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A Family of Unbiased Modified Linear Regression Estimators

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Abstract: In this paper a family of modified linear regression estimators has been proposed which are unbiased. The variance of the proposed estimators and the conditions for which the proposed estimators perform better than the classical ratio estimator and the existing modified ratio estimators have been obtained. Further we have shown that the classical ratio estimator, the existing modified ratio estimators and the usual linear regression estimator are the particular cases of the proposed estimators. It is observed from the numerical study that the proposed estimators perform better than the ratio estimator and the existing modified ratio estimators.

Keywords: Auxiliary variable, Efficiency, Mean squared error, Population Mean, Simple random sampling

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1. INTRODUCTION

Consider a finite population $U=\{U_{1,}U_{2},\ldots,U_{N}\}$ of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on $U_i, i=1,2,3,\ldots,N$ giving a vector $Y=\{Y_{1,}Y_{2},\ldots,Y_{N}\}$. The problem is to estimate the population mean $\bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i$ on the basis of a random sample selected from the population U. If there is no additional information on the auxiliary variable available the population mean is estimated by the sample mean obtained by simple random sampling without replacement. However if there exists an auxiliary variable X which is positively correlated with the study variable Y, one can use the usual ratio estimator or any one of the modified ratio

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estimators available in the literature to get a more efficient estimator than the usual simple random sample mean. So the problem is to get an efficient estimator compared to the existing estimators for estimation of the mean of a finite population.

Before discussing further about the modified ratio estimators and the proposed estimators, the notations to be used in this paper are described below:

- N – Population size
- n – Sample size
- $f = {}^{n}/_{N}$, Sampling fraction
- Y – Study variable
- X – Auxiliary variable
- $\bar{X}, \bar{Y}$ – Population means
- $\bar{x}, \bar{y}$ – Sample means
- S_x, S_y – Population standard deviations
- C_x, C_y – Coefficient of variations
- ρ – Correlation Coefficient
- $\beta_1 = \frac{N\sum_{i=1}^{N}(X_i - \bar{X})^3}{(N-1)(N-2)S^3}$, Skewness of the auxiliary variable
- $\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i - \bar{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$, Kurtosis of the auxiliary variable
- $B(.)$ – Bias of the estimator
- $MSE(.)$ – Mean squared error of the estimator
- $\hat{\bar{Y}}_{lr}$ – Linear regression estimator of $\bar{Y}$
- $\hat{\bar{Y}}_{i}$ – Existing modified ratio estimator of $\bar{Y}$
- $\hat{\bar{Y}}_{JS}$ – Proposed Modified Linear regression estimator

In case of SRSWOR, the sample mean $\bar{y}_r$ is used to estimate population mean $\bar{Y}$ which is an unbiased estimator and its variance is given below:

$$V(\bar{y}_r) = \frac{(1-f)}{n} S_y^2 \qquad (1.1)$$

Cochran (1940) has pioneered the ratio estimator for estimating the population mean $\bar{Y}$ of the study variable Y as given below:

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X} \text{ where } \hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{y}{x} \qquad (1.2)$$

The bias and MSE of $\hat{\bar{Y}}_R$ to the first order of approximation are given below:

$$B\left(\hat{\bar{Y}}_R\right) = \frac{(1-f)}{n}\bar{Y}\left(C_x^2 - \rho C_x C_y\right) \tag{1.3}$$

$$MSE\left(\hat{\bar{Y}}_R\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + C_X^2 - 2\rho C_x C_y\right) \tag{1.4}$$

Further improvements are achieved on the ratio estimator by adding the known parameters of the auxiliary variable like Co-efficient of Variation, Kurtosis, Skewness and Population Correlation Coefficient etc to the ratio estimators. A set of modified ratio estimators alone considered which are to be used for assessing the performance of the proposed estimators. It is to be noted that "the existing modified ratio estimators" means the list of modified ratio estimators to be considered in this paper unless otherwise stated. It does not mean the entire list of modified ratio estimators given in [9]. For a more detailed discussion on the ratio estimator, the linear regression estimator and its modifications one may refer to [1-11] and the references cited there in.

2. A FAMILY OF UNBIASED MODIFIED LINEAR REGRESSION TYPE ESTIMATORS

As we stated earlier further modifications have been obtained by adding the known values of the population parameters of the auxiliary variable, which is positively correlated with that of the study variable. The modified ratio estimators discussed above are biased but have smaller mean squared error compared to the classical ratio estimator under certain conditions. The existing modified ratio estimators use the known values of the parameters like $\bar{X}, C_x, \beta_1, \beta_2, \rho$ and their linear combinations.

In this section, a family of modified linear regression type estimators has been introduced. However the proposed estimators are unbiased and represented as a family of unbiased modified linear regression estimators.

The proposed family of modified linear regression type estimators for estimating the population mean $\bar{Y}$ is given below:

$$\hat{\bar{Y}}_{JS} = \bar{y} - \alpha e_1 \tag{2.1}$$

where $e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}$ and α is a suitably chosen scalar.

