

Note on Surface Wave in Fibre-Reinforced Medium

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Abstract: The wave velocity equations are derived for the particular cases of surface waves –Rayleigh, and Love types using direct method along with numerical results. Results are compared with results of classical theory when reinforced parameters tends to zero.

Keywords: Decoupling, fibre-reinforced media, transversely isotropic media, surface waves, half space.

1. Introduction

After the pioneer work of Rayleigh [9], many investigators have studied the problem extensively under different conditions. They have contributed in a wide range towards its application in various fields e.g. Seismology, geophysics, acoustics, telecommunications and environmental sciences etc. A good amount of literature is to be found in the standard books of Ben-menahem and Singh [3], Bullen [4], Ewing et al. [6] and Love [7].

In most of the previous investigations the effect of reinforcement has been neglected. This concept was introduced by Belfield et al. [2]. The characteristic property of a reinforced composite is that its components act together as a single anisotropic unit as long as they remain in the elastic conditions. The reinforcement introduces anisotropy in the medium which becomes transversely isotropic. For wave propagation in an isotropic homogeneous medium; the introduction of displacement potentials leads to the decoupling of P, SV & SH motions. The decoupling cannot be achieved for wave propagation in transversely isotropic media (see, e.g., Rahman & Ahmad [8]). Stresses produced in a fibre-reinforced half-space due to moving load was discussed by Chattopadhyay and Venkateswarlu [5].

For studying the propagation of surface waves in fiber-reinforced anisotropic elastic media, Sengupta & Nath [10] used the method of potential to decouple the P & SV motions which is not justified. Therefore Singh [11] pointed out that the results of Sengupta and Nath (2001) are in error regarding Rayleigh and Stoneley waves. Same mistake was done by Abd-Alla et al. [1] to investigate the surface waves in fibre-reinforced anisotropic elastic medium under gravity field.

In this paper, the author studies the propagation of surface waves in fibre-reinforced anisotropic elastic solid media for Rayleigh waves and Love waves using direct method. Numerical results for Rayleigh and Love surface waves are derived for specific materials and graphs are also drawn to show the effect of reinforced parameters on the speed of surface waves.

2. Basic Equations

The constitutive equations for a fibre-reinforced linearly elastic anisotropic medium with respect to the reinforcement direction \vec{a} are (Belfield et al. [2])

$$\begin{aligned} \sigma_{ij} = & \lambda e_{kk} \delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km} \delta_{ij} + e_{kk} a_i a_j) \\ & + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j, \end{aligned} \quad (2.1)$$

where σ_{ij} are components of stress; e_{ij} are components of strain; λ, μ_T are elastic constants; $\alpha, \beta, (\mu_L - \mu_T)$ are reinforcement parameters and $\vec{a} = (a_1, a_2, a_3), a_1^2 + a_2^2 + a_3^2 = 1$. We choose the fibre-direction as $\vec{a} = (1, 0, 0)$. The strain components can be expressed in terms of the displacements u_i as

$$e_{ij} = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \quad (2.2)$$

Equation (2.1) then yields

$$\begin{aligned} \sigma_{11} &= A_{11} \frac{\partial u_1}{\partial x_1} + A_{12} \frac{\partial u_2}{\partial x_2} + A_{13} \frac{\partial u_3}{\partial x_3}, \\ \sigma_{22} &= A_{12} \frac{\partial u_1}{\partial x_1} + A_{22} \frac{\partial u_2}{\partial x_2} + A_{23} \frac{\partial u_3}{\partial x_3}, \\ \sigma_{33} &= A_{12} \frac{\partial u_1}{\partial x_1} + A_{23} \frac{\partial u_2}{\partial x_2} + A_{22} \frac{\partial u_3}{\partial x_3}, \\ \sigma_{21} = \sigma_{12} &= \mu_L \left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right), \end{aligned}$$

