Math. J. Interdiscip. Sci..Vol. 7, No.1, Sep. 2018, pp.21–28

CHITKARA

Mathematical Journal of Interdisciplinary Sciences

Journal homepage: https://mjis.chitkara.edu.in/

Homotopy Analysis Approach of Boussinesq Equation for Infiltration Phenomenon in Unsaturated Porous Medium

Mahendra A Patel[1] and N B Desai[2*]

[1]*Government Engineering College, Gandhinagar, Gujarat*
[2]*A. D. Patel Institute of Technology, Karamsad, Gujarat*

*Email: drnbdesai@yahoo.co.in

ARTICLE INFORMATION

Received: 09 July 2018
Revised: 18 July 2018
Accepted: 10 August 2018

Published online: September 6, 2018

Keywords:
Groundwater, Infiltration, Unsaturated soil, Homotopy analysis method

https://doi.org/10.15415/mjis.2018.71004

ABSTRACT

Boussinesq's equation is one-dimensional nonlinear partial differential equation which represents the infiltration phenomenon. This equation is frequently used to study the infiltration phenomenon in unsaturated porous medium. Infiltration is the process in which the groundwater of the water reservoir has entered in the unsaturated soil through vertical permeable wall. An approximate analytical solution of nonlinear partial differential equation is presented by homotopy analysis method. The convergence of homotopy analysis solution is discussed by choosing proper value of convergence control parameter. The solution represents the height of free surface of infiltrated water.

1. Introduction

The fluid flow in porous media has great importance in various fields of engineering and science. In particular, the groundwater flow is very important part of fluid mechanics, hydrology, water resources engineering, irrigation engineering, etc. [15, 19, 20, 22, 23]. In this work, we examined the fluid flow problem in groundwater infiltration. Infiltration is the process in which water on the ground surface enters into unsaturated soils and pass into rocks through cracks and interstices. If the storage for the additional water had been available then the infiltration process can continue for a long time. The availability of additional water into the soil is dependent on the porosity of the soil. Once water has infiltrated into the soil it may stay in soil until it gradually evaporated, absorbed by plant roots and later transpired. The rate of infiltration process is dependent on different factors like as texture and structure of soil, storage capacity of soil, the depth of water reservoirs, the amount of plant over the region, etc.

Many researchers have been discussed various problems of groundwater infiltration like as Troch *et al.* [13] have derived an expression for mean water table height on the basis of hydraulic groundwater theory by means of Boussinesq equation, Govindaraju and Koelliker [3] have developed the expression for the flow rate from the stream to the aquifer, Hogarth *et al.* [4] have discussed an analytical approach for Boussinesq equation with constant and time dependent boundary conditions, Hogarth *et al.* [5] have obtained the approximate analytical solution of Boussinesq equation which is accurate solution by comparison with the numerical solution when the boundary conditions is a power to time, Wojnar [14] has discussed the Boussinesq equation for flow in the aquifer with time dependent porosity, Moutsopoulos [7] has discussed Boussinesq equation with nonlinear robin boundary condition, Basha [1] has discussed the traveling wave solution of the groundwater flow in horizontal aquifers.

The aim of current work is to obtain the solution of Boussinesq equation for infiltration phenomenon. The mathematical form of the infiltration phenomenon gives the nonlinear partial differential equation in the form of Boussinesq equation. This equation is solved using homotopy analysis method. The BVPh package for nonlinear equations is employed to interpret numerically and graphically solution. Liao [24] has employed the homotopy analysis method to solve nonlinear equations.

ISSN No.: 2278-9561 (Print) ISSN No.: 2278-957X(Online) Registration No. : CHAENG/2013/49583 Periodicity: Bi-Annually

It has been successfully employed to solve many nonlinear equations. The homotopy analysis solution is strongly dependent on convergence control parameter and its proper value chosen from the valid region of c_0. The valid region of c_0 is obtained from the c_0-curve. The line segment almost parallel to horizontal line in c_0-curve gives us the admissible range of c_0.

2. Mathematical formulation

Let volume V of the fluid in an aquifer bounded by the surface ∂V. The mass of fluid flowing in unit time through an element ndA of the surface is $\rho u\cdot ndA$. The mass conservation law is expressed by the equation

$$-\frac{\partial}{\partial t}\int_V P\rho dV=\int_{\partial V}\rho u\cdot ndA \tag{1}$$

where P is the porosity of the medium, ρ is the density of the fluid, u is the velocity of the fluid and n is the outward normal to the surface element. We assume that the fluid is incompressible then ρ is constant and by divergence theorem (1) becomes

$$-\frac{\partial}{\partial t}\int_V PdV=\int_V \nabla\cdot udV. \tag{2}$$

Let us assume the plane O_{xy} be situated at the horizontal bottom of an aquifer and the z-axis be vertical up. Let V be the volume of the irregular cylinder, $z=0$ is lower surface and the free surface $z=h(x,y,t)$ is the upper surface. Then

$$-\frac{\partial}{\partial t}\int_S\int_0^h PdzdS=\int_S\int_0^h \nabla\cdot udzdS \tag{3}$$

or

$$\int_S\left(\frac{\partial}{\partial t}\int_0^h Pdz+\int_0^h \nabla\cdot udz\right)dS=0. \tag{4}$$