Further we can write $\bar{y} = \bar{Y}(1 + e_0)$ and $\bar{x} = \bar{X}(1 + e_1)$ such that

$E(e_0) = E(e_1) = 0,$

$E(e_0^2) = \frac{(1-f)}{n} C_y^2,$

$E(e_1^2) = \frac{(1-f)}{n} C_X^2,$

$$E(e_0 e_1) = \frac{(1-f)}{n} \rho C_y C_x \text{ and } f = \frac{n}{N} \tag{2.2}$$

Taking expectation on both sides of equation (2.1), the expected value of the proposed estimators is obtained as:

$$E\left(\hat{\bar{Y}}_{JS}\right) = E(\bar{y} - \alpha e_1) = \bar{Y} \tag{2.3}$$

Since $E(e_0) = E(e_1) = 0,$ this shows that the proposed estimators are unbiased estimators. The corresponding variances of the proposed modified linear regression estimators are as given below:

$$V\left(\hat{\bar{Y}}_{JS}\right) = \frac{(1-f)}{n}\left(\bar{Y}^2 C_y^2 + \alpha^2 C_x^2 - 2\rho\alpha\bar{Y} C_x C_y\right) \tag{2.4}$$

Remark 2.1: If α = 0 in (2.1) then proposed estimator $\hat{\bar{Y}}_{JS}$ reduces to simple random sampling without replacement sample mean $\bar{y}_r$.

Remark 2.2: If $\alpha = \beta\bar{X}$ in (2.1) then proposed estimator $\hat{\bar{Y}}_{JS}$ reduces to the simple linear regression estimator $\hat{\bar{Y}}_{lr}$, β is the population regression coefficient.

3. EFFICIENCY COMPARISON

To the first degree of approximation, the MSE of the classical ratio estimator $\hat{\bar{Y}}_R$ is given below:

$$MSE(\hat{\bar{Y}}_R) = \frac{(1-f)}{n} \bar{Y}^2 \left(C_y^2 + C_x^2 - 2\rho C_x C_y\right) \tag{3.1}$$

Class 1: The MSE and the constants of the modified ratio estimators $\hat{\bar{Y}}_1$ to $\hat{\bar{Y}}_7$ are represented in a single class as given below:

$$\mathrm{MSE}(\hat{\bar{Y}}_i)=\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_i^2C_x^2-2\rho\theta_iC_xC_y\right);i=1,2,3,\ldots,7 \tag{3.2}$$

$$\text{where}\,\theta_1=\frac{\bar{X}}{\bar{X}+C_X},\theta_2=\frac{\bar{X}}{\bar{X}+\beta_2},\theta_3=\frac{\bar{X}}{\bar{X}+\beta_1},\theta_4=\frac{\bar{X}}{\bar{X}+\rho},\theta_S=\frac{C_x\bar{X}}{C_x\bar{X}+\beta_2},$$

$$\theta_6=\frac{\beta_1\bar{X}}{\beta_1\bar{X}+\beta_2}\,\text{and}\,\theta_7=\frac{C_x\bar{X}}{C_x\bar{X}+\beta_1}$$

Class 2: The MSE and the constants of the modified ratio estimators in $\hat{\bar{Y}}_8$ to $\hat{\bar{Y}}_{17}$ are represented in a single class as given below:

$$\mathrm{MSE}(\hat{\bar{Y}}_i)=\frac{(1-f)}{n}\left(R_i^2S_x^2+S_y^2\left(1-\rho^2\right)\right);i=8,9,10,\ldots17 \tag{3.3}$$

where

$$R_8=\frac{\bar{Y}}{\bar{X}},R_9=\frac{Y}{\bar{X}+C_X},R_{10}=\frac{Y}{\bar{X}+\beta_2},R_{11}=\frac{C_X\bar{Y}}{C_X\bar{X}+\beta_2},$$

$$R_{12}=\frac{\bar{Y}}{\bar{X}+\beta_1},R_{13}=\frac{\beta_1\bar{Y}}{\beta_1\bar{X}+\beta_2},R_{14}=\frac{\bar{Y}}{\bar{X}+\rho}R_{15}=\frac{C_X\bar{Y}}{C_X\bar{X}+\rho},$$

$$R_{16}=\frac{\rho\bar{Y}}{\rho\bar{X}+C_X}\,\text{and}\,R_{17}=\frac{\rho\bar{Y}}{\rho\bar{X}+\beta_2}$$

The variance of the proposed modified linear regression estimators is given below:

$$V\left(\hat{\bar{Y}}_{JS}\right)=\frac{(1-f)}{n}(\bar{Y}^2C_y^2+\alpha^2C_x^2-2\alpha\rho\bar{Y}C_xC_y) \tag{3.4}$$

By comparing (3.1) and (3.4), the proposed family of estimators $\hat{\bar{Y}}_{JS}$ is more efficient than the classical ratio estimator as given below:

$$V\left(\hat{\bar{Y}}_{JS}\right)<\mathrm{MSE}\left(\hat{\bar{Y}}_R\right)\text{if}\left(2\frac{c_y}{c_x}\rho-1\right)<\alpha<1\,\text{or}\,1<\alpha<\left(2\frac{c_y}{c_x}\rho-1\right) \tag{3.5}$$