$$\begin{aligned}\sigma_{13} = \sigma_{31} &= \mu_L \left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right), \\ \sigma_{23} = \sigma_{32} &= \mu_T \left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} \right),\end{aligned}\quad (2.3)$$

where $A_{11} = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta$, $A_{12} = \lambda + \alpha$,

$$A_{22} = \lambda + 2\mu_T, A_{23} = \lambda. \quad (2.4)$$

The equations of motion without body forces are

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2} \quad (ij=1,2,3), \quad (2.5)$$

where ρ is the density of the elastic medium. Using equation (2.3) in equation (2.5)

$$\begin{aligned}A_{11} \frac{\partial^2 u_1}{\partial x_1^2} + (A_{12} + \mu_L) \frac{\partial^2 u_2}{\partial x_1 \partial x_2} + (A_{12} + \mu_L) \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + \mu_L \frac{\partial^2 u_1}{\partial x_2^2} + \mu_L \frac{\partial^2 u_1}{\partial x_3^2} &= \rho \frac{\partial^2 u_1}{\partial t^2}, \\ A_{22} \frac{\partial^2 u_2}{\partial x_2^2} + (A_{12} + \mu_L) \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + (A_{23} + \mu_T) \frac{\partial^2 u_3}{\partial x_1 \partial x_2} + \mu_L \frac{\partial^2 u_2}{\partial x_1^2} + \mu_L \frac{\partial^2 u_2}{\partial x_3^2} &= \rho \frac{\partial^2 u_2}{\partial t^2}, \\ A_{22} \frac{\partial^2 u_3}{\partial x_3^2} + (A_{23} + \mu_T) \frac{\partial^2 u_2}{\partial x_2 \partial x_3} + (A_{12} + \mu_L) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + \mu_L \frac{\partial^2 u_3}{\partial x_1^2} + \mu_T \frac{\partial^2 u_3}{\partial x_2^2} &= \rho \frac{\partial^2 u_3}{\partial t^2},\end{aligned}\quad (2.6)$$

For plane strain deformation in the $x_1 x_2$ -plane, put $\frac{\partial}{\partial x_3} = 0, u_3 = 0$ in first two equations of (2.6) we get;

$$\begin{aligned}A_{11} \frac{\partial^2 u}{\partial x^2} + B_2 \frac{\partial^2 v}{\partial x \partial y} + B_1 \frac{\partial^2 u}{\partial y^2} &= \rho \frac{\partial^2 u}{\partial t^2}, \\ A_{22} \frac{\partial^2 v}{\partial y^2} + B_2 \frac{\partial^2 u}{\partial x \partial y} + B_1 \frac{\partial^2 v}{\partial x^2} &= \rho \frac{\partial^2 v}{\partial t^2}.\end{aligned}\quad (2.7)$$

The third equation of (2.6) is identically satisfied and here we have used the notation $x_1 = x, x_2 = y, u_1 = u, u_2 = v, B_1 = \mu_L, B_2 = A_{12} + \mu_L$.

3. Propagation of Rayleigh Waves

We consider a fibre-reinforced elastic half-space with free surface as x - z plane y -axis pointing into the half-space such that $-\infty < x, z < \infty; 0 \leq y < \infty$. For Rayleigh waves of circular frequency ω , wave number k and phase velocity C_R

propagating in the x-direction through the fibre-reinforced anisotropic half-space. We may assume the solution of equation (2.7) as

$$\begin{aligned} \mathbf{u} &= \mathbf{U}e^{-kqy} e^{ik(C_R t - x)}, \\ \mathbf{u} &= \mathbf{V}e^{-kqy} e^{ik(C_R t - x)}, \end{aligned} \quad (3.1)$$

where U and V are the amplitude factors, $i = \sqrt{-1}$ and q is assumed to be real and positive. Putting the values of the displacements in equation (2.7), we get

$$\begin{aligned} (\rho C_R^2 - A_{11} + B_1 q^2)U + iqB_2 V &= 0, \\ iqB_2 U + (\rho C_R^2 - B_1 + A_{22} q^2)V &= 0. \end{aligned} \quad (3.2)$$