The integrand under $\int_S dS$ must be zero because the equation hold for any surface S. So

$$\frac{\partial}{\partial t}\int_0^h Pdz=-\int_0^h \nabla\cdot udz. \tag{5}$$

Now we assume that the flow in unsaturated zone for which the pressure p_0 at the free surface $z=h(x,y,t)$ is constant. Thus the vertical component of the momentum equation gives

$$-\frac{\partial P}{\partial z}-\rho g=0. \tag{6}$$

Since the pressure p_0 at free surface $z=h(x,y,t)$ is constant. Then we get

$$p=p_0+\rho g(h-z) \tag{7}$$

it means that the vertical velocity component is neglected and the flow is plane.

The velocity u of the fluid obeys Darcy's law which relates with the pressure gradient [15, 16, 20]

$$u=-\frac{K}{\delta}\nabla p \tag{8}$$

where K is the permeability of porous medium, δ is the viscosity of water. Using (7) in (8) we get

$$u_x=-\frac{\rho gK}{\delta}\frac{\partial h}{\partial x},u_y=-\frac{\rho gK}{\delta}\frac{\partial h}{\partial y},u_z=0. \tag{9}$$

So,

$$u=-\frac{\rho gK}{\delta}\nabla h \tag{10}$$

where ∇h is the two dimensional gradient of h.

Since the flow is considered in the plane then u is independent of z and the porosity P of the porous medium is constant. Thus (5) gives us

$$\frac{\partial(Ph)}{\partial t}=-h\nabla\cdot u. \tag{11}$$

Combine (10) and (11) we get

$$P\frac{\partial h}{\partial t}=\frac{\rho gK}{\delta}h\nabla^2 h \tag{12}$$

where K is the constant permeability of the porous medium. We neglected squares of the slope angles, we get the approximation

$$h\nabla^2 h=\nabla(h\nabla h)-(\nabla h)^2\approx\nabla(h\nabla h). \tag{13}$$

Thus (12) write as

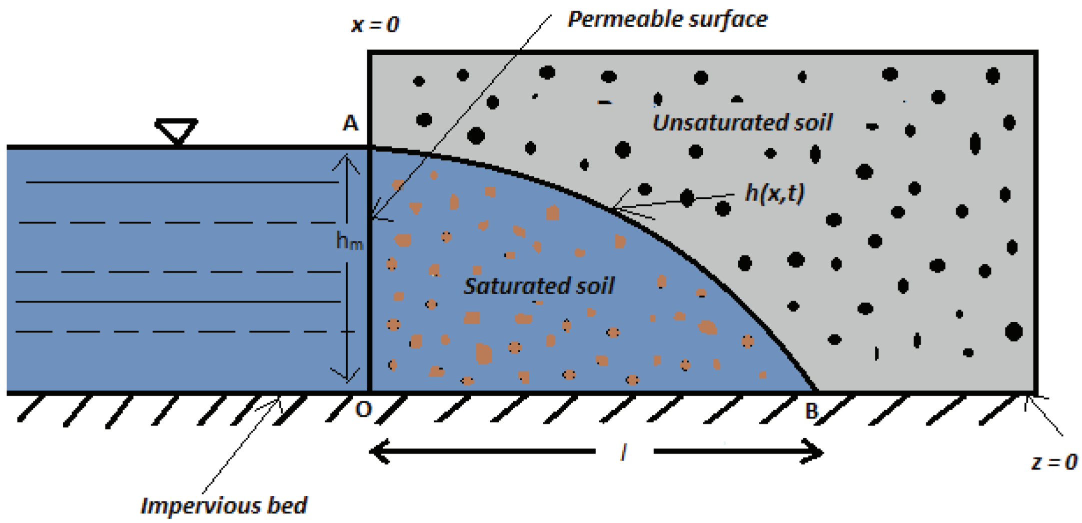

Figure 1. The infiltration phenomenon.

$$P\frac{\partial h}{\partial t}=\frac{\rho gK}{\delta}\nabla\left(h\nabla h\right) \tag{14}$$

we get the Boussinesq equation

$$P\frac{\partial h}{\partial t}=\frac{\rho gK}{2\delta}\nabla^2\left(h^2\right) \tag{15}$$

where ∇^2 is the two dimensional Laplacian operator.