Let us define the lower limit as α_L and upper limit as α_U, for the above case the values of α_L and α_U are

$$\alpha_L = \left(2\frac{c_y}{c_x}\rho - 1\right) \text{and}\, \alpha_U = 1 \tag{3.6}$$

At these limit points $\alpha_L = \left(2\frac{c_y}{c_x}\rho - \theta_i\right)\theta_i\frac{s_y}{c_y} \text{and}\, \alpha_U = \theta_i\frac{s_y}{c_y}$, the variance of proposed estimator $\hat{\bar{Y}}_{JS}$ is equal to the MSE of the classical ratio estimator $\hat{\bar{Y}}_R$. That is,

$$V\left(\hat{\bar{Y}}_{JS}\right) = MSE\left(\hat{\bar{Y}}_R\right) \text{at}\, \alpha_L = \left(2\frac{c_y}{c_x}\rho - 1\right) \text{and}\, \alpha_U = 1 \tag{3.7}$$

By comparing (3.2) and (3.4), the proposed estimators $\hat{\bar{Y}}_{JS}$ are more efficient than the existing modified ratio estimators $\hat{\bar{Y}}_i$;i=1,2,3,...,7 as given below:

$$V\left(\bar{Y}_{Js}\right) < MSE\left(\hat{\bar{Y}}_i\right) \text{if} \left(2\frac{c_y}{c_x}\rho - \theta_i\right) < \alpha < \theta_i \text{ or } \theta_i < \alpha < \left(2\frac{c_y}{c_x}\rho - \theta_i\right); i = 1,2,3,\ldots,7 \tag{3.8}$$

For the above case, the values of α_L and α_U are

$$\alpha_L = \left(2\frac{c_y}{c_x}\rho - \theta_i\right)\frac{s_y}{c_y}\bar{y} \,\text{and}\, \alpha_u = \theta_i\frac{s_y}{c_y}; i = 1,2,3,\ldots,7 \tag{3.9}$$

At these limit points $\alpha_L = \left(2\frac{c_y}{c_x}\rho - 1\right) \text{and}\, \alpha_U = 1$, the variance of proposed estimators $\hat{\bar{Y}}_{JS}$ is equal to the MSE of the existing modified ratio estimators given in Class 1.

$$V\left(\bar{\hat{Y}}_{Js}\right) = MSE\left(\bar{\hat{Y}}_{i}\right) at \alpha_L \left(2\frac{c_y}{c_x}\rho - \theta_i\right) and \alpha_U = \theta_i; i = 1,2,3,\ldots,7 \quad (3.10)$$

By comparing (3.3) and (3.4), the proposed estimators $\bar{\hat{Y}}_{JS}$ are more efficient than the existing modified ratio estimators $\bar{\hat{Y}}_{i}$;i=8,9,10,...,17 as given below:

$$V\left(\bar{\hat{Y}}_{JS}\right) < MSE\left(\bar{\hat{Y}}_{i}\right) if \left(\frac{S_y\rho + R_iS_X}{c_x}\right) < \alpha < \left(\frac{S_y\rho - R_iS_X}{c_x}\right) or \left(\frac{S_y\rho + R_iS_X}{c_x}\right) < \alpha < \left(\frac{S_y\rho + R_iS_X}{c_x}\right) \quad (3.11)$$

For the above case, the values of α_L and α_U are

$$\alpha_L = \left(\frac{S_y\rho + R_iS_X}{c_x}\right) and \alpha_U = \left(\frac{S_y\rho - R_iS_X}{c_x}\right); i = 8,9,10,\ldots,17 \quad (3.12)$$

At these limit points $\alpha_L = \left(\frac{S_y\rho + R_iS_X}{c_x}\right) and \alpha_U = \left(\frac{S_y\rho - R_iS_X}{c_x}\right)$, the variance of proposed estimators $\bar{\hat{Y}}_{JS}$ is equal to the MSE of the existing modified ratio estimators. That is,

$$V(\bar{Y}_{JS}) = MSE\left(\bar{Y}_i\right) at \alpha_L = \left(\frac{s_y\rho + R_iS_x}{c_x}\right) and \quad \alpha_U = \left(\frac{s_y\rho + R_iS_x}{c_x}\right); i = 8,9,10,\ldots,17 \quad (3.13)$$

By taking average of the limits in (3.5), (3.8) and (3.11), we can obtain optimum value (say α_A), where $\alpha_A = \beta\bar{X}$, at this point; the proposed estimators are reduced to the usual linear regression estimator and hence

$$V\left(\hat{\bar{\bar{Y}}}_{JS}\right) = V(\hat{\bar{\bar{Y}}}_{lr})\,\text{at}\,\alpha_A = \beta\bar{X} \quad (3.14)$$

The above results are summarized in Table 1 given below:

Table-1: Particular case of the proposed estimators.