The values of q may be obtained from

$$\begin{vmatrix} \rho C_R^2 - A_{11} + B_1 q^2 & iqB_2 \\ iqB_2 & \rho C_R^2 - B_1 + A_{22} q^2 \end{vmatrix} = 0,$$

which on simplification, we get

$$\begin{aligned} & -\{(A_{22} + B_1)\rho C_R^2 - A_{11}A_{22} - B_1^2 + B_2^2\} \pm \\ & q_1^2, q_2^2 \frac{-\left[\{(A_{22} + B_1)\rho C_R^2 - A_{11}A_{22} - B_1^2 + B_2^2\}^2 - 4B_1A_{22}(\rho C_R^2 - A_{11})(\rho C_R^2 - B_1)\right]^{1/2}}{2B_1A_{22}}. \end{aligned} \quad (3.3)$$

Therefore the solution (3.1) can be written as

$$\begin{aligned} \mathbf{u} &= (U_{11}e^{-q_1 ky} + U_{12}e^{-q_2 ky})e^{-ik(C_R t - x)}, \\ \mathbf{v} &= (V_{11}e^{-q_1 ky} + V_{12}e^{-q_2 ky})e^{-ik(C_R t - x)}, \end{aligned} \quad (3.4)$$

(U_{11}, U_{12}) and (V_{11}, V_{12}) are not independent but are connected by equation (3.2) for $q = q_1$ and q_2 . Taking second member of equation (3.2), we get

$$\frac{U_{11}}{V_{11}} = m_1, \frac{U_{12}}{V_{11}} = m_2, \quad (3.5)$$

where

$$m_1 = iM_1, m_2 = iM_2 \quad (3.6)$$

$$M_1 = \frac{\rho C_R^2 - B_1 + A_{22} q_1^2}{q_1 B_2}, \quad (3.7)$$

and

$$M_2 = \frac{\rho C_R^2 - B_1 + A_{22}q_2^2}{q_2 B_2}, \quad (3.8)$$

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where q_1 and q_2 are real and positive quantities defined in equation (3.3). Therefore the displacements in equation (3.4) can be written as

$$\begin{aligned} u &= \left(m_1 V_{11} e^{-q_1 k y} + m_2 V_{12} e^{-q_2 k y} \right) e^{ik(C_R t - x)}, \\ v &= \left(V_{11} e^{-q_1 k y} + V_{12} e^{-q_2 k y} \right) e^{ik(C_R t - x)}. \end{aligned} \quad (3.9)$$

4. Boundary Conditions

The displacements in equation (3.9) must satisfy the boundary conditions,

$$\sigma_{21} = \sigma_{22} = 0, \text{ at } y = 0, \quad (4.1)$$

where σ_{21} and σ_{22} are defined in equation (2.3). For plane deformation in the $x_1 x_2$ -plane, $\frac{\partial}{\partial x_3} = 0, u_3 = 0$. Equation (2.3) then yields

$$\begin{aligned} \sigma_{21} &= \mu_L \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \\ \sigma_{22} &= A_{12} \frac{\partial u}{\partial x} + A_{22} \frac{\partial v}{\partial y}, \end{aligned} \quad (4.2)$$

where we have used the notation $x_1 = x, x_2 = y, u_1 = u$ and $u_2 = v$.

