DOI: 10.15415/mjis.2014.31006

Selection of Generalized Two Plan System Using Minimum Angle Method

KK SURESH AND KX VINITHA

Department of Statistics, Bharathiar University, Coimbatore, Tamilnadu, India.

Received: August 1, 2013| Revised: February 12, 2014| Accepted: February 13, 2014

Published online: September 20, 2014

Abstract: This paper presents a new procedure and tables for the selection of Generalized Two Plan system through minimum angle criteria involving producer's and consumer's quality levels. The approach of minimum angle method by considering the tangent of the angle between the lines joining the points (AQL, 1-α) and (LQL, β). Illustrations are also provided for ready made selection of plan parameters.

Keywords: Two-Plan Switching System, Minimum Angle, Acceptable Quality Level, Limiting Quality Level, Producer's Risk, Consumer's Risk.

1. Introduction

Acceptance sampling is necessarily a defensive measure, instituted as protective device against the threat about deterioration in quality. When inspection is for the purpose of acceptance or rejection of a product, based on the adherence to a standard, the type of inspection procedure employed is usually called acceptance sampling. The primary objective of sampling inspection is to reduce the cost of inspection while at the same time assuring the customer of an adequate level of quality in the items being inspected.

Dodge [3] has proposed a new sampling inspection system namely Two-plan switching system. The Two-Plan system has normal and tightened plan which has stringent OC curve compared with that of normal plan. This system is largely incorporated in MIL-STD-105E [5] for designing of a sampling system.

The two-plan system together with the switching rules with the MIL-STD-105E [5] form an integrated sampling inspection system guaranteeing the consumer that the outgoing quality will be better than the AQL specified and at the same time assuring the producer that the risk of rejection will be small for products of AQL quality of better ones. Kuralmani [4] has designed

Mathematical Journal of Interdisciplinary Sciences
Vol. 3, No. 1,
September 2014
pp. 65–72

CHITKARA UNIVERSITY

Two-Plan Switching System involving acceptable and limiting quality levels. Suresh [10] has proposed procedures to select Multiple Deferred State Plan of type MDS and MDS-1 indexed through producer and consumer quality levels considering filter and incentive effects. The procedure with a pair of plans gives an overall OC curve that generally lies in between the OC curve of the normal and tightened plans in a Two-Plan Switching system. Deepa [2] has developed Special type Double Sampling Plan indexed through minimum angle method with producer quality level and consumer quality level. Balamurali and Chi-Hyuck Jun [1] have made contributions to designing of a variables two-plan system by minimizing the average sample number (ASN). Muthulakshmi [6] obtained SkSP-2 plans using minimum angle method. Sangeetha [9] has proposed selection procedure for one plan suspension system through minimum angle method.

Norman Bush *et* al. [7] have used different techniques involving comparison of some portion of the OC curve to that of the ideal OC curve. The techniques of minimum angle method by considering the tangent of the angle between the lines joining the points (AQL, 1-α) and (LQL,β). By employing this method one can get a better discriminating plan with the minimum angle.

2. Conditions for Application of a Two-Plan Switching System

1. The production is steady so that results on current and preceding lots are broadly indicative of a continuing process and submitted lots are expected to be of essentially the same quality
2. Lots are submitted substantially in the order of production.
3. The product comes from a source in which the consumer has confidence.

3. Operating Procedure for Two-Plan System

Switching rules for generalized Two-plan Systems are:

- Normal to Tightened

  When normal inspection is in effect, tightened shall be instituted when'*s'* out of *'m'* consecutive lots or batches have been rejected on original inspection $(s \leq m)$.

- Tightened to Normal

  When tightened inspection is in effect, normal shall be instituted when'*d*' consecutive lots or batches have been considered acceptable on original inspection.

4. Notation and Symbols

A number of important measures of performance to be determined and used in the evaluation of OC function which will be discussed.

P_N = the proportion of lots expected to be accepted under normal inspection.

