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Mathematical Model for Impact of Media on Cleanliness Drive in India

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ABSTRACT

A mathematical model on cleanliness drive in India is analysed for active cleaners and passive cleaners. Cleanliness and endemic equilibrium points are found. Local and global stability of these equilibrium points are discussed using Routh-Hurwitz criteria and Lyapunov function respectively. Impact of media (as a control) is studied on passive cleaners to become active. Numerical simulation of the model is carried out which indicates that with the help of media transfer rate to active cleaners from passive cleaners is higher.

1. Introduction

Maintaining a clean environment plays an important role in the health of every human being, because their health is fully based on the surrounding environment. The importance of cleanliness cannot be ignored by any society. Every religion and civilization emphasis the significance of cleanliness. It is a virtue of God. Woefully, this virtue of cleanliness not reflected in our society. People usually throw garbage in the roads, streets and gardens etc. Spitting anywhere is also common practice. Students of schools and colleges also throw waste in the classroom and the other premises. pitiful conditions about cleanliness are observed in our society. We must develop positive attitude to tackle this situation. In fact, there is a need to inspire people about importance of cleanliness in our life [4].

Taking in to the account the urgent need of cleanliness in India, the Prime Minister Narendra Modi had launched cleanliness drive as "Swachh Bharat Abhiyan" on 2nd October 2014, at Rajghat, New Delhi [3]. The objectives of the cleanliness drive in India are to clean up houses, streets, roads and infrastructure of India's cities, smaller towns, and as well as eliminating open defecation in rural areas [4]. The cleanliness drive aims to achieve the vision of 'Clean India' by 2nd October 2019. Each citizen of India should join in this cleanliness drive as 'cleaner' to achieve the goal of "Swachh Bharat Abhiyan".

There are two types of cleaners, namely 'active cleaners' and 'passive cleaners'. Active cleaners are involved in cleanliness activity with their own interest, while passive cleaners need an external force to engage in cleanliness related activity. It is observed that media plays vital role as motive force for passive cleaners to join in cleanliness drive and become active media happens to be the most readily available and potentially most economical means of spreading information about cleanliness drive. Media can effectively lift public awareness and concern about sanitation. Moreover, every day public see a lot of advertisements in electronic and print media, where they tell people to support this drive [7]. Eventually, the media works as stimulant in cleanliness drive. Shah *et al.* [9] developed a mathematical model for cleanliness drive and has discussed its stability through graph theory.

In this paper, we will analyse how passive cleaners turns to be active cleaners under impact of media. In section 2, a mathematical model is discussed with schematic diagram along with notations, description and parametric values as described in Table 1. Local and

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global stability of the equilibrium points are discussed in section 3.1 and 3.2 respectively. Section 4 studies optimal control on the system. In section 5, numerical simulation of the model is carried out and interpretations are work out.

2. Mathematical Model

Here, a mathematical model has been formulated to study the impact of media on cleanliness drive in India as an application of *SEIR* model. The notations along with its description and parametric values are given in Table 1. The transmission diagram for cleanliness drive is shown in figure 1.

Every susceptible individuals (S) either act as passive cleaners (P) or active cleaners (A). β and η are the rates described for the passive and active cleaners respectively who are indulge in cleanliness related activities. Gradually, passive cleaners (P) tries to become active cleaners (A) which is mentioned by the rate γ. To boost up this activity of cleanliness among individuals media campaign is essential which is described as the rate u_1. Many times, it happens that this active cleaners (A) are forced to lose their interest and thus becomes a passive cleaners (P) which is described by the rate σ. B represents new recruitment rate and μ represents escape rate in the model. Now, from the figure 1 a set of nonlinear ordinary differential equations for cleanliness has been obtained.

Table 1. Notations and its parametric values.