From equations (3.9) and (4.2), we get

$$\begin{aligned} (m_1 q_1 + i) V_{11} + (m_2 q_2 + i) V_{12} &= 0, \\ (A_{12} m_1 i + A_{22} q_1) V_{11} + (m_2 A_{12} i + A_{22} q_2) V_{12} &= 0. \end{aligned} \quad (4.3)$$

Eliminating V_{11} and V_{12} from equation (4.3), we get

$$\begin{vmatrix} m_1 q_1 + i & m_2 q_2 + i \\ A_{12} m_1 i + A_{22} q_1 & m_2 A_{12} i + A_{22} q_2 \end{vmatrix} = 0. \quad (4.4)$$

On simplification, equation (4.4) becomes

$$\left[(m_1 q_1 + i)(m_2 A_{12} i + A_{22} q_2) - (m_2 q_2 + i)(A_{12} m_1 i + A_{22} q_1) \right] = 0. \quad (4.5)$$

This is the velocity equation for Rayleigh-waves in a fibre-reinforced elastic medium.

5. Particular Case

If we put $\alpha = \beta = 0$ and $\mu_L = \mu_T = \mu$, the elastic coefficients become

$$\begin{aligned} A_{11} &= \lambda + 2\mu, \quad A_{12} = \lambda, \quad B_1 = \mu, \\ A_{22} &= \lambda + 2\mu, \quad A_{23} = \lambda, \quad B_2 = \lambda + \mu. \end{aligned} \quad (5.1)$$

From equations (3.7) and (3.8) we get

$M_1 = q_1$, $M_2 = \frac{1}{q_2}$, then

$$q_1^2 = \left(1 - \frac{C_R^2}{C_S^2}\right), \quad q_2^2 = \left(1 - \frac{C_R^2}{C_p^2}\right), \quad (5.2)$$

where

$$C_p^2 = \frac{\lambda + 2\mu}{\rho} \quad \text{and} \quad C_S^2 = \frac{\mu}{\rho}. \quad (5.3)$$

λ, μ are Lamé's constants. Using equations (5.1)-(5.3), the velocity equation (4.5) reduces to

$$\left(\frac{C_R^2}{C_S^2} - 2\right)^2 = 4 \left(1 - \frac{C_R^2}{C_p^2}\right)^2 \left(1 - \frac{C_R^2}{C_S^2}\right)^2. \quad (5.4)$$

which is the Rayleigh wave velocity equation in isotropic media.

6. Propagation of Love Waves

Consider an isotropic layer of fibre-reinforced elastic medium of thickness H over a homogeneous anisotropic fibre-reinforced elastic half space. The surface of contact in plane $x_2 = 0$ and x_2 axis is directed vertically downwards. The wave is assumed to propagate along the x_1 - direction. An antiplane strain equation of motion for Love waves, in the upper layer is obtained from third equation of (2.6) by putting $u_1 = u_2 = 0$, $u_3 = w$, $x_1 = x$, $x_2 = y$, $x_3 = z$ and $\frac{\partial}{\partial x_s} = 0$, as

$$\mu_L' \frac{\partial^2 w'}{\partial x^2} + \mu_T' \frac{\partial^2 w'}{\partial y^2} = \rho' \frac{\partial^2 w'}{\partial t^2}, \quad (6.1)$$

where ρ' is density, w' is displacement along z -axis and (μ_L', μ_T') are reinforced anisotropic elastic parameters for the layer. The equation of motion for the half space is

$$\mu_L \frac{\partial^2 w}{\partial x^2} + \mu_T \frac{\partial^2 w}{\partial y^2} = \rho \frac{\partial^2 w}{\partial t^2}, \quad (6.2)$$

where μ_L, μ_T and ρ are the corresponding quantities in the half space.

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7. Solution of the Equation of Motion

We assume a solution of equation (6.1) for plane harmonic waves propagating in the upper layer along the x –direction in the form

$$w' = W' e^{i\omega \left(t - \frac{x}{C_L} \right) - p' y}, \quad (7.1)$$

where W' is the amplitude factor, ω and C_L are circular frequency and phase velocity respectively in the layer. Substituting equation (7.1) in equation (6.1) we get

$$\frac{\omega^2}{C_L^2} \mu_L' - \mu_T' p'^2 = \rho' \omega^2. \quad (7.2)$$

From equation (7.2), p' can be written as $p' = \pm i p_1'$ where

$$p_1' = \frac{\omega}{C_L} \sqrt{\frac{\rho' C_L^2 - \mu_L'}{\mu_T'}} = K \sqrt{\frac{\rho' C_L^2 - \mu_L'}{\mu_T'}}, K = \frac{\omega}{C_L}. \quad (7.3)$$

