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Common Fixed Point Theorem For Mappings Satisfying (CLRg) Property

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Abstract: The aim of this paper is to establish a common fixed point theorem for two pairs of mappings satisfying (CLRg) property.

Keywords: Common fixed point, complex-valued metric space, (CLRg) property, weakly compatible mappings.

1. INTRODUCTION

Fixed point theory has fascinated hundreds of researchers since 1922 with the celebrated Banach's fixed point theorem. This is a very active field of research at present. In 2011, Azam et al [6] introduced the concept of complex-valued metric space. Recently, Sintunavarat and Kumam [15] introduced the concept of (CLRg) property. Many results are proved on existence of fixed points in complex-valued metric spaces, see [1,3-6,8,9,11,12,14,16,17]. An interesting and detailed discussion on (CLRg) property is given by Babu and Subhashini [7].

In this paper, we use the concept of (CLRg) property and prove a common fixed point theorem for mappings satisfying (CLRg) property in complex-valued metric space.

2. PRELIMINARIES

Let C be the set of complex numbers. Define a partial order $\precsim$ on C as follows:

$$z_1 \precsim z_2 \text{ if } \operatorname{Re}(z_1) \le \operatorname{Re}(z_2)\ ,\operatorname{Im}(z_1) \le \operatorname{Im}(z_2)\,,$$

$$z_1 \precsim z_2 \text{ if } z_1 \ne z_2 \text{ and either } \operatorname{Re}(z_1) < \operatorname{Re}(z_2)\,,\ \operatorname{Im}(z_1) < \operatorname{Im}(z_2)$$

$$\text{or } \operatorname{Re}(z_1) < \operatorname{Re}(z_2),\ \operatorname{Im}(z_1) = \operatorname{Im}(z_2)$$

$$\text{or } \operatorname{Re}(z_1) = \operatorname{Re}(z_2),\ \operatorname{Im}(z_1) < \operatorname{Im}(z_2)$$

Definition 2.1 ([6]). Let X be a nonempty set such that the map $d : X \times X \to \boldsymbol{C}$ satisfies the following conditions:

(c1) $0 \precsim d(x,y)$ for all $x, y \in X$ and $d(x,y) = 0$ iff $x = y$;

(c2) $d(x,y) = d(y,x)$ for all $x, y \in X$;

(c3) $d(x,y) \precsim d(x,z) + d(z,y)$ for all $x, y, z \in X$.

Then d is called a complex-valued metric on X and (X, d) is called complex-valued metric space.

Definition 2.2 ([6]). Let (X,d) be a complex-valued metric space and $x \in X$. Then the sequence $\{x_n\}$ is said to converge to x if for every $0 \prec c \in \boldsymbol{C}$, there is a natural number N such that $d(x_n, x) \prec c$ for all $n \in N$.

We write it as $\lim_{n\to\infty} x_n = x$.

Definition 2.3 ([13]). An element $(x, y) \in X \times X$ is called coupled coincidence point of the mappings $S : X \times X \to X$ and $T : X \to X$ if

$$S(x,y) = T(x),\ S(y,x) = T(y).$$

Definition 2.4 ([10]). An element $x \in X$ is called common fixed point of the mappings $S : X \times X \to X$ and $T : X \to X$ if

$$x = S(x,x) = T(x)$$

Definition 2.5 ([2]). The mappings $S : X \times X \to X$ and $T : X \to X$ are called w-compatible if $TS(x,y) = S(Tx,Ty)$, whenever $S(x,y) = Tx,\ S(y,x) = Ty$.

Definition 2.6 ([10]). The mappings $S : X \times X \to X$ and $T : X \to X$ are called commutative if $TS(x,y) = S(Tx,Ty)$, for all $x, y \in X$.

We note that the maps $S : X \times X \to X$ and $T : X \to X$ are weakly compatible if $S(x,y) = T(x), S(y,x) = T(y)$ implies $TS(x,y) = S(Tx,Ty),\ TS(y,x) = S(Ty,Tx)$ for all $x, y \in X$

Definition 2.7 ([15]). Let (X,d) be a metric space. Two mappings $f : X \to X$ and $g : X \to X$ are said to satisfy (CLRg) property if there exists a sequence $\{x_n\} \subset X$ such that

$$\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n) = g(p) \text{ for some } p \in X.$$