Values of α	Variance of Proposed estimator
$\alpha_L = \left(2\frac{C_y}{C_x}\rho - 1\right)$ and $\alpha_U = 1$	$V(\hat{\bar{\bar{Y}}}_{JS}) = MSE\left(\hat{\bar{Y}}_R\right)$
$\alpha_L = \left(2\frac{C_y}{C_x}\rho - \theta_i\right)$ and $\alpha_U = \theta_i$	$V(\hat{\bar{\bar{Y}}}_{JS}) = MSE\left(\hat{\bar{Y}}_i\right); i = 1,2,3,\ldots,7$
$\alpha_L = \left(\frac{S_y\rho + R_iS_x}{C_X}\right)$ and $\alpha_L = \left(\frac{S_y\rho - R_iS_x}{C_X}\right)$	$V(\hat{\bar{\bar{Y}}}_{JS}) = MSE\left(\hat{\bar{Y}}_i\right); i = 8,9,10,\ldots,17$
$\alpha_A = \beta\bar{X}$	$V(\hat{\bar{\bar{Y}}}_{JS}) = V\left(\hat{\bar{Y}}_{lr}\right)$

4. NUMERICAL COMPARISON

The performances of the proposed modified linear regression type estimators are assessed with that of the classical ratio estimator, the modified ratio estimators and the linear regression estimator for certain natural populations. The populations 1 and 2 are taken from ([4], page 228), population 3 is taken from ([1], page 152) and population 4 is taken from ([1], page 325).The parameters and the constants computed from the above populations are given below:

Population-1:

X= Fixed Capital and Y= Output for 80 factories in a region

$N = 80$	$n = 20$	$\bar{Y} = 51.8264$	$\bar{X} = 11.2646$
$\rho = 0.9413$	$S_y = 18.3569$	$C_y = 0.3542$	$S_x = 8.4563$
$C_X = 0.7507$	$\beta_2 = -0.06339$	$\beta_1 = 1.05$	

Population-2:

X= Data on number of workers and Y= Output for 80 factories in a region

$N = 80$	$n = 20$	$\bar{Y} = 51.8264$	$\bar{X} = 2.8513$
$\rho = 0.9150$	$S_y = 18.3569$	$C_y = 0.3542$	$S_x = 2.7042$
$C_x = 0.9484$	$\beta_2 = 1.3005$	$\beta_1 = 0.6978$	

Population-3:

X= Number of rooms and Y = Number of persons

$N = 10$	$n = 4$	$\bar{Y} = 101.1$	$\bar{X} = 58.8$
$\rho = 0.6515$	$S_y = 14.6523$	$C_y = 0.1449$	$S_x = 7.5339$
$C_x = 0.1281$	$\beta_2 = -0.3814$	$\beta_1 = 0.5764$	

Population-4:

X=Size of United States cities in 1920 and Y = Size of United States cities in 1930

$N = 49$	$n = 20$	$\bar{Y} = 116.1633$	$\bar{X} = 98.6734$
$\rho = 0.6904$	$S_y = 98.8286$	$C_y = 0.8508$	$S_x = 102.9709$
$C_x = 1.0435$	$\beta_2 = 5.9878$	$\beta_1 = 2.4224$	

The range of α in which $\hat{\bar{Y}}_{JS}$ is better than $\hat{\bar{Y}}_R$ and $\hat{\bar{Y}}_i$;i=1,2,3,...,17 is given in the following table:

Table-2: Range of α for which $\hat{\bar{Y}}_{JS}$ is better than $\hat{\bar{Y}}_R$ and $\hat{\bar{Y}}_i$;i=1,2,3,...,17.

Pro-posed	Existing Estimators	Population 1	Population 2	Population 3	Population 4
[1]	$\hat{\bar{Y}}_R = \bar{y}\frac{\bar{X}}{\bar{x}}$	(-5.7911, 51.8264)	(-16.4055, 51.8264)	(47.9098, 101.1000)	(14.6123, 116.1633)
[8]	$\hat{\bar{Y}}_1 = \bar{y}\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$	(-2.5530, 48.5884)	(-3.4697, 38.8906)	(48.1296, 100.8802)	(15.8279, 114.9476)
[7]	$\hat{\bar{Y}}_2 = \bar{y}\left[\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right]$	(-6.0844, 52.1197)	(-0.1716, 35.5924)	(47.2497, 101.7600)	(21.2581, 109.5174)
[11]	$\hat{\bar{Y}}_3 = \bar{y}\left[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right]$	(-1.3721, 47.4074)	(-6.2158, 41.6366)	(48.8912, 100.1185)	(17.3957, 113.3798)