Therefore equation (7.1) can be written as

$$w'(x, y, t) = W_1' e^{i\omega \left(t - \frac{x}{C_L} \right) - i p_1' y} + W_2' e^{i\omega \left(t - \frac{x}{C_L} \right) + i p_1' y}. \quad (7.4)$$

Similarly the solution of equation (6.2) for half-space is

$$w(x, y, t) = W_1 e^{i\omega \left(t - \frac{x}{C_L} \right) - i p_1 y} + W_2 e^{i\omega \left(t - \frac{x}{C_L} \right) + i p_1 y}, \quad (7.5)$$

where

$$p_1^2 = K^2 \left(\frac{\rho C_L^2 - \mu_L}{\mu_T} \right). \quad (7.6)$$

For the existence of Love type surface waves it is necessary that in equation (7.5), $w(x, y, t) \rightarrow 0$ as $y \rightarrow \infty$. Therefore p_1 should be purely imaginary.

$$\text{Let } p_1 = i p_2 = i \left(K \sqrt{\frac{\mu_L - \rho C_L^2}{\mu_T}} \right). \quad (7.7)$$

Equation (7.5) then shows that

$$W_1 = 0. \quad (7.8)$$

8. Boundary Conditions

The following boundary conditions must be satisfied

$$\begin{aligned} \text{I. } \sigma'_{23} &= 0 \text{ at } y = -H, \\ \text{II. } w' &= w_3 \text{ at } y = 0, \\ \text{III. } \sigma'_{23} &= \sigma_{23} \text{ at } y = 0. \end{aligned} \quad (8.1)$$

σ_{23} is defined in equation (2.3) and can be written as (putting, $u_2 = u_1 = 0, u_3 = w, x_1 = x, x_2 = y, x_3 = z$ and $\frac{\partial}{\partial x_3} = 0$)

$$\sigma_{23} = \mu_T \frac{\partial w}{\partial y};$$

$$\text{Similarly } \sigma'_{23} = \mu'_T \frac{\partial w'}{\partial y}. \quad (8.2)$$

From equations (7.4) – (8.2), we get

$$\begin{aligned} W'_1 e^{ip_1 H} - W'_2 e^{-ip_1 H} &= 0, \\ W'_1 + W'_2 - W_2 &= 0, \\ \mu'_T p_1 (W'_1 - W'_2) + \mu_T p_1 W_2 &= 0. \end{aligned} \quad (8.3)$$

Eliminating $W'_1, W'_2,$ and W_2 from equation (8.3). On simplification, we obtain

$$i \tan(p_1 H) = \frac{\mu'_T p_1}{p_1 \mu_T}. \quad (8.4)$$

Substituting for p_1 and p_1 from equations (7.4) and (7.8) in equation (8.4), we get

$$\tan \left[\left(\frac{\rho C_L^2 - \mu'_L}{\mu'_T} \right)^{1/2} KH \right] = \frac{(\mu_L - \rho C_L^2)^{1/2}}{\left(\frac{\rho C_L^2 - \mu'_L}{\mu'_T} \right)^{1/2}} \cdot \frac{\mu'_T}{\mu_T}. \quad (8.5)$$

This is the frequency equation for Love waves in an anisotropic fibre-reinforced elastic medium.

9. Particular Case

If we put $\mu_L = \mu_T = \mu$, in equation (8.5), we get

$$\tan \left[\left(\frac{C_L^2}{C_S^2} - 1 \right)^{1/2} KH \right] = \frac{\mu}{\rho} \cdot \frac{\left(1 - \frac{C_L^2}{C_S^2} \right)^{1/2}}{\left(\frac{C_L^2}{C_S^2} \right)^{1/2}}, \quad (9.1)$$

where $C_s^2 = \frac{\mu}{\rho}$ and $C_s^2 = \frac{\mu}{\rho}$,

which is the classical frequency equation of Love waves in a homogeneous elastic layer over a isotropic half space.