P_T = the proportion of lots expected to be accepted under tightened inspection

I_N = the expected proportion of lots inspected on normal inspection.

I_T = the expected proportion of lots inspected on tightened inspection.

Dodge [3] has provided a performance measure with a composite of function for the probability of acceptance

$$P_a(p) = I_N p_N + I_T P_T \tag{4.1}$$

The method of deriving various measures of performance for the Generalized Two-Plan system is also studied.

5. Minimum Angle Method

Bush *et* al. [7] have considered two points on the OC curve as (AQL, 1- α), and (IQL, 0.50) (where IQL is the indifference quality level and α is the producer's risk) for minimizing the consumer's risk. However, Peach and Littauer (1946) have considered two points on the OC curve as (p_1, 1- α) and (p_2, β) for ideal condition, to minimize the consumer's risk (β). In this paper, another approach of minimizing the angle between the lines that join the points (AQL, 1-α), (AQL, β) and (AQL, 1- α), (LQL, β) is given. Applying this method one can get a better plan, which has an OC curve approaching to the ideal OC curve.

The formula for tanθ is given as

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{P_2 - p1}{P_a(p_1) - pa(p_2)} \tag{5.1}$$

Where p_1 = AQL and p_2 = LQL.

This may be expressed as,

$$n\tan\theta = (np_2 - np_1)/(1-\alpha-\beta) \tag{5.2}$$

Hence, for two given points on the OC curve, the minimum values of tanθ can be calculated, as tabulated in table 1(a) for Two-Plan System. Designing procedures for the two plan switching system which is designated as TPS - (*n,kn; c*) is discussed.

6. Selection of Two-Plan System Using Minimum Angle

Table 1(a) can be used to select a two plan system for given values of AQL and LQL. To select a TPS (*n,kn; c*) system for given *s, m, d* ,c, *k*, p_1 and p_2 first calculate the operating ratio p_2/p_1. From table 1(a), locate the nearest OR value and the associated np_1 values. The other associated n tanθ, α% and β% can be recorded from the table 1(a).

Now,

$$\theta = \tan^{-1}(n\tan\theta)\,/\,n \qquad (6.1)$$

For Example,

For given $s = 2$, $m = 2$, $d= 4$, $c =10$, $k=5$, $p_1 = 0.05$, $p_2 = 0.25$

Then OR $=p_2/p_1 = 0.25 / 0.05 = 5$

The associated sets of values corresponding to the computed OR of 5 are

1. $np_1 = 300$, $n \tan\theta = 6.7654$, $c =10$, α % = 1.68, β % =2.34
2. $np_1 = 400$, $n \tan\theta= 8.5374$, $c =10$, α % = 2.24, β% =2.70
3. $np_1 = 500$, $n \tan\theta = 9.9353$, $c =10$, α % = 2.78, β% =3.44

From the above results one can find,

1. $n = np_1 / p_1 = 300/0.05 = 6000$

   $\theta= \tan^{-1}(n \tan\theta)/n$

   $= \tan^{-1}(6.7654)/6000 = 0.0136$
2. $n = np_1 / p_1 = 400/0.05 = 8000$

   $\theta =\tan^{-1}(n \tan\theta)/n$

   $= \tan^{-1}(8.5374)/8000 = 0.0104$
3. $n = np_1 / p_1 = 500/0.05 = 10000$

   $\theta = \tan^{-1}(n \tan\theta)/n$

   $= \tan^{-1}(9.9353)/10000 = 0.008425$

Now the minimum angle of the three choices is $\theta = 0.0084$. Hence the selected parameters for the two-plan system for given s=2, m=2, d=4, c=10, k=5, p_1 = 0.05 and p_2= 0.25, α= 0.05 and β =0.10 is TPS *(n,kn; c)* - (10000, 50000; 3) with minimum angle $\theta = 0.0084$.