Notations	Description	Parametric values
	State Variables	
$S(t)$	Number of susceptible individuals at some instant of time	30
$P(t)$	Number of passive cleaners at some instant of time t	16
$A(t)$	Number of active cleaners at some instant of time t	12
	Model Parameters	
B	New recruitment rate	12 %
β	Rate of individuals transferring to passive cleaners from susceptible compartment	46 %
γ	Rate of individuals transferring as active cleaners from passive cleaners	50 %
σ	Rate of individuals transferring again as passive cleaners from active cleaners	10 %
η	Rate of individuals transferring as active cleaners from susceptible compartment	40 %
μ	Escape rate	12 %
u_1	Rate of media as control on passive cleaners to become active cleaners to boost the cleanliness related activities	[0,1]

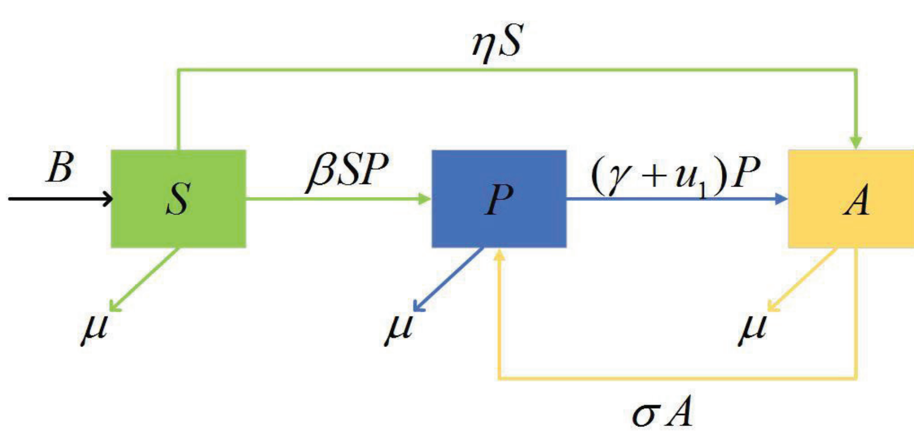

Figure 1. Transmission diagram of the model.

$$\begin{aligned}\frac{dS}{dt}&=B-\beta SP-\eta S-\mu S\\ \frac{dP}{dt}&=\beta SP-(\gamma+u_1)P-\mu P+\sigma A\\ \frac{dA}{dt}&=(\gamma+u_1)P+\eta S-\sigma A-\mu A\end{aligned} \tag{1}$$

Adding all above equations, we have

$$\frac{d}{dt}(S+P+A)=B-\mu(S+P+A)\geq 0$$

$$\therefore \lim_{t\to\infty}\sup(S+P+A)\leq\frac{B}{\mu}$$

$\therefore$ The feasible region of solution of (1) is

$$\Lambda=\left\{(S,P,A)/S+P+A\leq\frac{B}{\mu},S>0;P,A\geq 0\right\}$$

Thus, cleanliness equilibrium point $E_0=\left(\frac{B}{\mu},0,0\right)$.

Now, using next generation matrix method [1], a basic reproduction number is to be calculated. For this, let $\Im(X)$ denotes rate of new passive and active cleaners and $\nu(X)$ denotes rate of transfer of cleanliness activity among individuals.

Let $X=(P,A,S)$

$$\therefore \frac{dX}{dt}=\Im(X)-\nu(X)$$

where,

$$\Im(X)=\begin{bmatrix}\beta SP\\0\\0\end{bmatrix} \text{ and } \nu(X)=\begin{bmatrix}\gamma P+\mu P-\sigma A\\ -\gamma P-\eta S+\sigma A+\mu A\\ -B+\eta S+\mu S+\beta SP\end{bmatrix}$$

Now, the derivative of $\Im$ and ν at cleanliness equilibrium point E_0 gives matrices F and V of order 3×3 defined as,

$$F=\left[\frac{\partial\Im_i(E_0)}{\partial X_j}\right],\quad V=\left[\frac{\partial\nu_i(E_0)}{\partial X_j}\right] \text{ for } i,j=1,2,3$$

Hence,

$$F=\begin{bmatrix}\frac{\beta B}{\mu} & 0 & 0\\ 0&0&0\\0&0&0\end{bmatrix} \quad and \quad V=\begin{bmatrix}\gamma+\mu & -\sigma & 0\\ -\gamma & \sigma+\mu & -\eta\\ \frac{\beta B}{\mu} & 0 & \eta+\mu\end{bmatrix}$$

where, V is non-singular matrix.