10. Numerical Results and Discussion

To study the effect of reinforcement on the velocity of Rayleigh waves we use the following numerical values for the physical constant (Chattopadhyay, [5])

$$\begin{aligned} \lambda &= 5.65 \times 10^9 \text{ Nm}^{-2}, & \mu_L &= 5.65 \times 10^9 \text{ Nm}^{-2}, \\ \mu_T &= 2.46 \times 10^9 \text{ Nm}^{-2}, & \alpha &= -1.28 \times 10^9 \text{ Nm}^{-2}, \\ \beta &= 220.90 \times 10^9 \text{ Nm}^{-2} & \rho &= 7800 \text{ kg m}^{-3}. \end{aligned}$$

We obtain the numerical values of constants from equation (2.4) using above values as

$$\begin{aligned} A_{11} &= 241.71 \times 10^9 \text{ Nm}^{-2}, & A_{12} &= 4.387 \times 10^9 \text{ Nm}^{-2}, \\ A_{22} &= 10.57 \times 10^9 \text{ Nm}^{-2}, & A_{23} &= 5.65 \times 10^9 \text{ Nm}^{-2}, \\ B_1 &= 5.66 \times 10^9 \text{ Nm}^{-2}, & B_2 &= 10.03 \times 10^9 \text{ Nm}^{-2}. \end{aligned}$$

Making use of equations (3.3) and (3.5)-(3.8) in equation (4.5), the velocity equation for Rayleigh waves can be modified as

$$\begin{aligned} A_{22} \left(\frac{\rho C_R^2}{B_1} - 1 \right) & \left\{ \left(\frac{\rho C_R^2}{B_1} \right) + \frac{B_2 A_{12}}{B_1 A_{22}} - \frac{A_{12}}{A_{22}} - \frac{A_{11}}{B_1} \right\}^2 - B_1 \left(\frac{\rho C_R^2}{B_1} - \frac{A_{12}}{A_{22}} \right) \\ & \times \left(\frac{\rho C_R^2}{B_1} - \frac{A_{12}}{A_{22}} \right) \left(\frac{\rho C_R^2}{B_1} + \frac{B_2}{B_1} - \frac{A_{11}}{B_1} - 1 \right)^2 = 0. \end{aligned} \quad (10.1)$$

Using these values in equation (10.1), we obtain the value of wave velocity as $\frac{\rho C_R^2}{\mu_L}$ (dimensionless) = 42.38574 .

μ_L

$$\sqrt{\frac{\rho C_R^2}{\mu_L}} = 6.51043$$

$$C_R^2 = 307.56832 \times 10^5 \text{ m} / \text{s},$$

$$C_R = 5.545884 \text{ km} / \text{s},$$

The values calculated by Sengupta and Nath [10] for velocity of Rayleigh waves are not coincide of our results however same values of physical quantities are used for numerical calculations. It is clear the above numerical results of Rayleigh waves velocity in a fibre reinforced elastic medium is considerably higher than the Rayleigh waves velocity in isotropic medium. In this reference terrestrial Rayleigh waves is about 3 km /s(Love [7], p.160).