Definition 2.8 ([7]). Let (X,d) be a metric space. Two mappings $f: X\times X \to X$ and $g: X \to X$ are said to satisfy (CLRg) property if there exist sequences $\{x_n\},\{y_n\} \subset X$ such that

$$\lim_{n\to\infty} f(x_n, y_n) = \lim_{n\to\infty} g(x_n) = g(p),$$

$$\lim_{n\to\infty} f(y_n, x_n) = \lim_{n\to\infty} g(y_n) = g(q), \text{ for some } p,q \in X.$$

Definition 2.9 ([14]). The "max" function for the partial order relation "$\precsim$" defined by the

(1) $\max\{z_1, z_2\} = z_2$ if and only if $z_1 \precsim z_2$,

(2) If $z_1 \precsim \max\{z_2, z_3\}$, then $z_1 \precsim z_2$ and $z_1 \precsim z_3$,

(3) $\max\{z_1, z_2\} = z_2$ if the only if $z_1 \precsim z_2$ or $|z_1| \precsim |z_2|$.

Example 2.1. Let $X = [0,\infty)$ be a metric space under usual metric. Define mappings $f: X\times X \to X$ and $g: X \to X$ by

$$f(x,y) = x + y + 2, g(x) = 5 + x, \ \forall x,v \in X.$$

Let $\{x_n\}$ and $\{y_n\}$ be sequences in X where $x_n = 3 + \frac{1}{n}$ and $y_n = 3 - \frac{1}{n}$. Since

$$\lim_{n\to\infty} f(x_n, y_n) = \lim_{n\to\infty} (x_n + y_n + 2) = 8 = g(3),$$

$$\lim_{n\to\infty} g(x_n) = \lim_{n\to\infty} g\left(3 + \frac{1}{n}\right) = 3 + \frac{1}{n} + 5 = 8 = g(3)$$

and

$$\lim_{n\to\infty} f(y_n, x_n) = \lim_{n\to\infty} (y_n + x_n + 2) = 8 = g(3),$$

$$\lim_{n\to\infty} g(y_n) = \lim_{n\to\infty} g\left(3 - \frac{1}{n}\right) = 8 = g(3)$$

So, the maps *f* and *g* satisfy (CLRg) property.

3. MAIN RESULTS

Theorem 3.1. Let (X,d) be a complex valued metric-space and let $f,g: X\times X \to X$ and $\phi,\psi: X \to X$ are mappings such that

(1) $d(f(x,y),g(u,v)) \precsim p\max\{d(\phi x,\psi u),d(f(x,y),\phi x),d(g(u,v),\psi u),$
$d(f(x,y),\psi u),d(g(u,v),\phi x)\}$

for all $x,y,u,v \in X$ and $0<p<1$, (2) the pair (f,ϕ) and (g,ψ) are weakly compatible.
If the pair (f,ϕ) and (g,ψ) satisfy (CLRg) property then f,g,ϕ and ψ have a unique common fixed point, that is, there exists a unique x in X such that

$$f(x,x)=\psi x=g(x,x)=\phi x=x\ .$$

Proof. Let (f,ϕ) and (g,ψ) satisfy (CLRg) property then there exist sequences $\{x_n\},\{y_n\}$, $\{x'_n\}$ and $\{y'_n\}$ in X such that

$$\lim_{n\to\infty} f(x_n,y_n)=\lim_{n\to\infty}\phi(x_n)=\phi\alpha \tag{3.1}$$

$$\lim_{n\to\infty} f(y_n,x_n)=\lim_{n\to\infty}\phi(y_n)=\phi\beta \tag{3.2}$$

$$\lim_{n\to\infty} g(x'_n,y'_n)=\lim_{n\to\infty}\psi(x'_n)=\psi\alpha' \tag{3.3}$$

$$\lim_{n\to\infty} g(y'_n,x'_n)=\lim_{n\to\infty}\psi(y'_n)=\psi\beta' \tag{3.4}$$

for some $\alpha,\beta,\alpha',\beta' \in X$.

Now we will show that (f,ϕ) and (g,ψ) have common coupled coincidence point. For this, we will first show that $\phi\alpha=\psi\alpha'$.
Putting $x=x_n, y=y_n, u=x'_n, v=y'_n$ in condition (1) we get

$$d(f(x_n,y_n),g(x'_n,y'_n)) \precsim p\max\{d(\phi x_n,\psi x'_n),d(f(x_n,y_n),\phi x_n),d(g(x'_n,y'_n),\psi x'_n),$$
$$d(f(x_n,y_n),\psi x'_n),d(g(x'_n,y'_n),\phi x_n)\}.$$