[6]	$\hat{\bar{Y}}_4 = \bar{y}\left[\frac{\bar{X}+\rho}{\bar{x}+\rho}\right]$	(-1.7943, 47.8296)	(-3.8146, 39.2355)	(49.0177, 99.9920)	(15.4195, 115.3561)
[10]	$\hat{\bar{Y}}_5 = \bar{y}\left[\frac{C_x\bar{X}+\beta_2}{C_x\bar{x}+\beta_2}\right]$	(-6.1825, 52.2178)	(0.4249, 34.9960)	(42.5175, 106.4922)	(20.9960, 109.7796)
[11]	$\hat{\bar{Y}}_6 = \bar{y}\left[\frac{\beta_1\bar{X}+\beta_2}{\beta_1\bar{x}+\beta_2}\right]$	(-6.0703, 52.1057)	(4.0800, 31.3409)	(46.7591, 102.2507)	(17.4512, 113.3244)
[11]	$\hat{\bar{Y}}_7 = \bar{y}\left[\frac{C_x\bar{X}+\beta_1}{C_x\bar{x}+\beta_1}\right]$	(-0.0667, 46.1020)	(-5.7751, 41.1959)	(55.0964, 93.9134)	(17.2822, 113.4933)
[2]	$\hat{\bar{Y}}_8 = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\frac{\bar{X}}{\bar{x}}$	(-28.8085, 74.8438)	(-34.1165, 69.5373)	(-26.6020, 175.6414)	(-50.7755, 181.5511)
[2]	$\hat{\bar{Y}}_9 = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$	(-25.5705, 71.6058)	(-21.1805, 56.6014)	(-26.3822, 175.4216)	(-49.5598, 180.3354)
[2]	$\hat{\bar{Y}}_{10} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right]$	(-29.1018, 75.1371)	(-17.8824, 53.3032)	(-27.2622, 176.3016)	(-44.1297, 174.9052)
[2]	$\hat{\bar{Y}}_{11} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{C_x\bar{X}+\beta_2}{C_x\bar{x}+\beta_2}\right]$	(-29.1999, 75.2353)	(-17.2859, 52.7067)	(-31.9955, 181.0349)	(-44.3918, 175.1674)
[11]	$\hat{\bar{Y}}_{12} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right]$	(-24.3895, 70.4249)	(-23.9266, 59.3474)	(-25.6204, 174.6598)	(-47.9920, 178.7676)
[11]	$\hat{\bar{Y}}_{13} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\beta_1\bar{X}+\beta_2}{\beta_2\bar{x}+\beta_2}\right]$	(-29.0878, 75.1231)	(-13.6308, 49.0516)	(-27.7529, 176.7923)	(-47.9366, 178.7122)
[3]	$\hat{\bar{Y}}_{14} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}+\rho}{\bar{x}+\rho}\right]$	(-24.8118, 70.8471)	(-21.5254, 56.9463)	(-25.4939, 174.5333)	(-49.9683, 180.7439)
[3]	$\hat{\bar{Y}}_{15} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{C_x\bar{X}+\rho}{C_x\bar{x}+\rho}\right]$	(-23.6174, 69.6527)	(-21.0136, 56.4345)	(-18.5519, 167.5913)	(-50.0018, 180.7773)
[3]	$\hat{\bar{Y}}_{16} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right]$	(-25.3819, 71.4172)	(-20.2993, 55.7201)	(-26.2650, 175.3044)	(-49.0230, 179.7986)
[3]	$\hat{\bar{Y}}_{17} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\rho\bar{X}+\beta_2}{\rho\bar{x}+\beta_2}\right]$	(-29.1202, 75.1555)	(-16.8759, 52.2967)	(-27.6189, 176.6583)	(-41.3908, 172.1663)

The percent relative efficiencies (PRE's) of the proposed modified linear regression type estimators with respect to the existing estimators computed for different values of α like α_L, α_1, α_2, α_A, α_3, α_4 and α_U for the four populations are calculated and are presented in Tables 3 to 6,

where α_L = Lower Limit, α_U = Upper Limit

$$\alpha_A = \frac{(\alpha_L + \alpha_U)}{2}, \alpha_1 = \frac{(\alpha_L + \alpha_A)}{2}, \alpha_2 = \frac{(\alpha_1 + \alpha_A)}{2},$$

$$\alpha_3 = \frac{(\alpha_U + \alpha_A)}{2} \text{and}\, \alpha_4 = \frac{(\alpha_U + \alpha_2)}{2} \quad \text{PRE}(\hat{\bar{Y}}_{JS}) = \frac{\text{MSE(.)}}{V(\hat{\bar{Y}}_{JS})} = 100$$

Table 3: The percent relative efficiencies (PRE's) at different values of α for the population 1.

Proposed	Existing Estimators	α_L	α_1	α_2	α_A	α_3	α_4	α_U
[1]	$\hat{\bar{Y}}_R = \bar{y}\frac{\bar{X}}{\bar{x}}$	100.00	325.84	748.34	1318.10	325.84	167.87	100.00
[8]	$\hat{\bar{Y}}_1 = \bar{y}\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$	100.00	311.74	662.36	1059.66	311.74	165.62	100.00
[7]	$\hat{\bar{Y}}_2 = \bar{y}\left[\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right]$	100.00	326.96	755.81	1343.02	326.96	168.05	100.00
[11]	$\hat{\bar{Y}}_3 = \bar{y}\left[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right]$	100.00	305.73	629.52	973.07	305.73	164.62	100.00
[6]	$\hat{\bar{Y}}_4 = \bar{y}\left[\frac{\bar{X}+\rho}{\bar{x}+\rho}\right]$	100.00	307.94	641.35	1003.56	307.94	164.99	100.00
[10]	$\hat{\bar{Y}}_5 = \bar{y}\left[\frac{C_x\bar{X}+\beta_2}{C_x\bar{x}+\beta_2}\right]$	100.00	327.33	758.29	1351.43	327.33	168.10	100.00
[11]	$\hat{\bar{Y}}_6 = \bar{y}\left[\frac{\beta_1\bar{X}+\beta_2}{\beta_1\bar{x}+\beta_2}\right]$	100.00	326.91	755.43	1341.82	326.91	168.04	100.00
[11]	$\hat{\bar{Y}}_7 = \bar{y}\left[\frac{C_x\bar{X}+\beta_1}{C_x\bar{x}+\beta_1}\right]$	100.00	298.48	592.48	882.12	298.48	163.37	100.00
[2]	$\hat{\bar{Y}}_8 = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\frac{\bar{X}}{\bar{x}}$	100.00	372.36	1166.95	4042.13	372.36	174.42	100.00