Table 1(a): Values of Minimum Angle TPS (n, kn; c) System for Given OR and np_1 Values

OR	np_1	s	m	d	c	k	$n \tan\theta$	α%	β%
50	5	2	2	5	5	5	3.7843	0.36	5.05
	10	2	2	4	5	5	3.9753	0.49	2.77
	20	2	2	3	5	5	4.9844	0.58	3.58
	30	2	2	2	5	5	5.2033	0.62	1.93
	40	2	2	5	6	5	10.6114	0.67	3.71
	50	2	2	4	6	5	11.2506	0.75	4.88
	60	2	2	3	6	5	12.6008	0.77	2.21
45	10	2	2	5	7	5	3.2991	0.97	3.86
	20	2	2	2	6	5	4.0161	1.06	3.37
	30	2	2	4	7	5	4.7511	2.3	1.23
	40	2	2	3	7	5	5.5042	3.14	3.95
	50	2	2	3	7	5	6.1767	1.32	1.33
	60	2	2	3	6	5	7.9604	2.12	1.07
	70	2	2	2	6	5	9.7916	1.64	6.78
35	15	2	2	4	8	5	8.6247	0.23	3.42
	25	2	2	4	7	5	11.9811	0.35	5.29
	35	2	2	4	6	5	14.7510	0.51	6.13
	45	2	2	5	9	5	18.8542	1.05	2.12
	55	2	2	5	8	5	20.4351	0.86	1.64
	65	2	2	5	7	5	22.6843	1.93	3.25
	75	2	2	3	9	5	24.4765	1.44	2.36
30	20	2	2	3	8	5	6.4587	1.96	2.78
	30	2	2	3	7	5	8.2236	2.02	2.42
	40	2	2	4	10	5	16.8542	0.59	4.37
	50	2	2	4	9	5	18.4562	0.64	1.13
	60	2	2	5	8	5	23.7768	1.34	2.23
	70	2	2	5	9	5	25.7580	2.65	2.29
25	30	2	2	5	10	5	8.1780	0.43	3.52
	40	2	2	4	9	5	12.3579	0.37	4.19
	50	2	2	3	9	5	16.3847	0.93	3.01
	60	2	2	2	9	5	20.5204	1.39	1.57
	70	2	2	5	9	5	24.5364	2.34	3.26
	80	2	2	3	9	5	28.6042	1.96	2.83

OR	np_1	s	m	d	c	k	n tanθ	α%	β%
20	25	2	2	4	5	5	4.2215	1.35	1.86
	35	2	2	2	6	5	6.7848	0.85	1.24
	45	2	2	5	9	5	9.5624	0.64	2.42
	55	2	2	3	7	5	11.8246	0.58	2.02
	65	2	2	4	5	5	14.2536	0.42	1.54
	75	2	2	2	8	5	17.3568	0.33	3.15
15	40	2	2	5	9	5	5.8432	0.45	2.48
	60	2	2	4	6	5	8.5264	1.52	2.39
	80	2	2	2	7	5	10.5698	1.34	3.64
	100	2	2	3	5	5	12.1154	1.08	2.35
	120	2	2	5	8	5	14.3225	2.64	2.81
10	50	2	2	2	5	5	4.5378	0.94	1.32
	100	2	2	3	6	5	7.3245	1.94	2.51
	150	2	2	4	8	5	10.2469	1.56	2.47
	200	2	2	2	9	5	12.4765	2.02	3.78
	250	2	2	2	7	5	13.6524	2.23	3.93
5	100	2	2	5	9	5	3.7646	2.89	3.51
	200	2	2	4	10	5	5.5276	2.43	2.72
	300	2	2	4	10	5	6.7654	1.68	2.34
	400	2	2	4	10	5	8.5374	2.24	2.7
	500	2	2	4	10	5	9.9353	2.78	3.04