Assume that the one dimensional case for infiltration phenomenon. Let us consider the groundwater reservoir with impervious bottom and surrounding of the reservoir is unsaturated homogeneous soil. The maximum height of reservoir is considered h_m. Figure 1 shows that a vertical cross-section of the reservoir surrounded by unsaturated homogeneous soil. The height of free surface of infiltrated water is zero when $x=l$, the dotted portion below the curve is considered as saturated soil by infiltrated water and the above curve is the region of unsaturated soil. The bottom part of region is assumed as impervious bed, so water can't move in the downward direction. Here the problem is discussed for infiltration phenomenon in which the groundwater of reservoir enters into unsaturated soil through vertical permeable wall (see figure 1). For the one dimensional problem we ignore the transversal variable y and the region occupies by the infiltrated water described as $\left\{(x,z)\in R^2 : 0\le z\le h(x,t)\right\}$. We assume that the flow has an almost horizontal speed.

Thus, the one dimensional Boussinesq equation becomes

$$\frac{\partial h}{\partial t}=\frac{\rho gK}{2P\delta}\frac{\partial^2 h^2}{\partial x^2}. \tag{16}$$

Using dimensionless variables

$$\mathrm{H}=\frac{h}{l}, X=\frac{x}{l}, T=\frac{\rho gKt}{lP\delta}$$

(16) becomes

$$\frac{\partial \mathrm{H}}{\partial T}=\mathrm{H}\frac{\partial^2\mathrm{H}}{\partial X^2}+\left\{\frac{\partial \mathrm{H}}{\partial X}\right\}^2. \tag{17}$$

This one dimensional nonlinear partial differential equation is the governing equation for infiltration phenomenon. The homotopy analysis method is applied to solve this equation with the following boundary conditions. The solution $\mathrm{H}(X,T)$ of equation gives us the height of the free surface at a length X and time T. According to physical concepts of the problem we used the boundary conditions are as

$$\mathrm{H}(0,T)=h_m,\ \mathrm{H}(1,T)=\left(h_m-\frac{T}{2}\right)e^{-1}. \tag{18}$$

3. Homotopy Analysis Solution

Let us assume the nonlinear differential equation $\aleph\left[\phi(X,T;q)\right]=0$, where $\aleph$ is a nonlinear operator. Let us define the nonlinear operator according to (17) is as

$$\aleph\left[\phi(X,T;q)\right]=\phi(X,T;q)\frac{\partial^2\phi(X,T;q)}{\partial X^2}+\left\{\frac{\partial\phi(X,T;q)}{\partial X}\right\}^2-\frac{\partial\phi(X,T;q)}{\partial T} \tag{19}$$

where $\varphi(X,T;q)$ is an unknown function of spatial variable X and temporal variable T for $0 \le q \le 1$.

The zero order deformation equation is of the form [24]

$$(1-q)L\left[\phi(X,T;q)-\mathrm{H}_0(X,T)\right]=c_0 q\aleph\left[\phi(X,T;q)\right] \quad (20)$$

where $q\in[0,1]$ the homotopy parameter, L the linear operator, $\mathrm{H}_0(X,T)$ the initial approximation of the original solution H(X,T), c_0 the non-zero convergence control parameter.

Assume that the linear operator of the form $L\left[\phi(X,T;q)\right]=\dfrac{\partial^2\phi(X,T;q)}{\partial X^2}$ and the initial approximation of $\mathrm{H}(X,T)$ is of the form $\mathrm{H}_0(X,T)=\left(h_m-\dfrac{TX^2}{2}\right)e^{-X}$.

Thus when $q=0$, (20) becomes $\phi(X,T;0)=\mathrm{H}_0(X,T)$ and when $q=1$, we get $\varphi(X,T;1)=\mathrm{H}(X,T)$. Hence as q varies from 0 to 1, $\phi(X,T;q)$ continuously varies from initial approximation $\mathrm{H}_0(X,T)$ to the exact solution $\mathrm{H}(X,T)$. The series solution is assumed in the form of

$$\phi(X,T;q)=\mathrm{H}_0(X,T)+\sum_{m=1}^{\infty}\mathrm{H}_m(X,T)q^m \quad (21)$$

where

$$\mathrm{H}_m(X,T)=\frac{1}{m!}\frac{\partial^m\phi(X,T;q)}{\partial q^m}\bigg|_{q=0}. \quad (22)$$

Consider the linear operator, the initial approximation and the convergence control parameter in such a way that the series of $\phi(X,T;q)$ converges at $q=1$. Thus

$$\mathrm{H}(X,T)=\mathrm{H}_0(X,T)+\sum_{m=1}^{\infty}\mathrm{H}_m(X,T). \quad (23)$$