Taking limit as $n\to\infty$ and using (3.1), (3.2), (3.3) and (3.4), we have

$$d(\phi\alpha,\psi\alpha') \precsim p\max\{d(\phi\alpha,\psi\alpha'),d(\phi\alpha,\phi\alpha),d(\psi\alpha',\psi\alpha'),d(\phi\alpha,\psi\alpha'),d(\psi\alpha',\phi\alpha)\}$$

$$\Rightarrow \qquad d(\phi\alpha,\psi\alpha') \precsim p\, d(\phi\alpha,\psi\alpha')$$

$$\Rightarrow \qquad |d(\phi\alpha,\psi\alpha')| \le p\,|d(\phi\alpha,\psi\alpha')|$$

which is possible when $\phi\alpha = \psi\alpha'$.
So $\phi\alpha = \psi\alpha'$.
Similarly we can show that $\phi\beta = \psi\beta'$.
Now we will show that $\phi\beta = \psi\alpha'$.
For this, we put $x = y_n, y = x_n, u = x'_n, v = y'_n$ in condition (1), we get

$$d(f(y_n,x_n),g(x'_n,y'_n)) \precsim p\max\{d(\phi y_n,\psi x'_n),d(f(y_n,x_n),\phi y_n),d(g(x'_n,y'_n),\psi x'_n),$$
$$d(f(y_n,x_n),\psi x'_n),d(g(x'_n,y'_n),\phi y_n)\}$$

Taking limit as $n \to \infty$ and using (3.1), (3.2), (3.3) and (3.4), we have

$$d(\phi\beta,\psi\alpha') \precsim p\max\{d(\phi\beta,\psi\alpha'),d(\phi\beta,\phi\beta),d(\psi\alpha',\psi\alpha'),d(\phi\beta,\psi\alpha'),d(\psi\alpha',\phi\beta)\}$$

$$\Rightarrow \quad d(\phi\beta,\psi\alpha') \precsim p\, d(\psi\alpha',\phi\beta)$$

$$\Rightarrow \quad |d(\phi\beta,\psi\alpha')| \le p|d(\psi\alpha',\phi\beta)|$$

which is possible when $\phi\beta = \psi\alpha'$.
So $\phi\beta = \psi\alpha'$.
Similarly we can show that $\phi\alpha = \psi\beta'$.
Hence

$$\phi\alpha = \phi\beta = \psi\alpha' = \psi\beta' \tag{3.5}$$

Now we will show that $\phi\alpha = g(\alpha',\beta')$ and $\phi\beta = g(\beta',\alpha')$.
For this we put $x = x_n, y = y_n, u = \alpha', v = \beta'$ in condition (1), we get

$$d(f(x_n,y_n),g(\alpha',\beta')) \precsim p\max\{d(\phi x_n,\psi\alpha'),d(f(x_n,y_n),\phi x_n),d(g(\alpha',\beta'),\psi\alpha'),$$
$$d(f(x_n,y_n),\psi\alpha'),d(g(\alpha',\beta'),\phi x_n)\}$$

Taking limit as $n \to \infty$ and using (3.1), (3.2), (3.3), (3.4) and (3.5), we have

$$d(\phi\alpha,g(\alpha',\beta')) \precsim p\max\{d(\phi\alpha,\psi\alpha'),d(\phi\alpha,\phi\alpha),d(g(\alpha',\beta'),\psi\alpha'),$$
$$d(\phi\alpha,\psi\alpha'),d(g(\alpha',\beta'),\phi\alpha)\}$$

$$\Rightarrow \quad d(\phi\alpha, g(\alpha',\beta')) \precsim p\max\{0,0,d(g(\alpha',\beta'),\phi\alpha),0,d(g(\alpha',\beta'),\phi\alpha)\}$$

$$\Rightarrow \quad d(\phi\alpha, g(\alpha',\beta')) \precsim p\, d(\phi\alpha, g(\alpha',\beta'))$$

$$\Rightarrow \quad |d(\phi\alpha, g(\alpha',\beta'))| \le p|d(\phi\alpha, g(\alpha',\beta'))|$$

which is possible when as $\phi\alpha = g(\alpha',\beta')$ as $0 < p < 1$.
So $\phi\alpha = g(\alpha',\beta')$.
Similarly $\phi\beta = g(\beta',\alpha')$.

Now we will show that $\psi\alpha' = f(\alpha,\beta)$ and $\psi\beta' = f(\beta,\alpha)$.