[2]	$\hat{\bar{Y}}_9 = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\bar{X} + C_x}{\bar{x} + C_x}\right]$	100.00	368.95	1126.13	3564.91	368.95	173.98	100.00
[2]	$\hat{\bar{Y}}_{10} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\bar{X} + \beta_2}{\bar{x} + \beta_2}\right]$	100.00	372.64	1170.41	4086.87	372.64	174.46	100.00
[2]	$\hat{\bar{Y}}_{11} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{C_x\bar{X} + \beta_2}{C_x\bar{x} + \beta_2}\right]$	100.00	372.74	1171.57	4101.90	372.74	174.47	100.00
[11]	$\hat{\bar{Y}}_{12} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\bar{X} + \beta_1}{\bar{x} + \beta_1}\right]$	100.00	367.55	1110.05	3398.54	367.55	173.80	100.00
[11]	$\hat{\bar{Y}}_{13} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\beta_1\bar{X} + \beta_2}{\beta_1\bar{x} + \beta_2}\right]$	100.00	372.63	1170.23	4084.72	372.63	174.46	100.00
[3]	$\hat{\bar{Y}}_{14} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\bar{X} + \rho}{\bar{x} + \rho}\right]$	100.00	368.06	1115.86	3457.55	368.06	173.87	100.00
[3]	$\hat{\bar{Y}}_{15} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{C_x\bar{X} + \rho}{C_x\bar{x} + \rho}\right]$	100.00	366.59	1099.13	3291.96	366.59	173.67	100.00
[3]	$\hat{\bar{Y}}_{16} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\rho\bar{X} + C_x}{\rho\bar{x} + C_x}\right]$	100.00	368.73	1123.62	3538.08	368.73	173.95	100.00
[3]	$\hat{\bar{Y}}_{17} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\rho\bar{X} + \beta_2}{\rho\bar{x} + \beta_2}\right]$	100.00	372.66	1170.61	4089.69	372.66	174.46	100.00

Table 4: The percent relative efficiencies (PRE's) at different values of α for the population 2.

Proposed	Existing Estimators	α_L	α_1	α_2	α_A	α_3	α_4	α_U
[1]	$\hat{\bar{Y}}_R = \bar{y}\frac{\bar{X}}{\bar{x}}$	100.00	348.02	915.96	2008.61	348.02	171.15	100.00
[8]	$\hat{\bar{Y}}_1 = \bar{y}\left[\frac{\bar{X} + C_x}{\bar{x} + C_x}\right]$	100.00	294.33	572.44	835.63	294.33	162.64	100.00
[7]	$\hat{\bar{Y}}_2 = \bar{y}\left[\frac{\bar{X} + \beta_2}{\bar{x} + \beta_2}\right]$	100.00	270.18	470.25	624.37	270.18	158.09	100.00
[11]	$\hat{\bar{Y}}_3 = \bar{y}\left[\frac{\bar{X} + \beta_1}{\bar{x} + \beta_1}\right]$	100.00	310.36	654.66	1038.75	310.36	165.39	100.00