Define $\overrightarrow{\mathrm{H}_n(X,T)}=\left\{\mathrm{H}_0(X,T),\mathrm{H}_1(X,T),\ldots,\mathrm{H}_n(X,T)\right\}$. Differentiating (20) m times w.r.t. q and dividing them by $m!$ and then putting $q=0$, we have the m^{th}-order deformation equation

$$L\left[\mathrm{H}_m(X,T)-\chi_m\mathrm{H}_{m-1}(X,T)\right]=c_0\Re_m\left(\overrightarrow{\mathrm{H}_{m-1}}\right) \quad (24)$$

subject to the boundary conditions

$$\mathrm{H}_m(0,T)=0,\mathrm{H}_m(1,T)=0,\quad m\ge 1 \quad (25)$$

where

$$\Re_m\left(\overrightarrow{\mathrm{H}_{m-1}}\right)=\sum_{i=0}^{m-1}\mathrm{H}_i\frac{\partial^2\mathrm{H}_{m-1-i}}{\partial X^2}+\sum_{i=0}^{m-1}\frac{\partial\mathrm{H}_i}{\partial X}\frac{\partial\mathrm{H}_{m-1-i}}{\partial X}-\frac{\partial\mathrm{H}_{m-1}}{\partial T},\quad m\ge 1 \quad (26)$$

and

$$\chi_m=\begin{cases}0, & if\ m\le 1\\ 1, & if\ m>1.\end{cases} \quad (27)$$

So the special solution of (24) is of the form

$$\mathrm{H}_m(X,T)=\chi_m\mathrm{H}_{m-1}(X,T)+c_0\iint\Re_m\left(\overrightarrow{\mathrm{H}_{m-1}}\right)dXdX.$$

Thus the general solution (24) is of the form

$$\mathrm{H}_m(X,T)=\chi_m\mathrm{H}_{m-1}(X,T)+c_0\iint\Re_m\left(\overrightarrow{\mathrm{H}_{m-1}}\right)dXdX+C_1X+C_2 \quad (28)$$

where C_1 and C_2 are determined by boundary conditions (25). From (25) and (28), we now successively obtain

$$\mathrm{H}_0(X,T)=\left(h_m-\frac{TX^2}{2}\right)e^{-X}. \quad (29)$$

$$\mathrm{H}_1(X,T)=c_0\left\{\begin{array}{l}-3-\dfrac{h_m{}^2}{2}-\dfrac{h_m{}^2e^{-2}X}{2}+\dfrac{h_me^{-2}TX}{2}-\dfrac{e^{-2}T^2X}{8}-\dfrac{11e^{-1}X}{2}+3X+\dfrac{h_m{}^2X}{2}\\+\left(\dfrac{h_m{}^2}{2}-\dfrac{h_mTX^2}{2}+\dfrac{T^2X^4}{8}\right)e^{-2X}+\left(\dfrac{X^2}{2}+2X+3\right)e^{-X}\end{array}\right\}. \quad (30)$$

In this way we get $\mathrm{H}_2(X,T), \mathrm{H}_3(X,T), \ldots$. The solution of (17) is as

$$\mathrm{H}(X,T) = \mathrm{H}_0(X,T) + \mathrm{H}_1(X,T) + \mathrm{H}_2(X,T) + \cdots \quad (31)$$

which represents the height of free surface of infiltrated water at X for a given time T.

4. Result and Discussion

The solution contains the convergence control parameter c_0 and the proper value of the convergence control parameter c_0 gives us the convergent homotopy series solution. The proper value of c_0 is chosen from the c_0-curve [6, 8, 9, 10, 11, 12, 17, 21, 24]. The line segment almost parallel to horizontal axis in c_0-curve gives us the admissible range of c_0. We plotted the c_0-curves with the help of Mathematica BVPh package for homotopy analysis method [18]. Figure 2-3 show the c_0-curves of $\mathrm{H}_{XX}(0,0)$, $\mathrm{H}_{XX}(0,0.8)$ and $\mathrm{H}_{XX}(0,1)$ for 10th and 15th order of approximation, respectively. The admissible range of c_0 is $-1.3 \le c_0 \le -0.4$ from c_0-curves.

Table 1 shows the values of $\mathrm{H}_{XX}(0,0), \mathrm{H}_{XX}(0,0.8)$ and $\mathrm{H}_{XX}(0,1)$ for different values of c_0 and for different order of approximations using homotopy analysis solution.