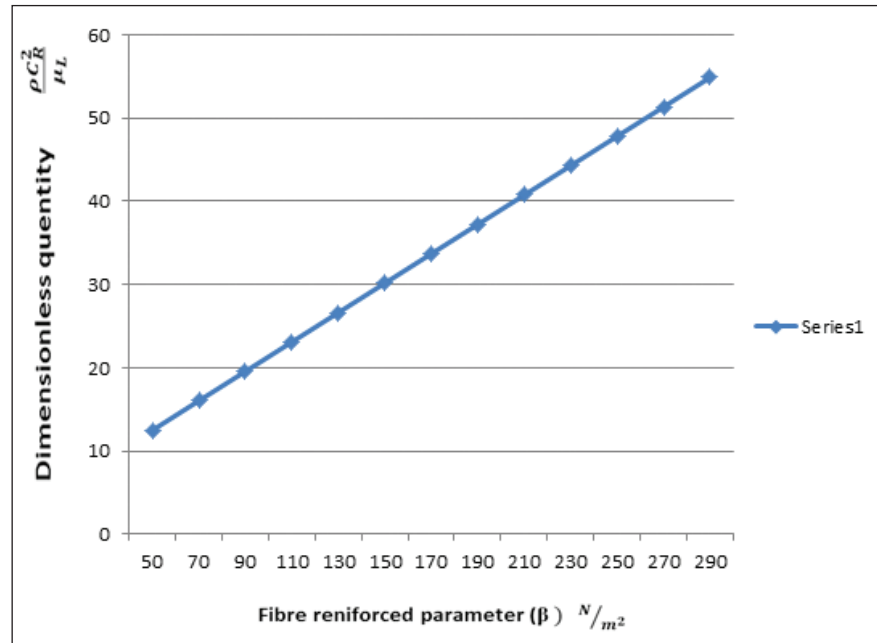
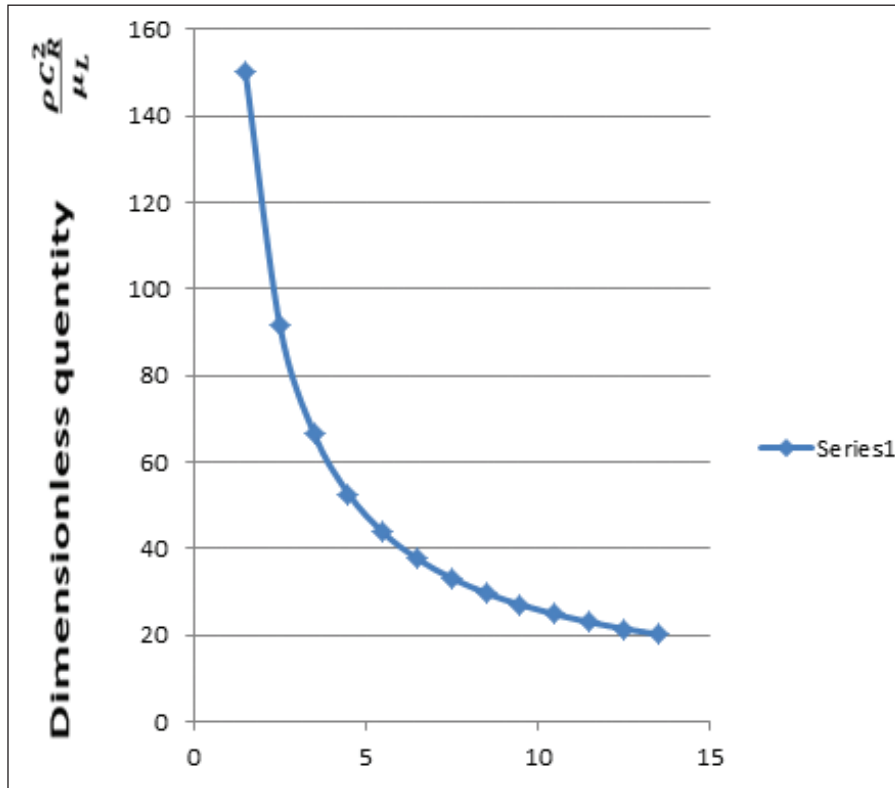


Figure 1: Variation of dimensionless quantity $\frac{\rho C_R^2}{\mu_L}$ with fibre reinforced parameter β .