Thus, the basic reproduction number (R_0) is given by

$$R_0=\frac{\beta B(\sigma+\mu)(\eta+\mu)}{B\beta\eta\sigma+(\eta+\mu)(\gamma+\mu+\sigma)\mu^2}$$

Now, on equating the set of equation (1) to zero, an endemic equilibrium point defined as E^* among individuals is obtained as

$E^*=\left(S^*,P^*,A^*\right)$

where,

$$S^*=\frac{1}{2\beta\mu(\eta+\mu+\sigma)}\times\left[\begin{array}{l}\beta B(\mu+\sigma)+(\eta\mu+\mu^2)(\gamma+\mu+\sigma)+\\ \left(\begin{array}{l}(\mu+\sigma)^2\left((\mu^2-B\beta)^2+\eta\mu^2(\eta+2\mu)\right)\\ +(\mu+\sigma)\left(\begin{array}{l}2\eta\gamma\mu^2(\eta+2\mu)\\ -2B\beta\mu\left(\eta(\gamma+\mu-\sigma)-\gamma\mu^2\right)\end{array}\right)\\ +\mu^2\left(\gamma^2(\eta+\mu)^2+\left(2\gamma\mu^3+\sigma^2\right)\right)\end{array}\right)^{\frac{1}{2}}\end{array}\right]$$

$$P^*=\frac{B-(\eta+\mu)S}{\beta S}=\frac{B-X_1S}{\beta S}\ ;\text{ where, } X_1=\eta+\mu$$

$$A^*=\frac{\gamma(B-X_1S)}{\beta X_2S}+\frac{\eta S}{X_2}\ ;\text{ where, } X_2=\sigma+\mu$$

3. Stability Analysis

In this section, the local and global stability of E_0 and E^* are to be studied.

3.1 Local stability

Theorem 1: (Stability of E_0) Cleanliness equilibrium point (E_0) is locally asymptotically stable.

Proof: At point E_0, the Jacobian matrix of the system (1) is,

$$J(E_0)=\begin{bmatrix}-(n+\mu) & -\frac{\beta B}{\mu} & 0\\ 0 & \frac{\beta B}{\mu}-(\gamma+\mu) & \sigma\\ \eta & \gamma & -(\sigma+\mu)\end{bmatrix}$$

The characteristic polynomial of above matrix is

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

where,

$$a_1 = \sigma + 3\mu + \gamma + \eta > 0$$
$$a_2 = \eta(\gamma + 2\mu + \sigma) + \mu(2\gamma + 3\mu + 2\sigma) > 0$$
$$a_3 = \frac{B\beta\eta\sigma + \eta\mu^2(\gamma+\sigma) + \mu^3(\eta+\gamma) + \mu^3(\mu+\sigma)}{\mu} > 0$$

Also $a_1 a_2 - a_3 > 0$.
$\therefore$ By Routh Hurwitz criteria [8], E_0 is locally asymptotically stable.

Theorem 2: (Stability of E^*) The endemic equilibrium point E^* is locally asymptotically stable if and only if
Proof: At point E^*, the Jacobian matrix of the system (1) is,

$$J(\mathrm{E}^*) = \begin{bmatrix} -\beta P^* - X_1 & -\beta S^* & 0 \\ \beta P^* & \beta S^* - X_3 & \sigma \\ \eta & \gamma & -X_2 \end{bmatrix}$$

where $X_3 = \gamma + \mu$; X_1 and X_2 stated earlier.
The characteristic polynomial of above matrix is

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

where,

$$a_1 = \beta(P^* - S^*) + X_1 + X_2 + X_3$$
$$a_2 = \beta X_2(P^* - S^*) + \beta(X_3 P^* - X_1 S^*) - \gamma\sigma + X_1X_2 + X_1X_3 + X_2X_3$$
$$a_3 = \beta\sigma(\eta S^* - \gamma P^*) + \beta X_2(X_3P^* - X_1S^*) - \gamma\sigma X_1 + X_1X_2X_3$$

Thus,
$a_1 > 0$ if and only if $P^* - S^* > 0$
$a_2 > 0$ if and only if $P^* - S^* > 0$ *and* $X_3P^* - X_1S^* > 0$
$a_3 > 0$ if and only if $\eta S^* - \gamma P^* > 0$ *and* $X_3P^* - X_1S^* > 0$
Hence collecting all conditions, we have,
$a_1, a_2, a_3 > 0$ if and only if $i)\ P^* - S^* > 0$,
$ii)\ X_3P^* - X_1S^* > 0, iii)\ \eta S^* - \gamma P^* > 0$
Also, we have $a_1a_2 - a_3 > 0$.
According to Routh-Hurwitz criteria, E^* is locally asymptotically stable if $a_1, a_2, a_3 > 0$ and $a_1a_2 - a_3 > 0$
Therefore, the endemic equilibrium point E^* is locally asymptotically stable if and only if
$i)\ P^* - S^* > 0, ii)\ X_3P^* - X_1S^* > 0, iii)\ \eta S^* - \gamma P^* > 0$.