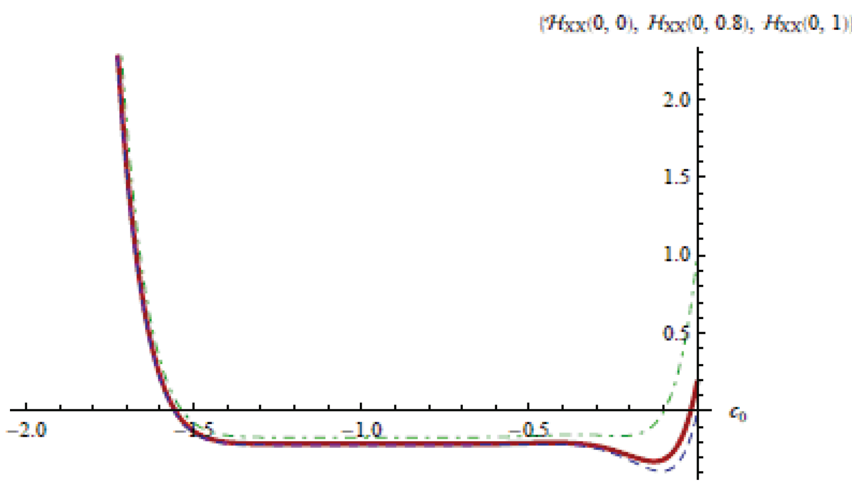

Figure 2. The c_0-curves of $\mathrm{H}_{XX}(0,0)$ (DotDashed line), $\mathrm{H}_{XX}(0,0.8)$ (Thick line) and $\mathrm{H}_{XX}(0,1)$ (Dashed line) for $\hbar_m = 0.98$

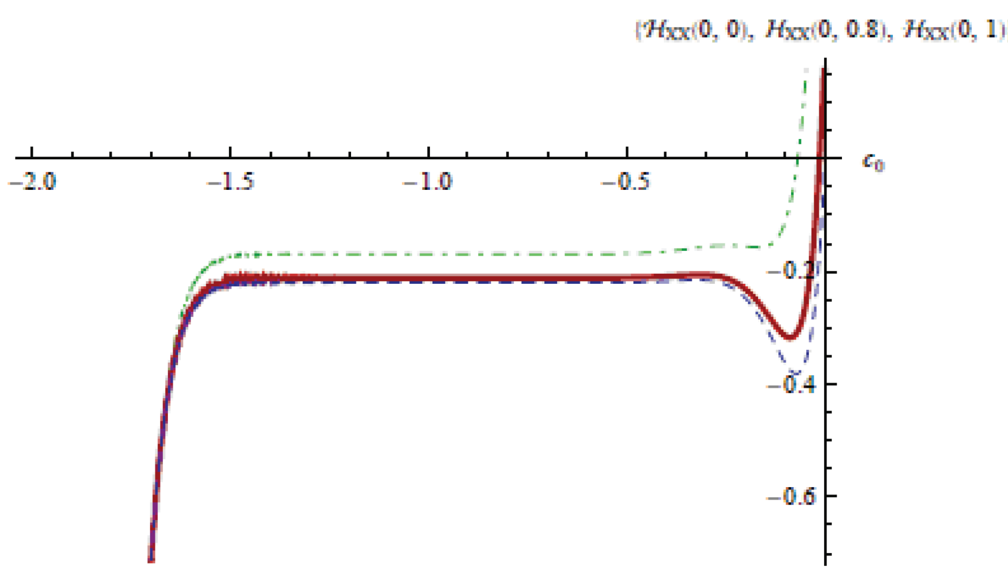

Figure 3. The c_0-curves of $\mathrm{H}_{XX}(0,0)$ (DotDashed line), $\mathrm{H}_{XX}(0,0.8)$ (Thick line) and $\mathrm{H}_{XX}(0,1)$ (Dashed line) for $\hbar_m = 0.98$.

ISSN No.: 2278-9561 (Print) ISSN No.: 2278-957X (Online) Registration No. : CHAENG/2013/49583

Table 1. Approximations of $\mathrm{H}_{XX}(0,0)$, $\mathrm{H}_{XX}(0,0.8)$ and $\mathrm{H}_{XX}(0,1)$ for different values of c_0 and for different order of approximations using homotopy analysis solution.

	$\mathrm{H}_{XX}(0,0)$		$\mathrm{H}_{XX}(0,0.8)$		$\mathrm{H}_{XX}(0,1)$	
	10th order	15th order	10th order	15th order	10th order	15th order
$c_0=-0.8$	−0.168942	−0.168869	−0.220129	−0.219842	−0.211775	−0.211551
$c_0=-0.9$	−0.168819	−0.168879	−0.219852	−0.219866	−0.211537	−0.211572
$c_0=-1$	−0.168820	−0.168876	−0.219750	−0.219877	−0.211466	−0.211578
$c_0=-1.1$	−0.168894	−0.168880	−0.219807	−0.219829	−0.211533	−0.211545
$c_0=-1.2$	−0.168912	−0.168877	−0.219896	−0.219909	−0.211609	−0.211600

Using the c_0-curve, we choose the proper value of $c_0=-1$ from admissible range of c_0. Table 2 indicates the numerical values of $\mathrm{H}(X,T)$ for 10th and 15th order of approximation of homotopy series solution. The graphical interpretation of the solution represents the height of free surface at length X for a time T. Figure 4 shows the graph of the height of free surface $\mathrm{H}(X,T)$ vs X for a fixed time $T=0.1,0.2,...,1$. The graph of $\mathrm{H}(X,T)$ vs X and T is shown in figure 5.