[6]	$\hat{\bar{Y}}_4 = \bar{y}\left[\frac{\bar{X}+\rho}{\bar{x}+\rho}\right]$	100.00	296.53	582.95	859.78	296.53	163.03	100.00
[10]	$\hat{\bar{Y}}_5 = \bar{y}\left[\frac{C_x\bar{X}+\beta_2}{C_x\bar{x}+\beta_2}\right]$	100.00	265.16	451.66	589.97	265.16	157.07	100.00
[11]	$\hat{\bar{Y}}_6 = \bar{y}\left[\frac{\beta_1\bar{X}+\beta_2}{\beta_1\bar{x}+\beta_2}\right]$	100.00	229.70	339.93	404.66	229.70	149.12	100.00
[11]	$\hat{\bar{Y}}_7 = \bar{y}\left[\frac{C_x\bar{X}+\beta_1}{C_x\bar{x}+\beta_1}\right]$	100.00	308.01	641.72	1004.49	308.01	165.00	100.00
[2]	$\hat{\bar{Y}}_8 = [\bar{y}+b(\bar{X}-\bar{x})]\frac{\bar{X}}{\bar{x}}$	100.00	375.02	1200.30	4504.66	375.02	174.76	100.00
[2]	$\hat{\bar{Y}}_9 = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$	100.00	358.34	1011.79	2580.27	358.34	172.58	100.00
[2]	$\hat{\bar{Y}}_{10} = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right]$	100.00	351.56	947.36	2177.42	351.56	171.65	100.00
[2]	$\hat{\bar{Y}}_{11} = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{C_x\bar{X}+\beta_2}{C_x\bar{x}+\beta_2}\right]$	100.00	350.17	934.88	2108.39	350.17	171.45	100.00
[11]	$\hat{\bar{Y}}_{12} = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right]$	100.00	363.00	1059.82	2942.90	363.00	173.20	100.00
[11]	$\hat{\bar{Y}}_{13} = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{\beta_1\bar{X}+\beta_2}{\beta_1\bar{x}+\beta_2}\right]$	100.00	340.32	852.50	1710.76	340.32	170.05	100.00
[3]	$\hat{\bar{Y}}_{14} = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{\bar{X}+\rho}{\bar{x}+\rho}\right]$	100.00	358.97	1018.10	2624.46	358.97	172.66	100.00
[3]	$\hat{\bar{Y}}_{15} = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{C_x\bar{X}+\rho}{C_x\bar{x}+\rho}\right]$	100.00	358.03	1008.71	2559.02	358.03	172.53	100.00
[3]	$\hat{\bar{Y}}_{16} = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right]$	100.00	356.66	995.33	2469.13	356.66	172.35	100.00
[3]	$\hat{\bar{Y}}_{17} = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{\rho\bar{X}+\beta_2}{\rho\bar{x}+\beta_2}\right]$	100.00	349.19	926.14	2061.60	349.19	171.31	100.00

Table 5: The percent relative efficiencies (PRE's) at different values of α for the population 3.

Proposed	Existing Estimators	α_L	α_1	α_2	α_A	α_3	α_4	α_U
[1]	$\hat{\hat{Y}}_R = \bar{y}\frac{\bar{X}}{\bar{x}}$	100.00	106.89	108.76	109.40	106.89	103.90	100.00
[8]	$\hat{\hat{Y}}_1 = \bar{y}\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$	100.00	106.77	108.61	109.24	106.77	103.84	100.00
[7]	$\hat{\hat{Y}}_2 = \bar{y}\left[\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right]$	100.00	107.22	109.20	109.87	107.22	104.09	100.00
[11]	$\hat{\hat{Y}}_3 = \bar{y}\left[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right]$	100.00	106.40	108.13	108.72	106.40	103.64	100.00
[6]	$\hat{\hat{Y}}_4 = \bar{y}\left[\frac{\bar{X}+\rho}{\bar{x}+\rho}\right]$	100.00	106.34	108.05	108.63	106.34	103.60	100.00
[10]	$\hat{\hat{Y}}_5 = \bar{y}\left[\frac{C_x\bar{X}+\beta_2}{C_x\bar{x}+\beta_2}\right]$	100.00	109.86	112.64	113.59	109.86	105.52	100.00
[11]	$\hat{\hat{Y}}_6 = \bar{y}\left[\frac{\beta_1\bar{X}+\beta_2}{\beta_1\bar{x}+\beta_2}\right]$	100.00	107.48	109.53	110.23	107.48	104.23	100.00
[11]	$\hat{\hat{Y}}_7 = \bar{y}\left[\frac{C_x\bar{X}+\beta_1}{C_x\bar{x}+\beta_1}\right]$	100.00	103.71	104.68	105.00	103.71	102.13	100.00
[2]	$\hat{\hat{Y}}_8 = [\bar{y}+b(\bar{X}-\bar{x})]\frac{\bar{X}}{\bar{x}}$	100.00	176.05	217.38	235.81	176.05	133.65	100.00
[2]	$\hat{\hat{Y}}_9 = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$	100.00	175.81	216.91	235.22	175.81	133.56	100.00
[2]	$\hat{\hat{Y}}_{10} = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right]$	100.00	176.80	218.79	237.59	176.80	133.89	100.00
[2]	$\hat{\hat{Y}}_{11} = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{C_x\bar{X}+\beta_2}{C_x\bar{x}+\beta_2}\right]$	100.00	182.11	229.13	250.69	182.11	135.64	100.00
[11]	$\hat{\hat{Y}}_{12} = [\bar{y}+b(\bar{X}-\bar{x})]\left[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right]$	100.00	174.95	215.28	233.19	174.95	133.27	100.00

[11]	$\hat{\bar{Y}}_{13} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\dfrac{\beta_1\bar{X} + \beta_2}{\beta_2\bar{x} + \beta_2}\right]$	100.00	177.35	219.85	238.92	177.35	134.08	100.00
[3]	$\hat{\bar{Y}}_{14} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\dfrac{\bar{X} + \rho}{\bar{x} + \rho}\right]$	100.00	174.81	215.01	232.85	174.81	133.23	100.00
[3]	$\hat{\bar{Y}}_{15} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\dfrac{C_x\bar{X} + \rho}{C_x\bar{x} + \rho}\right]$	100.00	167.02	200.63	215.05	167.02	130.52	100.00
[3]	$\hat{\bar{Y}}_{16} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\dfrac{\rho\bar{X} + C_x}{\rho\bar{x} + C_x}\right]$	100.00	175.68	216.66	234.91	175.68	133.52	100.00
[3]	$\hat{\bar{Y}}_{17} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\dfrac{\rho\bar{X} + \beta_2}{\rho\bar{x} + \beta_2}\right]$	100.00	177.20	219.56	238.56	177.20	134.03	100.00

Table 6: The percent relative efficiencies (PRE's) at different values of α for the population 4.