For this we put $x=\alpha, y=\beta, u=x_n'$ and $v=y_n'$ in condition (1), we get

$$d(f(\alpha,\beta), g(x_n',y_n')) \precsim p\max\{d(\phi\alpha,\psi x_n'), d(f(\alpha,\beta),\phi\alpha), d(g(x_n',y_n'),\psi x_n'), d(f(\alpha,\beta),\psi x_n'), d(g(x_n',y_n'),\phi\alpha)\}$$

Taking limit as $n \to \infty$ and using (3.1), (3.2), (3.3), (3.4) and (3.5), we have

$$d(f(\alpha,\beta),\psi\alpha') \precsim p\max\{d(\phi\alpha,\psi\alpha'), d(f(\alpha,\beta),\phi\alpha), d(\psi\alpha',\psi\alpha'), d(f(\alpha,\beta),\psi\alpha'), d(\psi\alpha',\phi\alpha)\}$$

$$\Rightarrow \quad d(f(\alpha,\beta),\psi\alpha') \precsim p\max\{0, d(f(\alpha,\beta),\psi\alpha'),0,d(f(\alpha,\beta),\psi\alpha'),0\}$$

$$\Rightarrow \quad d(f(\alpha,\beta),\psi\alpha') \precsim p\, d(f(\alpha,\beta),\psi\alpha')$$

$$\Rightarrow \quad |d(f(\alpha,\beta),\psi\alpha')| \le p\ |d(f(\alpha,\beta),\psi\alpha')|$$

possible when $f(\alpha,\beta) = \psi\alpha'$ as $0 < p < 1$.
So $f(\alpha,\beta) = \psi\alpha'$.
Similarly $f(\beta,\alpha) = \psi\beta'$.
Thus $\phi\alpha = \phi\beta = \psi\alpha' = \psi\beta' = f(\alpha,\beta) = f(\beta,\alpha) = g(\alpha',\beta') = g(\beta',\alpha')$

$$\Rightarrow \quad g(\alpha',\beta') = \phi\alpha = \psi\alpha' = f(\alpha,\beta)$$

$$\Rightarrow \quad g(\beta',\alpha') = \phi\beta = \psi\beta' = f(\beta,\alpha) .$$

Hence the pairs (f,ϕ) and (g,ψ) have common coupled coincidence point.
Now let $f(\alpha,\beta)=\phi\alpha=g(\alpha',\beta')=\psi\alpha'=x$
and $f(\beta,\alpha)=\phi\beta=g(\beta',\alpha')=\psi\beta'=y$.
Since (f,ϕ) and (g,ψ) are weakly compatible so

$$\phi f(\alpha,\beta)=f(\phi\alpha,\phi\beta)=f(x,y) \text{ and } \phi f(\beta,\alpha)=f(\phi\beta,\phi\alpha)=f(y,x),$$

but

$$f(\alpha,\beta)=x\Rightarrow \phi f(\alpha,\beta)=\phi x$$

$$f(\beta,\alpha)=y\Rightarrow \phi f(\beta,\alpha)=\phi y$$

Therefore $\phi x=f(x,y)$ and $\phi y=f(y,x)$.
Similarly $\psi x=g(x,y)$ and $\psi y=g(y,x)$.
Hence

$$\phi x=f(x,y),\ \phi y=f(y,x) \text{ and } \psi x=g(x,y),\ \psi y=g(y,x).$$

Now we will show that $x=y$.
Using condition (1), we get

$$d(x,y)=d(f(\alpha,\beta),g(\beta',\alpha'))$$

$$\precsim p\max\{d(\phi\alpha,\psi\beta'),d(f(\alpha,\beta),\phi\alpha),d(g(\beta',\alpha'),\psi\beta'),$$

$$d(f(\alpha,\beta),\psi\beta'),d(g(\beta',\alpha'),\phi\alpha)\}$$

$$\Rightarrow \quad d(x,y)\precsim p\max\{0,0,0,0,0\}$$

$$\Rightarrow \quad |d(x,y)|=0$$

$$\Rightarrow \quad x=y.$$

Now, we will prove that $\phi x=\psi x$.
Using condition (1), we have

$$d(\phi x,\psi x)=d(f(x,y),g(x,y))$$

$$\precsim p\max\{d(\phi x,\psi x),d(f(x,y),\phi x),d(g(x,y),\psi x),$$

$$d(f(x,y),\psi x),d(g(x,y),\phi x)\}$$

$\Rightarrow$ $$d(\phi x,\psi x)\precsim p\max\{d(\phi x,\psi x),0,0,d(\phi x,\psi x),d(\psi x,\phi x)\}$$

$\Rightarrow$ $$|d(\phi x,\psi x)|\leq p|d(\phi x,\psi x)|<|d(\phi x,\psi x)|$$

which is possible when $\phi x=\psi x$ as $0<p<1$.
So $\phi x=\psi x$.