3.2 Global stability

Theorem 3: (Stability of E_0) Cleanliness equilibrium point (E_0) is globally asymptotically stable.
Proof: Consider a Lyapunov function.
$L(t) = P(t) + A(t)$

$$\begin{aligned}\therefore L'(t) &= P'(t) + A'(t) \\ &= \beta SP - \gamma P - \mu P + \sigma A + \gamma P + \eta S - \sigma A - \mu A \\ &= \beta SP - \mu P + \eta S - \mu A \\ &= B - \eta S - \mu S - \mu P + \eta S - \mu A \\ &\le B - \mu\frac{B}{\mu} - \mu P - \mu A \\ &= -\mu(P + A) \le 0\end{aligned}$$

and $L'(t) = 0$ *if* $P = A = 0$
$\therefore$ By LaSalle's Invariance Principle [5], E_0 is globally asymptotically stable.
Theorem 4: (Stability of E^*) Endemic equilibrium point E^* is globally asymptotically stable.
Proof: Consider a Lyapunov function

$$L(t) = \frac{1}{2}\left[(S - S^*) + (P - P^*) + (A - A^*)\right]^2$$

$$\begin{aligned}\therefore L'(t) &= \left[(S - S^*) + (P - P^*) + (A - A^*)\right][S' + P' + A'] \\ &= \left[(S - S^*) + (P - P^*) + (A - A^*)\right]\left[B - \mu(S + P + A)\right]\end{aligned}$$

Taking $B = \mu S^* + \mu P^* + \mu A^*$

$$= -\mu\left[(S - S^*) + (P - P^*) + (A - A^*)\right]^2 \le 0$$

$\therefore E^*$ is globally asymptotically stable.

4. Optimal Control Model

Our aim is to involve maximum number of individuals in cleanliness drive. Media is effective medium to motivate all individuals in broad sense to join the drive actively. In this section, a control function is introduced to get maximum transfer rate from passive cleaners to active cleaners. The objective function with of the control variable is given by

$$J(u_1, \Omega) = \int_0^T \left(A_1S^2 + A_2P^2 + A_3A^2 + w_1u_1^2\right)dt \tag{2}$$

where, Ω is the set of all compartmental variables, A_1, A_2, A_3 and are non-negative weight constants for state variables S, P, A and control variable u_1 respectively.
Existence of optimal control variable is guaranteed since Ω is nonempty. Control set is convex and closed. One can easily see that right hand side of is bounded by linear function in state and control variables. The integrand of the objective function is convex and bounded below by $w_1 u_1^2 - 1$.
Now, we will calculate the value of control variable u_1 from $t = 0$ to $t = T$ such that

$$J\left(u_1(t)\right) = \min\left\{J\left(u_1^*, \Omega\right) / u_1 \in \phi\right\}$$

where ϕ = smooth function on the interval [0, 1].

Using Fleming and Rishel results [2], the optimal control denoted by u_1^* is obtained by collecting all the integrands of the objective function (2) using the lower bound and upper bound a_1 of the control variable respectively.

Now, using Pontrygin's principle [6], we construct a Lagrangian function consisting of state equations and adjoint variables $A_V = \left(\lambda_S, \lambda_P, \lambda_A\right)$ to maximize the objective function (2) as

$$\begin{aligned} L(\Omega, A_V) = {} & A_1 S^2 + A_2 P^2 + A_3 A^2 + w_1 u_1^2 \\ & + \lambda_S \left(B - \beta SP - \eta S - \mu S\right) \\ & + \lambda_P \left(\beta SP - (\gamma + u_1) P - \mu P + \sigma A\right) \\ & + \lambda_A \left((\gamma + u_1) P + \eta S - \sigma A - \mu A\right) \end{aligned} \tag{3}$$

Now, the partial derivative of the Lagrangian function with respect to each variable of the compartment gives us the adjoint equation such that

$$\begin{aligned} \overset{\circ}{\lambda}_S &= -\frac{\partial L}{\partial S} \\ &= -2A_1 S + \beta P\left(\lambda_S - \lambda_P\right) + \eta\left(\lambda_S - \lambda_A\right) + \mu \lambda_S \\ \overset{\circ}{\lambda}_P &= -\frac{\partial L}{\partial P} \\ &= -2A_2 P + \beta S\left(\lambda_S - \lambda_P\right) + \left(\gamma + u_1\right)\left(\lambda_P - \lambda_A\right) + \mu \lambda_P \\ \overset{\circ}{\lambda}_A &= -\frac{\partial L}{\partial A} \\ &= -2AA_3 + \sigma\left(\lambda_A - \lambda_P\right) + \mu \lambda_A \end{aligned}$$