Figure 1 Shows increase in β shows increase in $\frac{\rho C_R^2}{\mu_L}$ others parameter remains constant. There is linear relationship between dimensionless quantity $\frac{\rho C_R^2}{\mu_L}$ and fibre reinforced parameter β in m^{-2} .

Figure 2 shows the variation of dimensionless quantity $\frac{\rho C_R^2}{\mu_L}$ with elastic parameter μ_L in Nm^{-2} exponentially .

Equation (8.5) shows that C_L is dependent on the particular value of k and not a fixed constant. If k is small $C_L^2 \rightarrow \frac{\mu_L}{\rho}$ and while if k is large $C_L^2 \rightarrow \frac{\mu_L}{\rho}$.



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Figure 2: Variation of dimensionless quantity $\frac{\rho C_R^2}{\mu_L}$ with elastic parameter μ_L .

It is clear for the existence of Love waves p_1 should be imaginary and p_2 is real satisfied if $(\mu_L / \rho) < C_L^2 < (\mu_L / \rho)$ depends upon the reinforced parameters μ_L and ρ . The upper limit of C_L for the existence of Love waves in different elastic media is given as.

Fibre reinforced medium $\sqrt{\mu_L / \rho} = 0.851 \text{ km / s}$

Aluminum $\sqrt{\mu_L / \rho} = 4.5513 \text{ km / s}$

Steel $\sqrt{\mu / \rho} = 3.145 \text{ km / s}$

Gold(24 c) $\sqrt{\mu / \rho} = 4.6382 \text{ km / s}$

Copper(cast) $\sqrt{\mu / \rho} = 2.947 \text{ km / s}$

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$$\text{Copper (ore)} \quad \sqrt{\mu / \rho} = 4.238 \text{ km / s}$$

$$\text{Silicon} \quad \sqrt{\mu / \rho} = 5.649 \text{ km / s}$$

It is clear from the numerical values of upper limit of C_L for the existence of propagation of Love waves in the fibre- reinforced medium decreases in compression with the upper limits in the other elastic medium.

Figure 3 shows that upper limit of C_L in km/s for the existence of Love waves in the medium increases as μ_L increases, density remains constant .

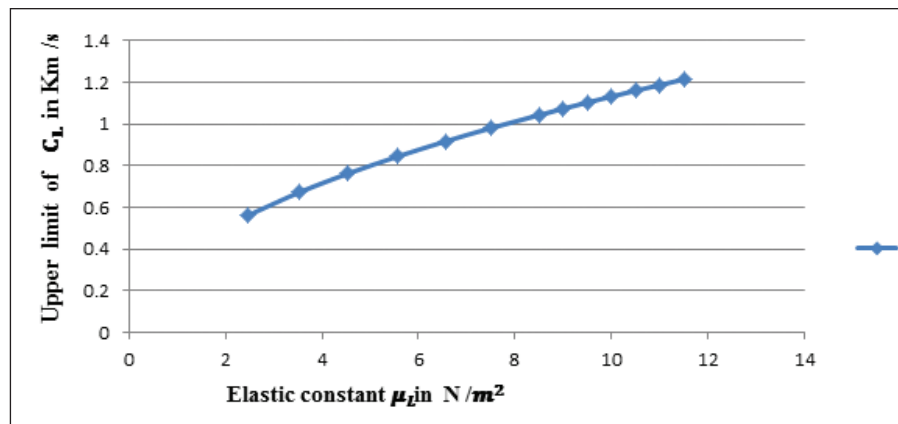


Figure 3: Variation of upper limit of C_L with elastic constant μ_L .

11. Conclusion

We conclude that the Rayleigh waves velocity in a fibre-reinforced elastic medium is higher than the Rayleigh wave velocity in isotropic media. The upper limit for the existence of Love waves in fibre-reinforced medium is less than other elastic medium .Hence the speed of Rayleigh waves and Love waves are affected by reinforced parameters. In the absence of reinforced parameters the velocity equation of each surface waves coincide with the classical results for isotropic elastic medium.

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