Table 2. Numerical values of the height of infiltrated groundwater H(*X*, *T*).

T	Approximation	$X=0.1$	$X=0.2$	$X=0.3$	$X=0.4$	$X=0.5$	$X=0.6$	$X=0.7$	$X=0.8$	$X=0.9$	$X=1$
0.1	10th order	0.93767	0.89326	0.84635	0.79639	0.74258	0.68378	0.61850	0.54433	0.45615	0.34213
	15th order	0.93767	0.89325	0.84633	0.79633	0.74252	0.68382	0.61864	0.54433	0.45603	0.34213
0.2	10th order	0.93695	0.89175	0.84397	0.79304	0.73812	0.67801	0.61112	0.53491	0.44369	0.32373
	15th order	0.93695	0.89175	0.84395	0.79298	0.73805	0.67806	0.61130	0.53491	0.44354	0.32373
0.3	10th order	0.93627	0.89031	0.84170	0.78984	0.73385	0.67247	0.60403	0.52584	0.43159	0.30534
	15th order	0.93626	0.89031	0.84168	0.78978	0.73379	0.67254	0.60425	0.52582	0.43138	0.30534
0.4	10th order	0.93562	0.88895	0.83954	0.78680	0.72978	0.66717	0.59723	0.51712	0.41985	0.28695
	15th order	0.93561	0.88894	0.83952	0.78673	0.72972	0.66727	0.59750	0.51707	0.41959	0.28695
0.5	10th order	0.93500	0.88765	0.83750	0.78391	0.72591	0.66212	0.59072	0.50878	0.40853	0.26855
	15th order	0.93500	0.88764	0.83747	0.78383	0.72584	0.66225	0.59106	0.50869	0.40819	0.26855
0.6	10th order	0.93441	0.88643	0.83556	0.78117	0.72225	0.65731	0.58452	0.50083	0.39767	0.25016
	15th order	0.93441	0.88642	0.83553	0.78109	0.72218	0.65750	0.58494	0.50069	0.39722	0.25016
0.7	10th order	0.93387	0.88527	0.83373	0.77859	0.71879	0.65277	0.57864	0.49330	0.38729	0.23176
	15th order	0.93386	0.88526	0.83370	0.77850	0.71871	0.65300	0.57916	0.49308	0.38672	0.23176
0.8	10th order	0.93335	0.88419	0.83202	0.77617	0.71553	0.64848	0.57308	0.48621	0.37746	0.21337
	15th order	0.93335	0.88418	0.83198	0.77607	0.71546	0.64877	0.57371	0.48588	0.37672	0.21337
0.9	10th order	0.93287	0.88318	0.83042	0.77391	0.71249	0.64445	0.56785	0.47958	0.36822	0.19498
	15th order	0.93287	0.88317	0.83038	0.77380	0.71242	0.64482	0.56862	0.47911	0.36728	0.19497
1	10th order	0.93242	0.88224	0.82893	0.77181	0.70966	0.64068	0.56295	0.47345	0.35964	0.17658
	15th order	0.93242	0.88223	0.82889	0.77169	0.70959	0.64115	0.56389	0.47279	0.35843	0.17658

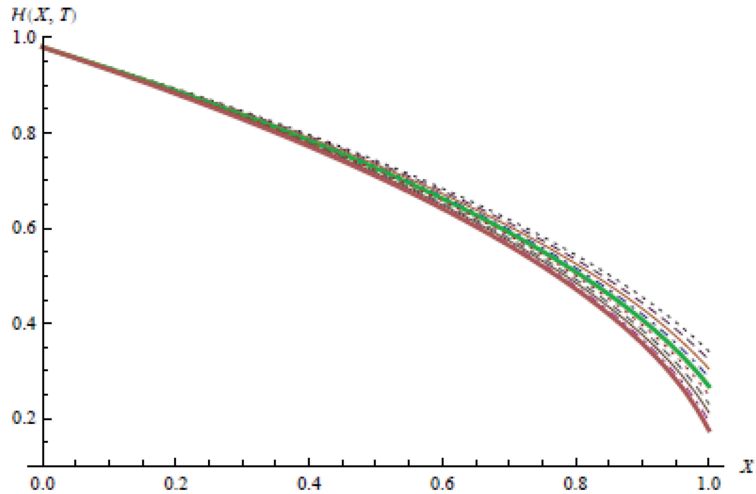

Figure 4. The graph of the height of infiltrated groundwater $H(X,T)$ v/s length X for fixed time $T = 0.1$ (uppermost graph),0.2,...,1 (lowermost graph) for $h_m = 0.98$.