The necessary condition for Lagrangian function to be optimal for control is

$$\frac{\partial L}{\partial u_1} = 2w_1 u_1 + P\left(\lambda_A - \lambda_P\right) = 0 \tag{4}$$

By solving equation (4) we have, $u_1 = \dfrac{P\left(\lambda_P - \lambda_A\right)}{2w_1}$.

So, the required optimal control condition is obtained as

$$u_1^* = \max\left(a_1, \min\left(b_1, \frac{P\left(\lambda_P - \lambda_A\right)}{2w_1}\right)\right)$$

where a_1 = lower bound and b_1 = upper bound.

5. Numerical Simulation

In this section, we will study numerical results of all compartments. Effect of media as control are studied on all compartments.

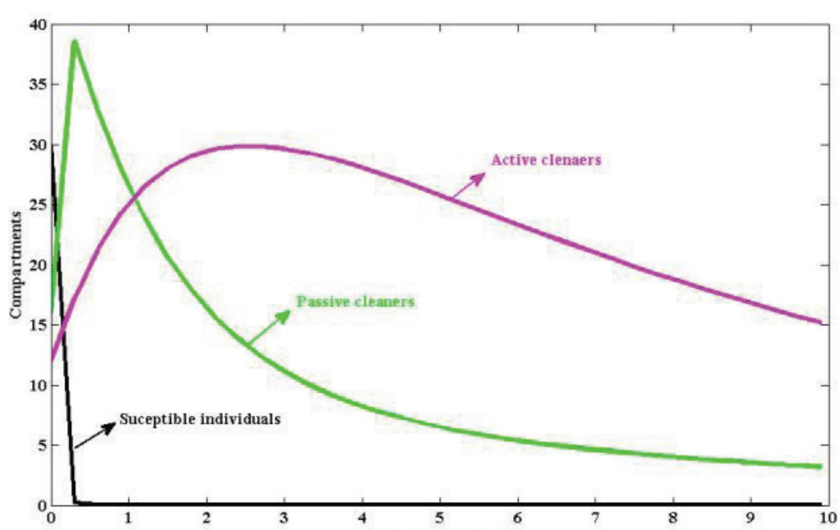

Figure 2. Dynamics of compartments without control.

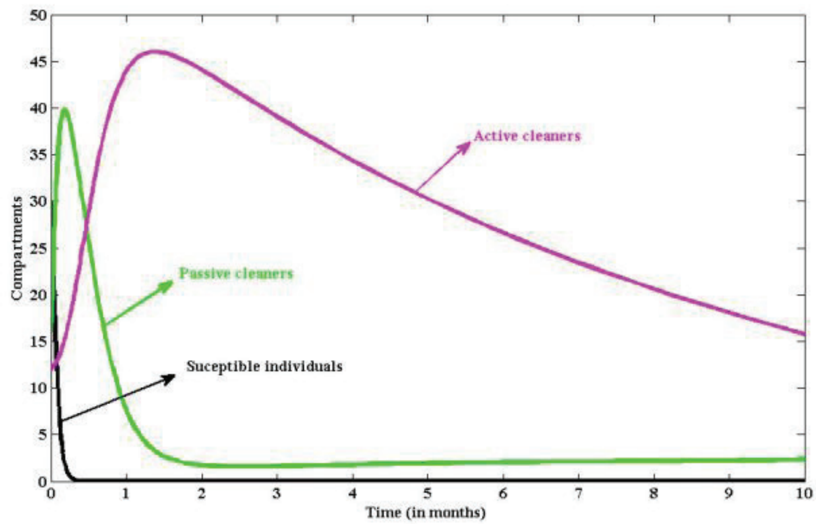

Figure 3. Dynamics of compartments with control.

Figure 2 shows that in first month of cleanliness drive, susceptible individuals become passive cleaners as well as active cleaners. But, there are more passive cleaners then active cleaners in first month. It also indicates that it takes more than one month to have more active cleaners then passive cleaners in cleanliness drive. It reflects laziness of individuals in cleanliness drive. Figure 3 shows that under influence of media as control, susceptible individuals quickly join cleanliness activities as active and passive cleaners. It indicates that almost all individuals become active cleaners in a month and remains active for longer time.