7. Construction of Tables

The OC function for Two-Plan System is

$$P_a(p) = \frac{\mu P_N + \tau P_T}{\mu + \tau} \tag{7.1}$$

Where

$$P_N = P(d \le c / n, p) \tag{7.2}$$

$$P_T = P(d \le c / kn, p) \tag{7.3}$$

$$\mu = \frac{1 + (1-a)^{s-2}(2a - a^{m-s+2} - 1)}{a(1 - a^{m-s+2})(1-a)^{s-1}} \tag{7.4}$$

$$\tau = \frac{1-b^d}{(1-b)^{b^d}} \quad (7.5)$$

If the operating ratio p_2/p_1 and np_1 are known then np_2 can be obtained as

$$np_2 = (np_1)(p_2 / p_1) \quad (7.6)$$

The following search procedure is used to obtain the optimum value of c which minimizes the tangent angle for certain specific values of np_1 and np_2 by keeping the producer's risk below 5% and consumer's risk below 10%.

1. Set c=0
2. Compute α and β using μ, τ, P_N and P_T for given s, m, d, c, k, np_1 and OR.
3. If $P_a(p) \geq 1-\alpha$, go to step (6)
   If $P_a(p) \leq 1-\beta$, go to step (6)
4. Find $n\tan\theta$ using np_1, α, β and computed np_2 = OR x np_1
5. Record minimum of $n\tan\theta$.
6. Increase 'c' by 1, go to step 2.
7. If the current value of $c \geq 15$, stop the process, otherwise repeat steps 2 to 7.
8. Record minimum of $n\tan\theta$ and c.

8. Conclusion

Acceptance Sampling is the technique, which deals with the procedures in which a decision either to accept or reject lots or process which are based on the examination of past history or knowledge of samples. The Two-Plan Switching System studied here has an advantage of having normal and tightened plan which has stringent OC curve compared with normal plan. This method is applicable to destructive testing. The work presented in this paper are mainly related to the new procedure for the selection of Generalized Two Plan System through minimum angle criteria involving producer's and consumer's quality levels. Tables are provided here which are tailor made for industrial shop floor applications towards quality of a product.

References

[1] Balamurali, S; and Chi-Hyuck Jun; (2009); Designing of a variables two-plan system by minimizing the average sample number; *Journal of Applied Statistics*, **36**; pp.1159-1172. http://dx.doi.org/10.1080/02664760802562514

[2] Deepa, O.S; (2002); Studies on certain classical and Bayesian acceptance sampling Plan; Ph.D. Thesis, Bharathiar University, Tamil Nadu, India.

[3] Dodge, H.F; (1969); Notes on the Evaluation of Acceptance Sampling Plans - Part I; *Journal of Quality Technology*, **Vol.1**,No.2; pp. 77-78.

[4] Kuralmani, V; (1992); Studies on Designing Minimum Inspection Attribute Acceptance Sampling Plans; Ph.D. Thesis, Bharathiar University, Tamil Nadu, India.

[5] MIL -STD- 105E; (1989); Sampling Procedures and Tables for Inspection by Attributes; US Government Printing Office, Washington, DC.

[6] Muthulakshmi, S; (2009); Designing of Minimum angle Skiplot sampling Plans; Res. High, JADU, 19,32.

[7] Norman Bush; Leonard, E.J; Marvin, Q.M; Marchant, Jr; (1953); A Method of Discrimination for Single and Double Sampling OC curves utilizing the Tangent of the Point of Inflexion; Engineering Agency Special Report (ENSAR); No, PR-7, 1-77.

[8] Peach, P; and Littauer, S.B; (1946); A Note on Sampling Inspection; *Annals of Institute of Statistical Mathematics*, **Vol.17**; pp.81-84. http://dx.doi.org/10.1214/aoms/1177731027

[9] Sangeetha,V; (2012); Some contributions of the study on special purpose plans and suspension system indexed through quality regions and risk minimization; Ph.D. Thesis, Bharathiar University, Tamil Nadu, India.

[10] Suresh, K.K; (1993); A study on Acceptance Sampling Plan using Acceptable and Limiting Quality Levels; Ph.D.Thesis, Bharathiar University, Tamilnadu, India