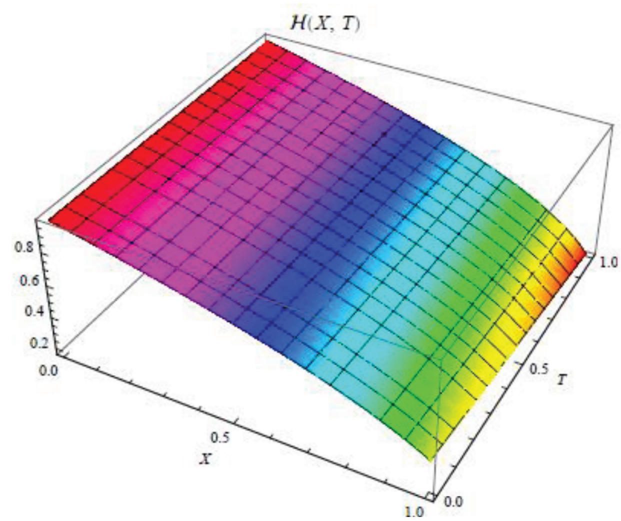

Figure 5. The graph of the height of infiltrated groundwater $H(X, T)$ v/s X and T for $h_m = 0.98$.

Table 3: Difference between 15th order and 10th order approximation of $H(X, T)$.

T	15^{th} order of appro. of $H(X,T)$-10^{th} order appro. of $H(X,T)$									
	$X = 0.1$	$X = 0.2$	$X = 0.3$	$X = 0.4$	$X = 0.5$	$X = 0.6$	$X = 0.7$	$X = 0.8$	$X = 0.9$	$X = 1$
0.1	-2.0811E-06	-5.6006E-06	-1.9778E-05	-5.3297E-05	-5.9018E-05	3.5795E-05	1.4286E-04	9.3097E-06	-1.1770E-04	3.8568E-08
0.2	-2.2808E-06	-6.0730E-06	-2.1647E-05	-5.8219E-05	-6.1785E-05	5.3768E-05	1.7946E-04	-2.2971E-06	-1.5546E-04	-3.4624E-07
0.3	-2.2401E-06	-6.4685E-06	-2.3392E-05	-6.3701E-05	-6.4585E-05	7.6369E-05	2.2372E-04	-2.0635E-05	-2.0450E-04	2.7049E-08
0.4	-2.4973E-06	-7.2382E-06	-2.5791E-05	-6.9799E-05	-6.7105E-05	1.0417E-04	2.7761E-04	-4.8023E-05	-2.6678E-04	-5.9482E-07
0.5	-2.8283E-06	-7.7823E-06	-2.8170E-05	-7.6489E-05	-6.9601E-05	1.3886E-04	3.4192E-04	-8.8526E-05	-3.4708E-04	-2.6285E-07
0.6	-3.1310E-06	-8.0655E-06	-3.0407E-05	-8.3826E-05	-7.0837E-05	1.8183E-04	4.2106E-04	-1.4450E-04	-4.4807E-04	7.9133E-08
0.7	-3.4175E-06	-8.8059E-06	-3.2993E-05	-9.1978E-05	-7.3097E-05	2.3376E-04	5.1631E-04	-2.2257E-04	-5.7639E-04	-8.2077E-07
0.8	-3.2422E-06	-9.3882E-06	-3.5428E-05	-1.0099E-04	-7.4488E-05	2.9797E-04	6.3157E-04	-3.2762E-04	-7.4042E-04	-7.0717E-07
0.9	-2.9964E-06	-9.4567E-06	-3.8050E-05	-1.1047E-04	-7.5582E-05	3.7676E-04	7.7115E-04	-4.6969E-04	-9.4501E-04	-1.1996E-06
1	-2.8044E-06	-9.9191E-06	-4.1949E-05	-1.2155E-04	-7.4969E-05	4.7115E-04	9.3911E-04	-6.5811E-04	-1.2037E-03	-1.3547E-07

The difference of 15th order approximation of solution $\mathrm{H}(X,T)$ and 10th order approximation of solution $\mathrm{H}(X,T)$ are mentioned in table 3 for different values of *X* and *T*.

Conclusions

The Boussinesq equation is discussed for infiltration phenomenon in unsaturated soil. The homotopy analysis solution of the governing equation is obtained with boundary condition. The convergence of homotopy analysis solution is discussed by c_0-curve. The solution represents the height of free surface which is discussed graphically and numerically.