It is observed from the figure 4 that with control, transmission rate of passive cleaners into active is higher compared to without control. It indicates that passive cleaners are inspired by media and they get actively involved in the drive. Media campaign increase passive cleaners by 12% as shown in figure 5.

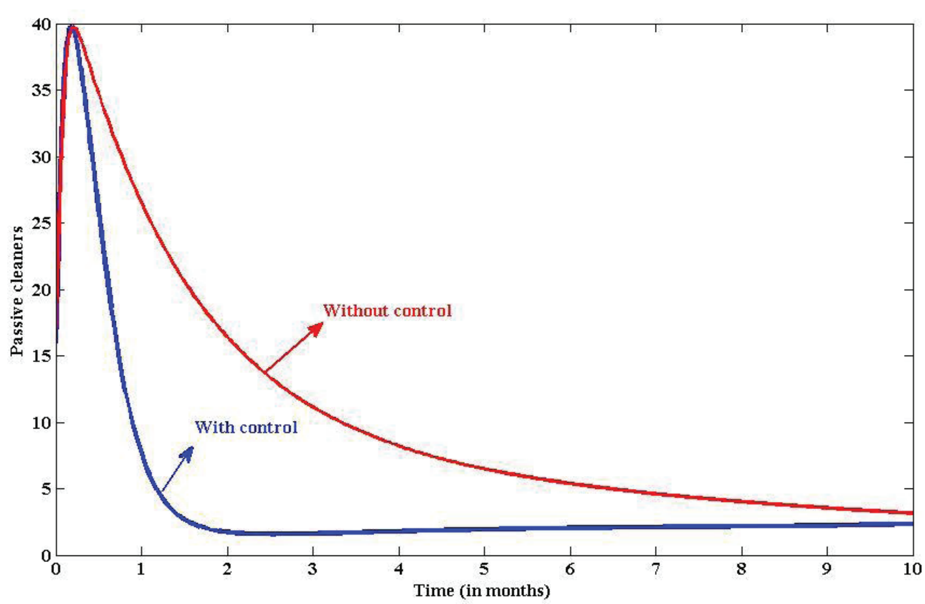

Figure 4: Impact of media on passive cleaners.

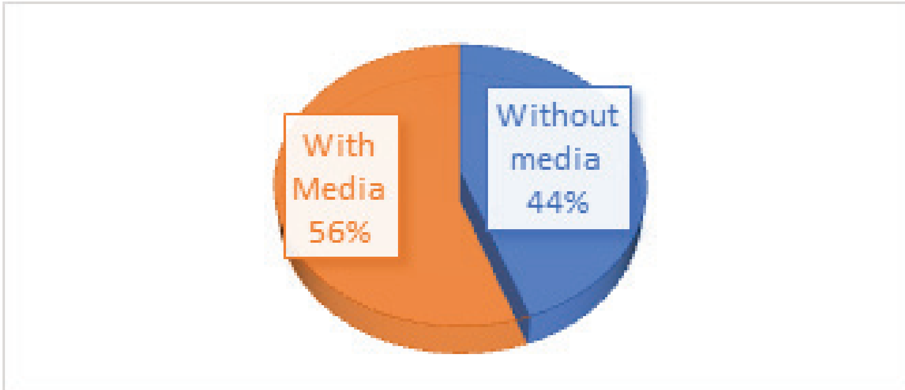

Figure 5. Effect of media on passive cleaners.