$\Rightarrow$ $$f(x,y)=\phi x=\psi x=g(x,y)$$

Similarly $\phi y=\psi y$ and $f(y,x)=g(y,x)$.
Now we will show that $\phi x=x$.
Using condition (1), we get

$$d(x,\phi x)=d(f(\alpha,\beta),g(x,y))$$

$$\precsim p\max\{d(\phi\alpha,\psi x),d(f(\alpha,\beta),\phi\alpha),d(g(x,y),\psi x),$$

$$d(f(\alpha,\beta),\psi x),d(g(x,y),\phi\alpha)\}$$

$\Rightarrow$ $$d(x,\phi x)\precsim p\max\{d(x,\psi x),d(f(\alpha,\beta),\phi\alpha),d(\phi x,\psi x),d(f(\alpha,\beta),\psi x),d(g(x,y),\phi\alpha)\}$$

$\Rightarrow$ $$d(x,\phi x)\precsim p\max\{d(x,\phi x),d(x,x),d(\phi x,\phi x),d(\psi x,x),d(\phi x,x)\}$$

$\Rightarrow$ $$d(x,\phi x)\precsim p\max\{d(x,\phi x),0,0,d(\phi x,x),d(\phi x,x)\}$$

$\Rightarrow$ $$|d(x,\phi x)|\leq p\max|d(x,\phi x)|$$

which is possible when $x=\phi x$ as $0<p<1$.
So $x=\phi x$.
Hence $f(x,x)=\psi x=g(x,x)=\phi x=x$.
Thus f,g,ϕ and ψ have a common fixed point.
Now to prove uniqueness, let y be any other common fixed point of f,g,ϕ and ψ .

$\Rightarrow$ $$f(y,y)=\psi y=g(y,y)=\phi y=y\cdot$$

Then $d(x,y) = d(f(x,x), g(y,y))$

$$\precsim p\max\{d(\phi x,\psi y), d(f(x,x),\phi x), d(g(y,y),\psi y),$$

$$d(f(x,x),\psi y), d(g(y,y),\phi x)\}$$

$$\Rightarrow \quad d(x,y) \precsim p\max\{d(x,y), d(x,x), d(y,y), d(x,y), d(y,x)\}$$

$$\Rightarrow \quad |d(x,y)| \le p\,|d(x,y)|$$

which is possible when $x = y$ as $0 < p < 1$.
So $x = y$.
Hence f, g, ϕ and ψ have unique common fixed point.

Example 3.1. Let $X = R$ be a complex valued metric space equipped with the complex valued metric space $d(x,y) = |x-y|\,i$.
Let $f : X \times X \to X$ and $g : X \times X \to X$ be defined for all $x, y \in X$ as

$$f(x,y) = \begin{cases} \dfrac{x-y}{8} & \text{if } x \ge y \\ 0 & \text{if } x < y \end{cases}, \cdots g(x,y) = \begin{cases} \dfrac{x-y}{10} & \text{if } x \ge y \\ 0 & \text{if } x < y \end{cases}$$

Let $\psi : X \to X$ and $\phi : X \to X$ be defined as

$$\psi(x) = \frac{x}{2}, \cdots \phi(x) = \frac{x}{30}, \cdots, \quad \text{for all } x \in X.$$

It is easy to check that all conditions of Theorem 3.1 are satisfied for all $x, y, u, v \in X$. Thus, we have $x = 0$ is the unique common fixed point of f, g, ϕ and ψ.
If $g = f$ and $\psi = \phi$ in Theorem 3.1 then we have the following corollary:

Corollary 3.1. Let (X,d) be a complex-valued metric-space and let $f : X^2 \to X$ and $\phi : X \to X$ are mappings such that

(1) $$d(f(x,y), f(u,v)) \le p\max\{d(\phi x,\phi u), d(f(x,y),\phi x), d(g(u,v),\phi u),$$

$$d(f(x,y),\phi u), d(f(u,v),\phi x)\}$$

for all $x, y, u, v \in X$ and $0 < p < 1$,

(2) the pair (f,ϕ) is weakly compatible.

If the pair (f,ϕ) satisfy (CLRg) property then there exists a unique x in X such that $f(x,x)=\phi x=x$.

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