Pro-posed	Existing Estimators	α_L	α_1	α_2	α_A	α_3	α_4	α_U
[1]	$\hat{\bar{Y}}_R = \bar{y}\dfrac{\bar{X}}{\bar{x}}$	100.00	136.22	149.79	154.93	136.22	118.36	100.00
[8]	$\hat{\bar{Y}}_1 = \bar{y}\left[\dfrac{\bar{X} + C_x}{\bar{x} + C_x}\right]$	100.00	134.71	147.51	152.33	134.71	117.69	100.00
[7]	$\hat{\bar{Y}}_2 = \bar{y}\left[\dfrac{\bar{X} + \beta_2}{\bar{x} + \beta_2}\right]$	100.00	128.20	137.92	141.49	128.20	114.72	100.00
[11]	$\hat{\bar{Y}}_3 = \bar{y}\left[\dfrac{\bar{X} + \beta_1}{\bar{x} + \beta_1}\right]$	100.00	132.78	144.64	149.08	132.78	116.83	100.00
[6]	$\hat{\bar{Y}}_4 = \bar{y}\left[\dfrac{\bar{X} + \rho}{\bar{x} + \rho}\right]$	100.00	135.22	148.27	153.20	135.22	117.91	100.00
[10]	$\hat{\bar{Y}}_5 = \bar{y}\left[\dfrac{C_x\bar{X} + \beta_2}{C_x\bar{x} + \beta_2}\right]$	100.00	128.50	138.36	141.99	128.50	114.86	100.00

[11]	$\hat{\bar{Y}}_6 = \bar{y}\left[\frac{\beta_2\bar{X}+\beta_2}{\beta_2\bar{x}+\beta_2}\right]$	100.00	132.72	144.54	148.96	132.72	116.80	100.00
[11]	$\hat{\bar{Y}}_7 = \bar{y}\left[\frac{C_x\bar{X}+\beta_2}{C_x\bar{x}+\beta_2}\right]$	100.00	132.92	144.84	149.31	132.92	116.89	100.00
[2]	$\hat{\bar{Y}}_8 = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\frac{\bar{X}}{\bar{x}}$	100.00	225.46	328.49	387.51	225.46	148.06	100.00
[2]	$\hat{\bar{Y}}_9 = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$	100.00	223.93	324.44	381.53	223.93	147.67	100.00
[2]	$\hat{\bar{Y}}_{10} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right]$	100.00	216.95	306.59	355.56	216.95	145.87	100.00
[2]	$\hat{\bar{Y}}_{11} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{C_x\bar{X}+\beta_2}{C_x\bar{x}+\beta_2}\right]$	100.00	217.29	307.44	356.78	217.29	145.96	100.00
[11]	$\hat{\bar{Y}}_{12} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right]$	100.00	221.93	319.25	373.90	221.93	147.17	100.00
[11]	$\hat{\bar{Y}}_{13} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\beta_1\bar{X}+\beta_2}{\beta_2\bar{x}+\beta_2}\right]$	100.00	221.86	319.07	373.63	221.86	147.15	100.00
[3]	$\hat{\bar{Y}}_{14} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}+\rho}{\bar{x}+\rho}\right]$	100.00	224.44	325.80	383.53	224.44	147.80	100.00
[3]	$\hat{\bar{Y}}_{15} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{C_x\bar{X}+\rho}{C_x\bar{x}+\rho}\right]$	100.00	224.48	325.91	383.70	224.48	147.81	100.00
[3]	$\hat{\bar{Y}}_{16} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right]$	100.00	223.24	322.66	378.91	223.24	147.50	100.00
[3]	$\hat{\bar{Y}}_{17} = \left[\bar{y}+b(\bar{X}-\bar{x})\right]\left[\frac{\rho\bar{X}+\beta_2}{\rho\bar{x}+\beta_2}\right]$	100.00	213.36	297.73	342.94	213.36	144.91	100.00

From the PRE's of the proposed estimators given in Table 3 to Table 6, it is clear that the proposed estimators are more efficient than the existing estimators.

5. CONCLUSION

In this paper, a family of unbiased modified linear regression type estimators is proposed together with its variance. It has been shown that the ratio estimator, modified ratio estimators and the linear regression estimator are particular cases of the proposed estimators. The performance of the proposed estimators are assessed theoretically with that of the ratio estimator and the existing modified ratio estimators. Further, it is observed that the proposed estimators perform better than the ratio estimator and the existing modified ratio estimators for certain natural populations. Hence the proposed estimators can be viewed as a generalized class of estimators for estimating population mean and can be recommended for the practical use based on the numerical comparisons.

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