_{2t}
1	30	15	0x8, 1x7	0x2, 1x7, 2x4,3x2	0.0170	0.0085
2	50	25	0x13, 1x11, 2x1	0x4, 1x13 , 2x6,3x2		
3	50	35	0x19, 1x13, 2x3	0x2, 1x7,2x5, 3x1		
			θ_1 = 0.2	**θ_2 = 0.5**		
4	30	15	0x14, 1x1	0x7, 1x5, 2x3	0.0013	0.0208
5	50	25	0x21,1x4,	0x13, 1x9, 2x3		
6	50	35	0x29,1x6	0x8, 1x5, 2x2		

Table 2: Bayes Estimate of m, θ_1 and θ_2 under SEL

Sample No.	n	Bayes Estimates of m (Posterior Mean)	Bayes Estimates of θ_1 *and* θ_2 (Posterior Mean)	
			Posterior mean of θ_1	Posterior mean of θ_2
1	30	15	0.47	0.0085
2	50	25	0.47	0.0085
3	50	35	0.47	0.0085
4	30	15	0.2	0.021
5	50	25	0.2	0.021
6	50	35	0.2	0.021

We also compute the Bayes estimates of change point *and* θ_1 *and* θ_2 Using the results given in section 4 for the data given in table 1 and for different values of shape parameter q_1 and q_3, the results are shown in Tables 3 and 4.

Table 3 shows that for small values of $|q|$, q_1 = 0.007, 0.12, 0.23 the values of the Bayes estimate under a Linex loss function is near by the posterior mean. Table 3 also shows that, for q_1 1.5, 1.2, Bayes estimate are less than actual value of m = 15.

Table 3: The Bayes estimates using Linex Loss Function for sample 1

$\theta_1 = 0.47, \theta_2 = 0.8$	q_1	m_L^*	Posterior mean of θ_1^*	Posterior mean of θ_2^*
Log Inverse Gamma Prior	0.007	15	0.47	0.83
	0.12	15	0.47	0.82
	0.23	15	0.46	0.80
	1.2	14	0.44	0.73
	1.5	13	0.43	0.72
	-1.0	16	0.52	0.84
	-2.0	17	0.54	0.83

For $q_1 = q_3$=-1,-2,Bayes estimates are quite large than actual value m = 15. It can be seen from Table 3 and 4 that if we take the value of shape parameters of loss function negative , underestimation can be solved.

Table 4 shows that, for small values of $|q|$, q_3 = 0.007, 0.12, 0.23 General Entropy loss function, the values of the Bayes estimate under a loss is near by the posterior mean. Table 4 also shows that, for q_3 = 1.5, 1.2, Bayes estimates are less than actual value of m = 15.

Table 4: The Bayes estimates using General Entropy Loss Function for sample 1.

$\theta_1 = 0.47, \theta_2 = 0.8$	q_3	m_E^*	θ_{1E}^*	θ_{2E}^*
	0.007	15	0.47	0.81
Log Inverse Gamma Prior	0.12	15	0.46	0.82
	0.23	15	0.45	0.84
	1.2	14	0.42	0.74
	1.5	13	0.40	0.73
	-1.0	15	0.53	0.85
	-2.0	17	0.55	0.87

6. Sensitivity of Bayes Estimates

In this paper, we are studying five Bayes estimator of change pont and other parameters of theproposed change point model based on particular case of burr type III distribution, a part from that we can consider posterior mean is more appealing. .Table 5 shows that, when prior mean $\mu_{\psi_1} = 0.017$ actual value of $\psi_{1t}, \mu_{\psi_2} = 0.006$ and0.0095 (far from the true value of $\psi_{2t} = 0.0085$), it means correct choice of prior ψ_{1t} and wrong choice of prior of ψ_{2t} ,the value of Bayes estimator posterior mean of m does not differ.

Table 5: Bayes Estimate of m for Sample 1

$\mu_{\psi_{1t}}$	$\mu_{\psi_{2t}}$	m^*
0.017	0.006	15
0.017	0.0085	15
0.017	0.0095	15
0.012	0.0085	15
0.017	0.0085	15
0.022	0.0085	15

7. Simulation Study

we have also generated 10,000 different random samples with m=15, n=30 $\theta_1 =$ 0.47, $\theta_2 = 0.8, q_1 = q_3 = 0.1, \psi_{1t} = 0.017, \psi_{2t} = 0.0085$. and obtained the frequency distributions of posterior mean, m_L^*, m_E^*, with the same prior consideration explained as in numerical study. The result is shown in Table-8.

Table 6: Bayes Estimate of m for Sample 5

$\mu_{\psi_{1t}}$	$\mu_{\psi_{2t}}$	m^*
0.0013	0.011	25
0.0013	0.021	25
0.0013	0.041	25
0.0001	0.021	25
0.0013	0.021	25
0.0025	0.021	25

Table 7: Bayes Estimate of m for Sample 6

$\mu_{\psi_{1t}}$	$\mu_{\psi_{2t}}$	m^*
0.0013	0.012	35
0.0013	0.021	35
0.0013	0.043	35
0.0001	0.021	35
0.0013	0.021	35
0.0024	0.021	35

Table 8: Frequency distributions of the Bayes estimates of the change point

Bayes estimate	% Frequency for		
	01-13	14-17	17-30
Posterior mean	11	79	10
m_L*	20	65	15
m_E*	22	66	12

8. Conclusion

We conclude that if we are interested for avoiding overestimation as well as underestimation Linex and General Entopy loss function are more appropriate.

References

[1] Burr, I. W. (1942), “cumulative frequency function”, *Ann. Math. Statistics*, **Vol 1**. No. 2, pp 215-232. http://dx.doi.org/10.1214/aoms/1177731607

[2] Fry, R. L. Tin (1993), Univariate and multivariate Burr distributions-A survey, *Pak. J Staist*. , **9A**, 1-24.

[3] Nair, N. U. and Asha, G. (2004), Characterizations using failure and reversed failure rates, *J. Ind. Soc. Probab. And statist*, **8**, 45-56.

[4] Sreehari M.. (2008) "On a Class of Discrete Distributions Analogous to Burr Family", *Journal of the Indian Statistical Association*, **Vol.46**, 2, pp 263-181.

[5] Broemeling, L. D. and Tsurumi, H. (1987). Econometrics and structural change, Marcel Dekker, New York.

[6] Jani, P. N. and Pandya, M. (1999). Bayes estimation of shift point in left Truncated Exponential Sequence, *Communications in Statistics (Theory and Methods)*, **28**(11), 2623-2639. http://dx.doi.org/10.1080/03610929908832442

[7] Pandya, M. (2013). "Bayesian Estimation of AR (1) with Change Point under Asymmetric Loss Functions ", *Statistics Research Letters (SRL)* **Volume 2** Issue 2, May 2013.

[8] Pandya, M. Pandya, S and Andharia, P. (2014). " Bayes Estimation of Generalized Compound Rayleigh Distribution with Change Point" *Statistics Research Letters (SRL)* **Volume 3** Issue 2, May 2014 www.srl-journal.org

[9] Varian, H. R. (1975). "A Bayesian approach to real estate assessment," Studies in Bayesian econometrics and Statistics in Honor of Leonard J. Savage, (Feigner and Zellner, Eds.) North Holland Amsterdam, 195-208.

[10] Calabria, R. and Pulcini, G. (1996). "Point estimation under asymmetric loss functions for left-truncated exponential samples", *Communication in Statistics (Theory and Methods)*, **25**(3), 585-600. http://dx.doi.org/10.1080/03610929608831715