References

[1] H. A. Basha. Traveling wave solution of the Boussinesq equation for groundwater flow in horizontal aquifers, *Water Resources Research*, **49,** 1668–1679 (2013).

[2] J. Boussinesq. Recherches theoriques sur l'ecoulement des nappes d'eau infiltrees dans le sol et sur le debit des sources, *Journal de Mathematiques Pures et Appliquees*, **10,** 5–78 (1904).

[3] R. S. Govindaraju & J. K. Koelliker. Applicability of linearized Boussinesq equation for modeling bank storage under uncertain aquifer parameters, *Journal of Hydrology*, **157,** 349–366 (1994).

[4] W. L. Hogarth, R. S. Govindaraju, J. Y. Parlange & J. K. Koelliker. Linearised Boussinesq equation for modelling bank storage - a correction, *Journal of Hydrology*, **198,** 377–385 (1997).

[5] W. L. Hogarth, J. Y. Parlange, M. B. Parlange & D. Lockington. Approximate analytical solution of the Boussinesq equation with numerical validation, *Water Resources Research*, **35(10),** 3193–3197 (1999).

[6] H. Kheiri, N. Alipour, R. Dehghani. Homotopy analysis and homotopy pade methods for the modified Burgers-Korteweg-de Vries and the Newell-Whitehead equations, *Mathematical Sciences*, **5(1),** 33–50 (2011).

[7] K. N. Moutsopoulos. Solutions of the Boussinesq equation subject to a nonlinear Robin boundary condition, *Water Resources Research*, **49,** 7–18 (2013).

[8] M. A. Patel & N. B. Desai. Homotopy analysis solution of countercurrent imbibition phenomenon in inclined homogeneous porous medium, *Global Journal of Pure and Applied Mathematics*, **12(1),** 1035–1052 (2016).

[9] M. A. Patel & N. B. Desai. Homotopy analysis method for fingero-imbibition phenomenon in heterogeneous porous medium, *Nonlinear Science Letters A: Mathematics, Physics and Mechanics*, **8(1)**, 90–100 (2017).

[10] M. A. Patel & N. B. Desai. Mathematical modelling of fingero-imbibition phenomenon in heterogeneous porous medium with magnetic field effect, *PRAJNA - Journal of Pure and Applied Sciences*, **24-25,** 15–22 (2017).

[11] M. A. Patel & N. B. Desai. Homotopy analysis method for nonlinear partial differential equation arising in fluid flow through porous medium, *International Journal of Computer & Mathematical Sciences*, **6(5),** 14–18 (2017).

[12] M. M. Rashidi, G. Domairry & S. Dinarvand. Approximate solutions for the Burger and regularized long wave equations by means of the homotopy analysis method, *Communications in Nonlinear Science and Numerical Simulation*, **14,** 708–717 (2009).

[13] P. A. Troch, F. P. De Troch & W. Brutsaert. Effective water table depth to describe initial conditions prior to storm rainfall in humid regions, *Water Resources Research*, **29(2),** 427–434 (1993).

[14] R. Wojnar. Boussinesq equation for flow in an aquifer with time dependent porosity, *Bulletin of Polish Academy of Sciences: Technical Sciences*, **58(1),** 165–170 (2010).

[15] J. Bear. Dynamics of fluids in porous media, A*merican Elsevier Publishing Company*, Inc., New York. (1972).

[16] H. Darcy. Les Fontaines Publiques de la Ville de Dijon, *Dalmont, Paris* (1856).

[17] S. J. Liao. Beyond perturbation: Introduction to the homotopy analysis method, Chapman and Hall/CRC Press, *Boca Raton* (2003).

[18] S. J. Liao. Homotopy analysis method in nonlinear differential equations, *Higher Education Press, Beijing and Springer-Verlag Berlin Heidelberg* (2012).

[19] P. Y. Polubarinova-Kochina. Theory of Groundwater Movement, *Princeton Univ. Press, Princeton* (1962).

[20] A. E. Scheidegger. The Physics of flow through porous media, Revised edition, *University of Toronto Press, Toronto* (1960).

[21] K. Vajravelu & R. A. Van Gorder. Nonlinear flow phenomena and homotopy analysis: Fluid flow and Heat transfer, *Higher Education Press, Beijing and Springer-Verlag Berlin Heidelberg* (2012).

[22] J. L. Vazquez. The porous medium equation: mathematical theory, *Oxford University Press* (2007).

[23] N. B. Desai. The study of problems arises in single phase and multiphase ow through porous media, Ph.D. *Thesis, South Gujarat University, Surat, India* (2002).

[24] S. J. Liao. The proposed homotopy analysis technique for the solution of nonlinear problems, Ph.D. *Thesis, Shanghai Jiao Tong University, Shanghai, China* (1992).