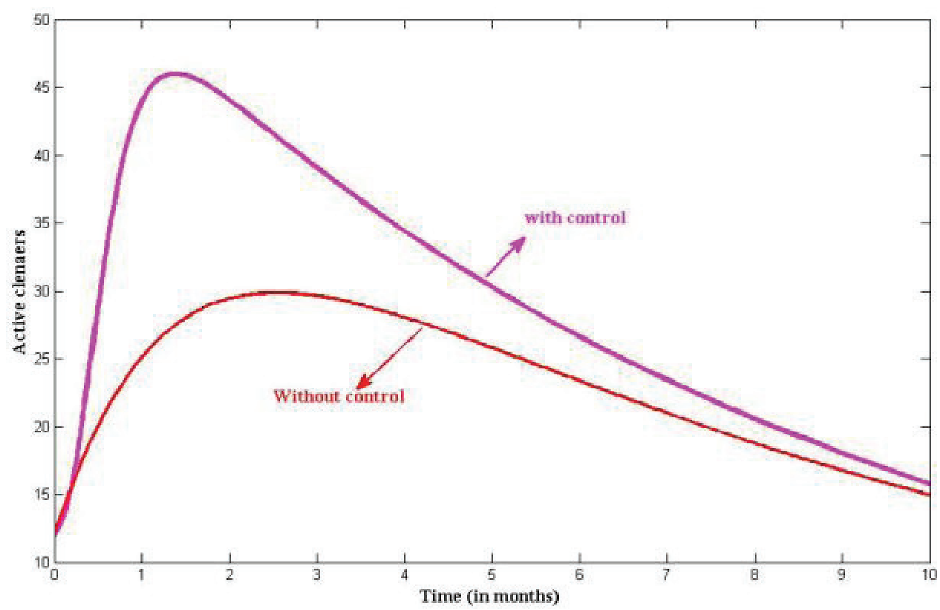

Figure 6. Impact of media on active cleaners.

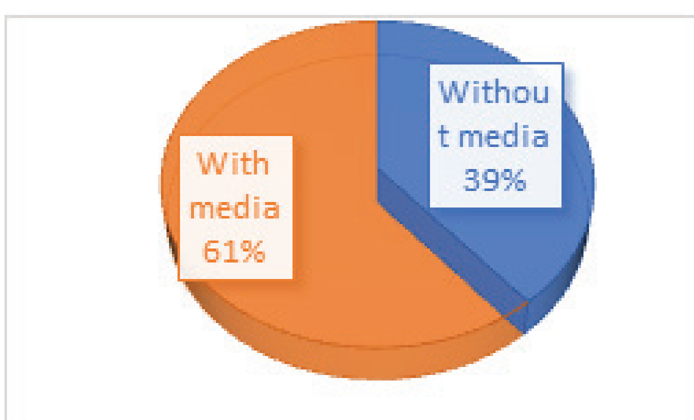

Figure 7. Effect of media on active cleaners.

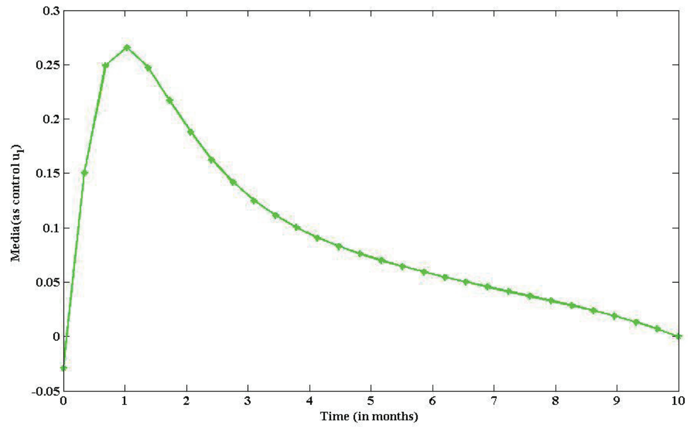

Figure 8. Control variable verses Time (in months).

Figure 6 indicates increment of active cleaners with control compare to without control. It shows that active cleaner increased by 100% in mid days of the first month of the drive with media campaign. Figure 7 shows that effect of media plays a vital role as it increases the active cleaners from 39% to 61%. One must put there step forward in making our country clean. Figure 8 shows the significance of media in the cleanliness drive. It can be seen from the figure that 27% awareness is required among individuals through media campaigning. It also indicates that after awareness about cleanliness among individuals, control can be gradually decreased.

6. Conclusion

In this paper, a mathematical model of cleanliness drive in India is studied. One can conclude that media is essential for the awareness of cleanliness among individuals since passive cleaners become active cleaners in cleanliness related activities comparatively at a higher rate when media impact is observed. Our first contribution for the clean India is to keep our house clean by proper wastage management of our house as per the instruction of the corporation. We should also actively participate in the cleanliness drive to clean our streets, school, college, office, public place etc. to achieve vision of 'Clean India'.

This model can be extended to various social and health related issues. Infectious diseases usually spread in unhygienic environment. Cleanliness play vital role to control it. Hence, the model can be extended to study the effect of cleanliness in controlling infectious diseases.

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